

Straight Aluminum Fin

EXAMPLE 2-9

An aluminum fin [$k = 200 \text{ W/m} \cdot ^\circ\text{C}$] 3.0 mm thick and 7.5 cm long protrudes from a wall, as in Figure 2-9. The base is maintained at 300°C , and the ambient temperature is 50°C with $h = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$. Calculate the heat loss from the fin per unit depth of material.

Solution:

Let neglecting the heat lost from the end

$$q = \sqrt{hPkA} \theta_0 \tanh mL$$

$$P = 2(W + t) = 2(1 + 0.003) = 2.006 \text{ m}$$

$$A = W \cdot t = 1 \times 0.003 = 0.003 \text{ m}^2$$

$$\theta_0 = T_0 - T_\infty = 300 - 50 = 250^\circ\text{C}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{10 \times 2.006}{200 \times 0.003}} \cong 5.774$$

\therefore

$$q = \sqrt{10 \times 2.006 \times 200 \times 0.003} \times 250 \times \tanh(5.774 \times 0.075)$$

$$q = 357 \text{ W/m depth}$$

Example: An experimental device that produce excess heat is supplied with pin fins to increase the rate of cooling consider a copper pin fin, 0.25cm in diameter that produces from a wall at 95°C into ambient air at 25°C . $h=10 \text{ W/m}^2 \cdot \text{K}$, $k=396 \text{ W/mK}$. Calculate the heat loss assuming that:

- 1- The fin is infinitely long.
- 2- The fin is 2.5cm long and heat is convected from the end.
- 3- How long would the fin have to be for the infinitely long solution to be corrected within 5%.

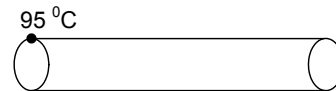
Solution:

1)

$$q = \sqrt{hPkA} \theta_0$$

$$P = \pi D, \quad A = \frac{\pi D^2}{4}$$

$$q = \sqrt{10 \times \pi \times 0.0025 \times 396 \times \pi / 4 \times (0.0025)^2} \times (95 - 25) = 0.865 \text{ W}$$



2)

$$q = \sqrt{hPkA} \theta_0 \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL} = 0.14 \text{ W}$$

3) For only 5% error

$$\frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL} \geq 0.95$$

Which gives $L \geq 28.3 \text{ cm}$

Fin Effectiveness

The effectiveness of the fin in transferring heat is given by the *fin efficiency* (η_f).

$$\eta_f = \frac{\text{actual heat transferred}}{\text{heat that would be transferred if entire fin area were at base temperature}} = \frac{q_{fin}}{q_{\max}}$$

$q_{\max} = h.A_f.\theta_0$, $A_f = P.L$, $A_f = \text{surface area of the fin}$

For case 2 :

$$q_{fin} = \sqrt{hPkA} \theta_0 \tanh mL \Rightarrow$$

$$\eta_f = \frac{\sqrt{hPkA} \theta_0 \tanh mL}{hPL\theta_0} = \frac{\tanh mL}{mL}$$

Fin Performance (€)

$$\varepsilon = \frac{q \text{ with fin}}{q \text{ without fin}} = \frac{\eta_f A_f h \theta_0}{h A_b \theta_0}$$

$$A_f = \text{surface area of the fin} = P.L$$

$$A_b = \text{base area of the fin} = A$$

Corrected Length (Lc)

Lc is used in all equations that apply for the case of the fin with an insulated tip (**case 2**).

$$Lc = L + \frac{t}{2} \quad (\text{For general})$$

$$Am = L.t$$

Example: The outer surface of an oil heater at a uniform temperature of 150°C is to be filled with straight rectangular fins having a uniform thermal conductivity of 25 W/m.K. The ambient air temperature is 20°C and the heat transfer coefficient is 570 W/m².K. Determine the thickness and fin efficiency if the length of each fin is 10mm and each to remove 900W per meter length of primary surface.

Solution.

$$T_0 = 150^\circ\text{C}, \quad k = 25 \text{ W/m.K}, \quad h = 570 \text{ W/m}^2.\text{K}, \quad L = 10 \text{ mm}, \quad T_\infty = 20^\circ\text{C}$$

$$q = \sqrt{hPkA}\theta_0 \tanh mL$$

$$P = 2(w + t) = 2(t + 1), \quad A = wt, \quad w = 1$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{570 \times 2(t+1)}{25 \times t}} = 6.75 \sqrt{\frac{t+1}{t}}$$

$$q = \sqrt{570 \times 2(t+1) \times 25 \times t} (150 - 20) \tanh \left(0.0675 \sqrt{\frac{t+1}{t}} \right)$$

$$900 = 21964.5 \sqrt{t^2 + t} \tanh \left(0.0675 \sqrt{\frac{t+1}{t}} \right)$$

$$\text{by trial and error} \Rightarrow t \cong 2.07 \text{ mm}$$

$$\eta_f = \frac{\tanh mL}{mL} = 60.7\%$$

Example 2: A very long 1cm diameter copper rod ($k = 377 \text{ W/m.K}$) is exposed to an environment at 22°C. The base temperature is maintained at 150°C and the heat transfer coefficient between the rod and surrounding air is 11 W/m².K. Determine the heat transfer rate from the rod to the surrounding air.

Solution.

$$q = \sqrt{hPkA}\theta_0$$

$$q = \sqrt{h(\pi D) \times k \times \pi D^2 / 4} (T_0 - T_\infty)$$

$$q = 12.95 \text{ W}$$

Example 3: Repeat example 2 for finite length of 2, 4, 8,128cm, a surrounding heat loss at the end (**case 3**).

Solution.

For case 3

$$q = \sqrt{hPkA} \theta_0 \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{h(\pi D)}{k(\frac{\pi D^2}{4})}} = \sqrt{\frac{4h}{kD}} = 3.416$$

$$\text{At } L=2\text{cm}, mL=0.06832, \sinh mL=0.06837, \cosh mL=1.00233 \Rightarrow q=0.993W$$