

Heat Exchanger Effectiveness (NTU method)

If more than one of the inlet and outlet temperature of the heat exchanger is unknown, LMTD may be obtained by trial and errors solution. Another approach introduce the definition of heat exchanger effectiveness (ϵ), which is a dimensionless with ranging between 0 to 1.

$$\epsilon = \frac{q_{act}}{q_{max}}$$

Where, q_{max} is the maximum possible heat transfer for the exchanger. The maximum value could be attained if one of the fluids were to undergo a temperature change equal to the maximum temperature difference present in the exchanger, which is the difference in the entering temperatures for the hot and cold fluids.

Let $C = mC_p$

$$q_{act} = C_h (Th_i - Th_o) = C_c (Tc_o - Tc_i)$$

The maximum possible heat transfer when the fluid of small C undergoes the maximum temperature difference available

$$q_{max} = C_{min} (Th_i - Tc_i)$$

$$q_{act} = \epsilon C_{min} (Th_i - Tc_i)$$

For parallel flow H.E with combining the last three equations, we get two expressions for effectiveness

$$\epsilon = \frac{C_h (Th_i - Th_o)}{C_{min} (Th_i - Tc_i)} = \frac{C_c (Tc_o - Tc_i)}{C_{min} (Th_i - Tc_i)}$$

For $C_h < C_c$:

$$\epsilon_h = \frac{Th_i - Th_o}{Th_i - Tc_i}$$

For $C_h > C_c$:

$$\epsilon_c = \frac{Tc_o - Tc_i}{Th_i - Tc_i}$$

Using the following equation,

$$\ln \frac{Th_o - Tc_o}{Th_i - Tc_i} = -UA \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

We get

$$\frac{Th_o - Tc_o}{Th_i - Tc_i} = \exp\left[-\frac{UA}{C_h}\left(1 + \frac{C_h}{C_c}\right)\right]$$

From energy balance,

$$Tc_o = Tc_i + \frac{C_h}{C_c}(Th_i - Th_o)$$

By using the last two equations, we obtained

$$\epsilon_h = \frac{1 - \exp[(-UA/C_h)(1 + (C_h/C_c))]}{1 + (C_h/C_c)}$$

and

$$\epsilon_c = \frac{1 - \exp[(-UA/C_c)(1 + (C_c/C_h))]}{1 + (C_c/C_h)}$$

The last two equations may be written as

$$\epsilon = \frac{1 - \exp[(-UA/C_{\min})(1 + C_{\min}/C_{\max})]}{1 + C_{\min}/C_{\max}}$$

The terms UA/C_{\min} is called the number of transfer units (NTU) since it is indicative of the size of the heat exchanger, i.e

$$NTU = \frac{UA}{C_{\min}}$$

Figures 10.12 to 10.17 are available to obtain the value ϵ instead of using equations.

Note, in a boiler or condenser, $C_{\min}/C_{\max} \rightarrow 0$ and all the heat-exchanger effectiveness relations approach a single simple equation,

$$\epsilon = 1 - e^{-NTU}$$

Examples 10.10, 10.11, 10.13 and 10.14 are requested.

Example: A double pipe parallel flow H.E. use oil ($c_p = 1.88$ kJ/kg.K) at an initial temperature of 205°C to heat water, flowing at 225kg/hr from 16°C to 44°C . The oil flow rate is 270 kg/hr. a) what is the heat transfer area required for an overall heat transfer coefficient of 340 W/m².K. b) Determine the number of transfer unit (NTU). c) Calculate the effectiveness of the H.E.

Solution:

$$(mcp\Delta T)_{oil} = (mcp\Delta T)_{water}$$

$$cp_{water} = 4.18 \text{ kJ/kg.K}$$

$$\therefore 270 \times 1.88 \times (205 - Th_o) = 225 \times 4.18 \times (44 - 16)$$

$$\Rightarrow Th_o = 153^\circ\text{C}$$

$$\therefore \Delta T_1 = 205 - 16 = 189^\circ\text{C}, \Delta T_2 = 153 - 44 = 109^\circ\text{C},$$

$$\therefore \Delta T_{LM} = \frac{189 - 109}{\ln \frac{189}{109}} = 145.4^\circ\text{C}$$

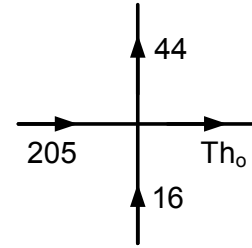
$$a) A = q / U \cdot \Delta T_{LM} = m_w cp_w \Delta T_w = 0.148 \text{ m}^2$$

$$b) (mcp)_{water} = 225 \times 4.18 = 9.405 \times 10^5 \text{ J/hr.K}, (mcp)_{oil} = 270 \times 1.88 = 5.076 \times 10^5 \text{ J/hr.K}$$

$$\therefore C_{min} = 5.076 \times 10^5 \text{ J/hr.K} = 141 \text{ W/K}$$

$$NTU = UA / C_{min} = 340 \times 0.148 / 141 = 0.36$$

$$c) \epsilon = \frac{1 - \exp[(-UA/C_{min})(1 + C_{min}/C_{max})]}{1 + C_{min}/C_{max}} = 28\%$$



