



Tikrit University

Chemical Engineering Department

Mass Transfer

Third Year

By

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Title: Mass Transfer

Course Instructor: Assist. Prof. Dr. Ahmed Daham Wiheeb

Textbook:

1. Coulson J.M. & Richardson J.F., Chemical Engineering, Volume 1, six edition, ELBS, Pergamon Press. 2002.
2. Coulson J.M. & Richardson J.F., Chemical Engineering, Volume 2, fifth edition, ELBS, Pergamon Press. 2002.

References:

1. Treybal R.E., Mass Transfer Operations, McGraw Hill
2. McCabe W.L., Smith J.C. & Harriott P., Unit Operations in Chemical Engineering, McGraw Hill.
3. Seader J.D. & Henley E.J., Separation Process Principles.
4. Rousseau R.W., Handbook of Separation Process Technology, John Wiley
5. Foust A.S. et al, Principles of Unit Operations, John Wiley

Course description:

This course covers the fundamentals of the basic concepts of mass transport and understanding about diffusion theory, gas absorption, liquid-liquid extraction, leaching, distillation, humidification, drying and evaporation.

Objectives:

To give the students a application of diffusion theory to simple mass transfer problems. Analysis of chemical engineering unit operations involving mass transfer. Design principles for mass transfer equipment. Solids/Liquids separation processes. liquids/Liquids separation processes. gases/Liquids separation processes.

Grading:

<u>No.</u>	<u>Assessment</u>	<u>Number</u>	<u>% each</u>	<u>% total</u>	<u>Dates</u>
1	Homework (HW), Quizzes(Q)		10 %	10	
2	Test 1	1	7.5 %	7.5	
3	Test 2	1	7.5 %	7.5	
4	Test 3	1	7.5 %	7.5	
5	Test 4	1	7.5 %	7.5	
6	Final Exam (F)	1	60 %	<u>60</u>	
Overall Total				100	

Syllabus:

<u>No</u>	<u>Topic</u>	<u>Hours</u>
1	Diffusion , flick's law, modes of diffusion	4
2	Multi-components mixture, correction of diffusivity	4
3	Diffusion in varying cross section area	4
4	Diffusivity coefficient in liquid and gas	2
5	Mass transfer theory	2
6	Mass transfer coefficient, wetted wall column	4
7	Absorption, equilibrium of gas and liquid	4
8	Packed tower	6
9	Tray tower	6
10	Calculation of tower diameter, stripping	2
11	Extraction , differential type	4
12	Completely immiscible	4
13	Partly miscible	4
14	Leaching, batch leaching	4
15	Continuous leaching, constant under flow	4
16	Continuous leaching, variable under flow	4
17	Distillation , vapor-liquid equilibrium	4
18	Differential type	2
19	flash distillation	2
20	Continuous distillation (binary system)	6
21	Calculation the number of stages and reflux ration in continuous distillation	4
22	Multi-feeds and side stream , lewis sorial method	6
23	Ponchon-savarit method	4
24	Batch distillation with constant reflux ratio and with constant product composition	6
25	Multi-component distillation	4
26	Humidification , humidity	2
27	Cooling tower calculation	2
28	Height of cooling tower, carey method	4
29	Drying process	6
30	Evaporation	4

