Flow Across Cylinder and Spheres

A) Cylinder

1) Circular Cylinder

The most comprehensive relation for the flow across circular cylinder in which applicable over the complete range of available data

$$\mathrm{Nu}_d = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$\text{for } 10^2 < \mathrm{Re}_d < 10^7;\ \mathrm{Pe}_d > 0.2$$

For more accurate value of Nud in the midrange of Reynolds numbers between 20,000 and 400,000, it is recommended to use the following relation

$$\mathrm{Nu}_d = 0.3 + \frac{0.62\,\mathrm{Re}_d^{1/2}\mathrm{Pr}^{1/3}}{\left[1+(0.4/\mathrm{Pr})^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_d}{282{,}000}\right)^{1/2}\right]$$

$$\text{for } 20{,}000 < \mathrm{Re}_d < 400{,}000;\ \mathrm{Pe}_d > 0.2$$

2) Non-Circular Cylinder

For the flow across non-circular cylinder, the following equation may be used

$$\mathrm{Nu}_{df} = \frac{hd}{k_f} = C\left(\frac{u_\infty d}{\nu_f}\right)^n \mathrm{Pr}_f^{1/3}$$

Where the constants C and n are tabulated in Table 6-2 below. Properties for using this equation are evaluated at the film temperature as indicated by the subscript *f*.

Table 6-2 | Constants for use with Equation (6-17), based on References 8 and 9.

Re_{df}	C	n
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.0266	0.805

B) Sphere

For the heat transfer from spheres to a flowing gas across it, the following relation may be used:

$$\frac{hd}{k_f} = 0.37 \left(\frac{u_\infty d}{\nu_f}\right)^{0.6} \qquad \text{for } 17 < \mathrm{Re}_d < 70{,}000$$

A more important relation is a single equation for gases and liquids flowing across spheres:

$$\mathrm{Nu} = 2 + \left(0.4\,\mathrm{Re}_d^{1/2} + 0.06\,\mathrm{Re}_d^{2/3}\right)\mathrm{Pr}^{0.4}\,(\mu_\infty/\mu_w)^{1/4}$$

Which is valid for the range $3.5 < \mathrm{Red} < 8 \times 10^4$ and $0.7 < \mathrm{Pr} < 380$. Properties in this equation are evaluated at the free-stream temperature $T\infty$.

EXAMPLE 6-7 Airflow Across Isothermal Cylinder

Air at 1 atm and 35°C flows across a 5.0-cm-diameter cylinder at a velocity of 50 m/s. The cylinder surface is maintained at a temperature of 150°C. Calculate the heat loss per unit length of the cylinder.

■ Solution

We first determine the Reynolds number and then find the applicable constants from Table 6-2 for use with Equation (6-17). The properties of air are evaluated at the film temperature:

$$T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 35}{2} = 92.5°\mathrm{C} = 365.5\ \mathrm{K}$$

$$\rho_f = \frac{p}{RT} = \frac{1.0132 \times 10^5}{(287)(365.5)} = 0.966\ \mathrm{kg/m^3} \quad [0.0603\ \mathrm{lb}_m/\mathrm{ft}^3]$$

$$\mu_f = 2.14 \times 10^{-5}\ \mathrm{kg/m\cdot s} \quad [0.0486\ \mathrm{lb}_m/\mathrm{h\cdot ft}]$$

$$k_f = 0.0312\ \mathrm{W/m\cdot °C} \quad [0.018\ \mathrm{Btu/h\cdot ft\cdot °F}]$$

$$\mathrm{Pr}_f = 0.695$$

$$\mathrm{Re}_f = \frac{\rho u_\infty d}{\mu} = \frac{(0.966)(50)(0.05)}{2.14 \times 10^{-5}} = 1.129 \times 10^5$$

From Table 6-2

$$C = 0.0266 \qquad n = 0.805$$

so from Equation (6-17)

$$\frac{hd}{k_f} = (0.0266)(1.129 \times 10^5)^{0.805}(0.695)^{1/3} = 275.1$$

$$h = \frac{(275.1)(0.0312)}{0.05} = 171.7\ \mathrm{W/m^2\cdot °C} \qquad [30.2\ \mathrm{Btu/h\cdot ft^2\cdot °F}]$$