Example: water enters a counter flow double pipe H.E at 38°C flowing at the rate of 0.75 kg/s . it is heated by oil ($cp = 1.884 \text{ kJ/kg.K}$) flowing at the rate 1.5 kg/s from an inlet temperature of 116°C . For an area of 13 m^2 and overall heat transfer coefficient of $340 \text{ W/m}^2.\text{K}$, determine the total heat transfer rate?

Solution:

$$mcp_{oil} = 1.5 \times 1.88 = 2.82 \text{ kJ/s.K}$$

$$mcp_{water} = 0.75 \times 4.18 = 3.14 \text{ kJ/s.K}$$

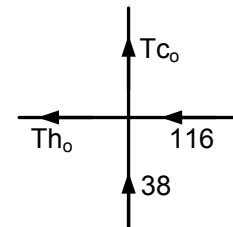
$$\therefore C_{min} = 2.82 \text{ kJ/s.K}$$

$$C = C_{min}/C_{max} = 0.9$$

$$NTU = UA/C_{min} = 1.57$$

From figure 10.13, $\epsilon = 0.62$

$$\therefore q_{act} = \epsilon \times C_{min} \times (Th_i - Tc_i) = 0.62 \times 2820 \times (116 - 38) = 136400 \text{ W}$$



Example: water enters a cross flow H.E (both fluid unmixed) at 16°C and flow at the rate 7.5 kg/s to cool 10 kg/s of air from 120°C. For an overall heat transfer coefficient of 225 W/m².K and an exchanger surface area of 240m². What is the exit air temperature?

Solution:

$$mcp_{\text{air}} = 10 \times 1.014 = 10.14 \text{ kJ/s.K} = 10140 \text{ W/K}$$

$$mcp_{\text{water}} = 7.5 \times 4182 = 31370 \text{ W/K}$$

$$\therefore C_{\text{min}} = 10.14 \text{ kJ/s.K}$$

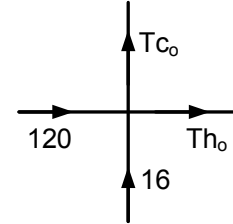
$$C = C_{\text{min}}/C_{\text{max}} = 10.14/31370 = 0.32$$

$$NTU = UA/C_{\text{min}} = 240 \times 225 / 10140 = 5.32$$

From figure 10.15, $\epsilon = 0.94$

$$\therefore q_{\text{act}} = 0.94 \times 10140 \times (120-16) = 9.91 \times 10^5 \text{ W}$$

$$9.91 \times 10^5 = 10140 (120 - T_{\text{ho}}) \Rightarrow T_{\text{ho}} = 32.6 \text{ } ^\circ\text{C}$$



Example: Hot water at 80°C enters the tube of two shell pass, eight tubes pass H.E at the rate 0.375 kg/s heating helium from 20°C. The overall heat transfer coefficient is 155 W/m².K and the exchanger area is 10m². If the water exits at 44°C, determine the exit temperature of the helium and its mass flow rate.

Solution:

Since the flow rate of helium is not given, we first assume that the min fluid is water with $c_p = 4.185 \text{ kJ/kg.K}$.

$$mcp_{\text{water}} = 0.375 \times 4185 = 1570 \text{ W/K}$$

$$NTU = UA/C_{\text{min}} = 155 \times 10 / 1570 = 0.99$$

The effectiveness can be obtained by the equation

$$\epsilon_h = \frac{T_{h_i} - T_{h_o}}{T_{h_i} - T_{c_i}} = \frac{80 - 44}{80 - 20} = 0.6$$

From figure 10.17, $C_{\text{min}}/C_{\text{max}} = 0.25$, which validates the initial assumption of the water as minimum fluid

$$C_{\text{min}}/C_{\text{max}} = 0.25 \Rightarrow C_{\text{max}} = C_{\text{helium}} = 6280 \text{ W/K}$$

$$c_{p_{\text{helium}}} = 5.2 \text{ kJ/kg.K} \Rightarrow m_{\text{helium}} = 1.21 \text{ kg/s}$$

$$(mcp\Delta T)_{\text{water}} = (mcp\Delta T)_{\text{He}} \Rightarrow (T_{\text{helium}})_{\text{out}} = 29^\circ\text{C}$$