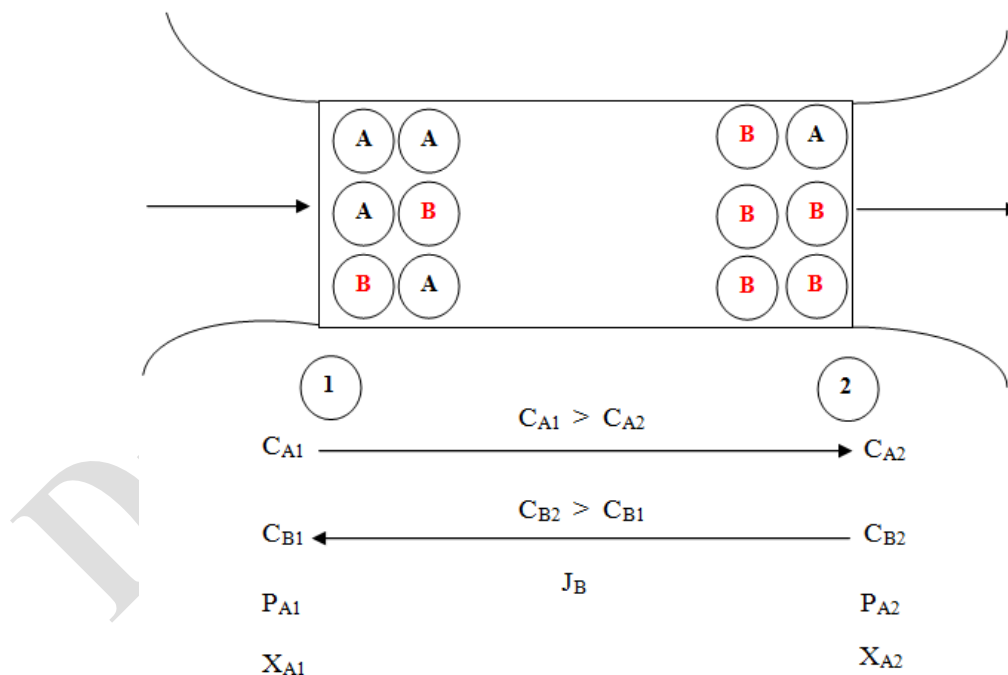
Chapter (10) in Volume (1)

((Diffusion))

The term diffusion (mass transfer) is used to denote the transference of a component in a mixture from a region where its concentration is high to a region where the concentration is lower. Diffusion process can take place in a gas or vapour or in a liquid, and it can result from the random velocities of the molecules (**molecular diffusion**) or from the circulating or eddy currents present in a turbulent fluid (**eddy diffusion**).

Diffusion depends on:

1. Driving force (ΔC), moles per unit volume (kmol/m^3).
2. The distance in the direction of transfer (Δz), meter (m).
3. Diffusivity coefficient, unit area per unit time (m^2/s).



Fick's Law of diffusion:

The rate of diffusion is governed by Fick's Law, first proposed by Fick in 1855 which expresses the mass transfer rate as a linear function of the molar concentration gradient. In a mixture of two gases **A** and **B**, assumed ideal, Fick's Law for steady state diffusion may be written as:

$$J_A \propto \frac{\Delta C_A}{\Delta z}$$

$$J_A = -D_{AB} \frac{dC_A}{dz} \dots \dots \dots \text{Fick's first law of steady state diffusion}$$

Where:

J_A : is the molecular diffusion flux of **A** , (moles per unit area per unit time) $\left[\frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}\right]$.

C_A : is the concentration of **A** (moles of A per unit volume) $\left[\frac{\text{kmol}}{\text{m}^3}\right]$.

D_{AB} : is known as the diffusivity or diffusion coefficient for **A** in **B** (unit area per unit time) $\left[\frac{\text{m}^2}{\text{s}}\right]$

z : is distance in the direction of transfer (m).

Diffusion with bulk of mass in motion:

The Fick's first law of diffusion describes the mass transfer from the random movement of molecules of a stationary medium or a fluid in streamline flow. If circulating currents or eddies are present, then the molecular mechanism will be reinforced and the total mass transfer rate may be written as:

$$\text{Total diffusion} = \text{Molecular diffusion} + \text{Convection term}$$

Convection term = Eddy diffusion = Molar flux due to convection

$$\text{Convection term} = \text{Concentration} * \text{mass transfer velocity} = C_A \cdot V$$

Where:

$$\text{mass transfer velocity (V)} = \frac{\text{mass flux}}{\text{concentration}} = \frac{N_A + N_B}{C_T} = \frac{\frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}}{\frac{\text{kmol}}{\text{m}^3}} = \frac{\text{m}}{\text{s}}$$

$$\text{Total diffusion} = N_A = J_A + C_A \cdot V$$

$$N_A = -D_{AB} \frac{dC_A}{dz} + \frac{C_A}{C_T} (N_A + N_B) \dots \dots \dots (1)$$



Total diffusion equation in the form of concentration (normally used for liquids)