The heat transfer per unit length is therefore

$$\begin{aligned}\frac{q}{L} &= h\pi d(T_w - T_\infty)\\ &= (171.7)\pi(0.05)(150 - 35)\\ &= 3100\ \mathrm{W/m} \quad [3226\ \mathrm{Btu/ft}]\end{aligned}$$

Heat Transfer from Electrically Heated Wire — EXAMPLE 6-8

A fine wire having a diameter of 3.94×10^{-5} m is placed in a 1-atm airstream at 25°C having a flow velocity of 50 m/s perpendicular to the wire. An electric current is passed through the wire, raising its surface temperature to 50°C. Calculate the heat loss per unit length.

Solution

We first obtain the properties at the film temperature:

$$T_f = (25 + 50)/2 = 37.5^\circ\text{C} = 310 \text{ K}$$

$$\nu_f = 16.7 \times 10^{-6} \text{ m}^2/\text{s} \qquad k = 0.02704 \text{ W/m} \cdot {}^\circ\text{C}$$

$$\text{Pr}_f = 0.706$$

The Reynolds number is

$$\text{Re}_d = \frac{u_\infty d}{\nu_f} = \frac{(50)(3.94 \times 10^{-5})}{16.7 \times 10^{-6}} = 118$$

The Peclet number is $\text{Pe} = \text{Re Pr} = 83.3$, and we find that Equations (6-17), (6-21), or (6-19) apply. Let us make the calculation with both the simplest expression, (6-17), and the most complex, (6-21), and compare results.

Using Equation (6-17) with $C = 0.683$ and $n = 0.466$, we have

$$\text{Nu}_d = (0.683)(118)^{0.466}(0.705)^{1/3} = 5.615$$

and the value of the heat-transfer coefficient is

$$h = \text{Nu}_d \left(\frac{k}{d}\right) = 5.615 \frac{0.02704}{3.94 \times 10^{-5}} = 3854 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

The heat transfer per unit length is then

$$q/L = \pi\, dh(T_w - T_\infty) = \pi(3.94 \times 10^{-5})(3854)(50 - 25)$$
$$= 11.93 \text{ W/m}$$

Using Equation (6-21), we calculate the Nusselt number as

$$\text{Nu}_d = 0.3 + \frac{(0.62)(118)^{1/2}(0.705)^{1/3}}{\left[1 + (0.4/0.705)^{2/3}\right]^{1/4}}\left[1 + (118/282{,}000)^{5/8}\right]^{4/5}$$
$$= 5.593$$

and

$$h = \frac{(5.593)(0.02704)}{3.94 \times 10^{-5}} = 3838 \text{ W/m}^2 \cdot {}^\circ\text{C}$$

and

$$q/L = (3838)\pi(3.94 \times 10^{-5})(50 - 25) = 11.88 \text{ W/m}$$

EXAMPLE 6-9 Heat Transfer from Sphere

Air at 1 atm and 27°C blows across a 12-mm-diameter sphere at a free-stream velocity of 4 m/s. A small heater inside the sphere maintains the surface temperature at 77°C. Calculate the heat lost by the sphere.

■ Solution

Consulting Equation (6-30) we find that the Reynolds number is evaluated at the free-stream temperature. We therefore need the following properties: at $T_\infty = 27°\text{C} = 300$ K,

$$\nu = 15.69 \times 10^{-6}\ \text{m}^2/\text{s} \qquad k = 0.02624\ \text{W/m}\cdot°\text{C},$$

$$\text{Pr} = 0.708 \qquad \mu_\infty = 1.8462 \times 10^{-5}\ \text{kg/m}\cdot\text{s}$$

At $T_w = 77°\text{C} = 350$ K,

$$\mu_w = 2.075 \times 10^{-5}$$

The Reynolds number is thus

$$\text{Re}_d = \frac{(4)(0.012)}{15.69 \times 10^{-6}} = 3059$$

From Equation (6-30),

$$\overline{\text{Nu}} = 2 + [(0.4)(3059)^{1/2} + (0.06)(3059)^{2/3}](0.708)^{0.4}\left(\frac{1.8462}{2.075}\right)^{1/4}$$
$$= 31.40$$