The total diffusion equation can be write in another forms:

- Partial pressure for gases.
- Mole fraction for gases and liquids.

a. Total diffusion equation in the partial pressure form:

If **A** and **B** are ideal gases in a mixture, the ideal gas law may be applied to each gas separately and to the mixture:

$$P V = n R T \quad \Rightarrow \quad P = \frac{n}{V} R T$$

$$P = C R T$$

$$P_A = C_A R T \quad \text{and} \quad P_T = C_T R T$$

$$C_A = \frac{P_A}{R T}$$

$$dC_A = \frac{1}{R T} dP_A$$

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} (N_A + N_B) \quad \dots \dots \dots (2)$$

Total diffusion equation in the form of partial pressure (normally used for gases)

b. Total diffusion equation in the mole fraction form:

$$X_A = \frac{P_A}{P_T} \quad \text{or} \quad X_A = \frac{C_A}{C_T}$$

$$P_T X_A = P_A \quad \text{and} \quad C_T X_A = C_A$$

$$P_T dX_A = dP_A \quad \text{and} \quad C_T dX_A = dC_A$$

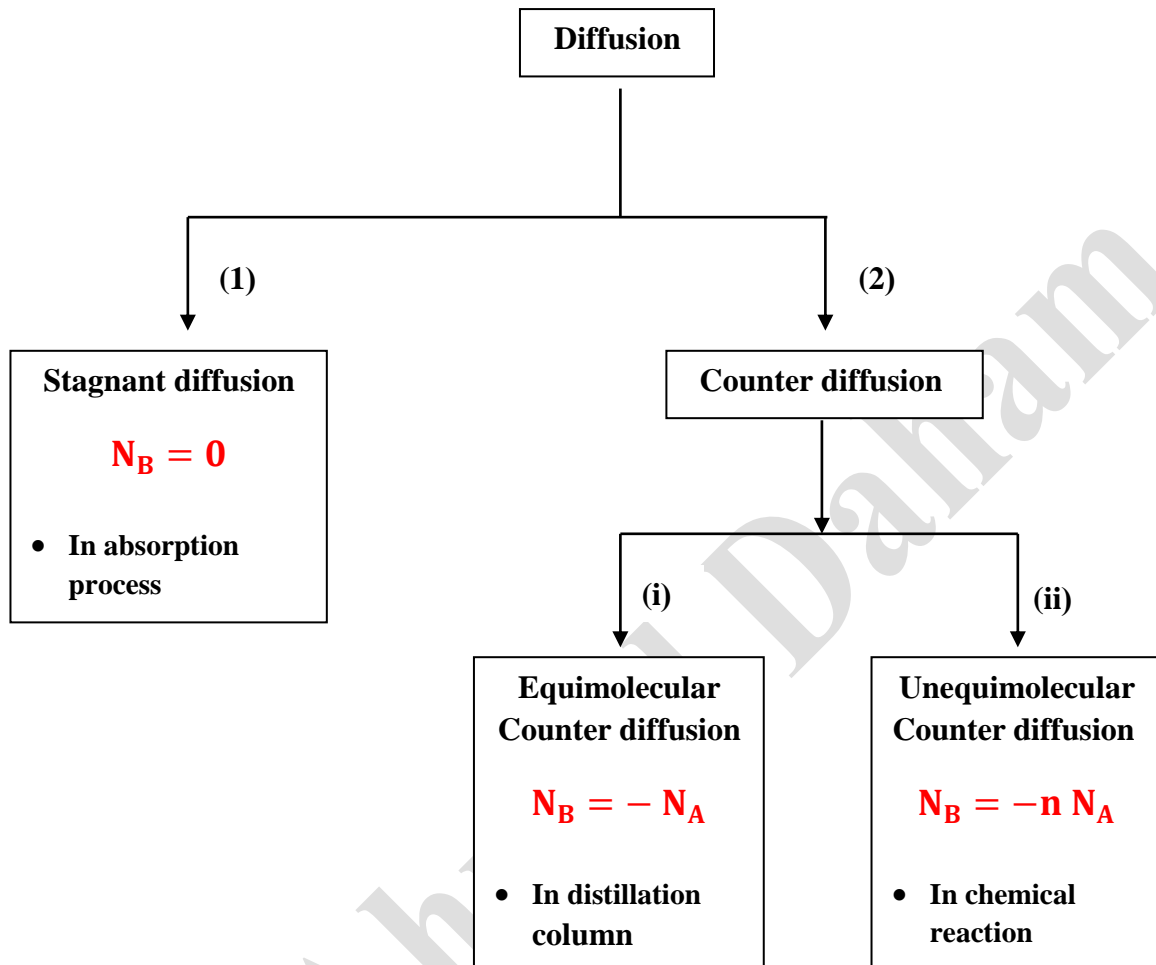
Then:

$$N_A = \frac{-D_{AB}}{RT} \frac{dX_A}{dz} + X_A (N_A + N_B) \quad \dots \dots \dots (3)$$

Total diffusion equation in the form of mole fraction (used for gases and liquids)

Modes of diffusion

There are two modes of diffusion:



1. Stagnant diffusion (Mass transfer through a stationary second component):

In several important processes, one component in a gaseous mixture will be transported relative to a fixed plane, such as a liquid interface, for example, and the other will undergo no net movement. In gas absorption a soluble gas **A** is transferred to the liquid surface where it dissolves, whereas the insoluble gas **B** undergoes no net movement with respect to the interface. Similarly, in evaporation from a free surface, the vapour moves away from the surface but the air has no net movement. The mass transfer process therefore:

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} (N_A + N_B) \quad \dots \dots \dots (1)$$

Since stagnant diffusion layer: $N_B = 0$

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} N_A \quad \dots \dots \dots (2)$$

$$N_A \left(1 - \frac{P_A}{P_T}\right) = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} \quad \dots \dots \dots (3)$$

$$N_A = \frac{-D_{AB}}{RT} \frac{1}{\left(1 - \frac{P_A}{P_T}\right)} \frac{dP_A}{dz} \quad \dots \dots \dots (4)$$

$$N_A = \frac{-D_{AB}}{RT} \frac{P_T}{dz} \frac{dP_A}{(P_T - P_A)} \quad \dots \dots \dots (5)$$

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{(z_2 - z_1)} \ln \left[\frac{P_T - P_{A2}}{P_T - P_{A1}} \right]$$

Example 10.1: Ammonia gas is diffusing at a constant rate through a layer of stagnant air 1 mm thick. Conditions are such that the gas contains 50 percent by volume ammonia at one boundary of the stagnant layer. The ammonia diffusing to the other boundary is quickly absorbed and the concentration is negligible at that plane. The temperature is 295 K and the pressure atmospheric, and under these conditions the diffusivity of ammonia in air is 0.18 cm²/s. Estimate the rate of diffusion of ammonia through the layer.

Solution:

If the subscripts 1 and 2 refer to the two sides of the stagnant layer and the subscripts **A** and **B** refer to ammonia and air respectively, then the rate of diffusion through a stagnant layer is given by:

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{(z_2 - z_1)} \ln \left[\frac{P_T - P_{A2}}{P_T - P_{A1}} \right]$$

Where:

$$P_T = 101.3 \text{ kPa} \quad , \quad P_{A2} = 0 \quad , \quad P_{A1} = y_A P_T = 0.5 * 101.3 = 50.65 \text{ kPa}$$

$$\Delta z = z_2 - z_1 = 1 \text{ mm} = 1 * 10^{-3} \text{ m}$$

$$R = 8.314 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}} \quad , \quad T = 298 \text{ K} \quad \text{and} \quad D_{AB} = 0.18 \frac{\text{cm}^2}{\text{s}} = 1.8 * 10^{-5} \frac{\text{m}^2}{\text{s}}$$

$$N_A = \frac{1.8 * 10^{-5}}{8.314 * 295} \frac{101.3}{1 * 10^{-3}} \ln \left[\frac{101.3 - 0}{101.3 - 50.65} \right] = 5.153 * 10^{-4} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

2. Counter diffusion:

i. Equimolecular counter diffusion:

When the mass transfer rates of the two components are equal and opposite the process is said to be one of equimolecular counter diffusion. Such a process occurs in the case of the box with a movable partition. It occurs also in a distillation column when the molar latent heats of the two components are the same ($\lambda_A = \lambda_B$). At any point in the column a falling stream of liquid is brought into contact with a rising stream of vapour with which it is not in equilibrium. The less volatile component is transferred from the vapour to the liquid and the more volatile component is transferred in the opposite direction. If the molar latent heats of the components are equal, the condensation of a given amount of less volatile component releases exactly the amount of latent heat required to volatilize the same molar quantity of the more volatile component. Thus at the interface, and consequently throughout the liquid and vapour phases, equimolecular counter diffusion is taking place ($N_B = -N_A$).

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} (N_A + N_B) \dots \dots \dots (1)$$

Since equimolecular counter diffusion:

$$N_B = -N_A$$

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} (N_A - N_A) \dots \dots \dots (2)$$

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} \dots \dots \dots (3)$$

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} \dots \dots \dots (4)$$

$$N_A = \frac{-D_{AB}}{RT} \left(\frac{P_{A2} - P_{A1}}{z_2 - z_1} \right) \dots \dots \dots (5)$$

$$N_A = \frac{D_{AB}}{RT} \left(\frac{P_{A1} - P_{A2}}{z_2 - z_1} \right)$$

Drift Factor:

For stagnant diffusion:

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right]$$

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \left[\frac{(P_T - P_{A_2}) - (P_T - P_{A_1})}{(P_T - P_{A_2}) - (P_T - P_{A_1})} \right] \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right]$$

From Dalton's Law of partial pressures: $P_T = P_A + P_B$

By definition, P_{Bm} , the logarithmic mean of P_{B_1} and P_{B_2} , is given by:

$$\frac{(P_T - P_{A_2}) - (P_T - P_{A_1})}{\ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right]} = \frac{P_{B_2} - P_{B_1}}{\ln \left[\frac{P_{B_2}}{P_{B_1}} \right]} = P_{Bm}$$

$$N_A = \frac{D_{AB}}{RT} \frac{1}{\Delta z} \left[\frac{P_T}{P_{Bm}} \right] (P_{A_1} - P_{A_2})$$

Where: $\left[\frac{P_T}{P_{Bm}} \right]$ is known as the **drift factor**.

If the **drift factor** = $\left[\frac{P_T}{P_{Bm}} \right] = 1$ (this happens when the concentration of component A being transferred is low)

Then,

$$N_A = \frac{D_{AB}}{RT} \left(\frac{P_{A_1} - P_{A_2}}{z_2 - z_1} \right)$$

* Thus the bulk flow enhances the mass transfer rate by a factor $\frac{P_T}{P_{Bm}}$, known as the **drift factor**.

Example: In an air-carbon dioxide mixture at 298 K and 202.6 kPa, the concentration of CO₂ at two planes (3 mm) apart are 15 vol.% and 25 vol.%. The diffusivity of CO₂ in air at 298 K and 202.6 kPa is $8.2 \times 10^{-6} \text{ m}^2/\text{s}$. Calculate the rate of transfer of CO₂ across the two planes, assuming:

- Equimolecular counter diffusion.
- Diffusion of CO₂ through a stagnant air layer.

Solution:

$$P_{A1} = y_{A1} \cdot P_T = (0.15)(202.6) = 30.39 \text{ kPa}$$

$$P_{A2} = y_{A2} \cdot P_T = (0.25)(202.6) = 50.65 \text{ kPa}$$

- Equimolecular counter diffusion.

$$N_A = \frac{D_{AB}}{RT} \left(\frac{P_{A1} - P_{A2}}{z_2 - z_1} \right)$$

$$N_A = \frac{8.2 \times 10^{-6}}{(8.314)(298)(3 \times 10^{-6})} (50.65 - 30.39) = 2.23 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

- Stagnant diffusion.

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \ln \left[\frac{P_T - P_{A2}}{P_T - P_{A1}} \right]$$

$$N_A = \frac{8.2 \times 10^{-6}}{(8.314)(298)} \frac{202.6}{3 \times 10^{-6}} \ln \left[\frac{202.6 - 30.39}{202.6 - 50.65} \right] = 2.79 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

ii. Unequimolecular counter diffusion:

When the mass transfer rates of the two components are unequal and opposite, the process is said to be the unequimolecular diffusion, such a process occurs in a chemical reaction.

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} (N_A + N_B) \quad \dots \dots \dots (1)$$

Since unequimolecular counter diffusion:

$$N_B = -n N_A$$

$$N_A = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} + \frac{P_A}{P_T} (N_A - n N_A) \quad \dots \dots \dots (2)$$

$$N_A \left(1 - \left[\frac{P_A}{P_T} \right] (1 - n) \right) = \frac{-D_{AB}}{RT} \frac{dP_A}{dz} \quad \dots \dots \dots (3)$$

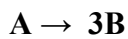
$$N_A = \frac{-D_{AB}}{RT} \frac{P_T}{dz} \frac{dP_A}{\left(1 - \left[\frac{P_A}{P_T} \right] (1 - n) \right)} \quad \dots \dots \dots (4)$$

$$N_A = \frac{-D_{AB}}{RT} \frac{1}{\Delta z} \frac{dP_A}{(P_T - P_A (1 - n))} \quad \dots \dots \dots (5)$$

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \frac{1}{(1 - n)} \ln \left[\frac{P_T - (1 - n) P_{A2}}{P_T - (1 - n) P_{A1}} \right]$$

Example: Species **A** in a gaseous mixture diffuses through a (3 mm) thick film and reaches a catalyst surface where the reaction $A \rightarrow 3B$ takes place. If the partial pressure of **A** in the bulk of the gas is 8.5 kN/m² and the diffusivity of **A** is 2×10^{-5} m²/s. Find the mole flux of **A**, given the pressure and temperature of the system are 101.3 kPa and 297 K, respectively.

Solution:



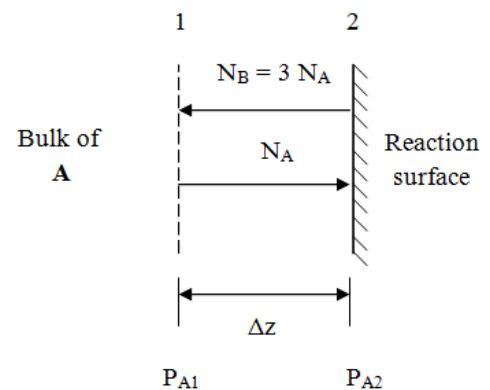
$$n = \frac{N_B}{N_A} = \frac{3}{1} = 3$$

Given:

$$D_{AB} = 2 \times 10^{-5} \frac{m^2}{s}, \quad P_T = 101.3 \text{ kPa}$$

$$T = 297 \text{ K}, \quad P_{A1} = 8.5 \text{ kPa}$$

$$P_{A2} = 0$$



$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \frac{1}{(1 - n)} \ln \left[\frac{P_T - (1 - n) P_{A2}}{P_T - (1 - n) P_{A1}} \right]$$

$$N_A = \frac{2 \times 10^{-5}}{8.314 \times 297} \frac{101.3}{3 \times 10^{-3}} \frac{1}{(1 - 3)} \ln \left[\frac{101.3 + 2(0)}{101.3 + 2(8.5)} \right] = 2.12 \times 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

Maxwell's Law for multicomponent mass transfer

This argument can be applied to the diffusion of a constituent of a multicomponent gas. Considering the transfer of component **A** through a *stationary gas* consisting of components B, C, D, ... etc, if the total partial pressure gradient can be regarded as being made up of a series of terms each representing the contribution of the individual component gases. The mass transfer rate can be calculated from the previous equations using the effective diffusivity of A in the mixture (D_{Am}).

Calculation of the effective diffusivity of (A) in the mixture (D_{Am}):

Let **A** be the diffusing species through *stagnant* mixture of **B, C, D** etc.

$$N_A = -D_{Am} C_T \frac{dX_A}{dz} + X_A (N_A + N_B + N_C + N_D) \quad \dots \dots \dots (1)$$

Where: D_{Am} is the effective diffusivity of **A** in the mixture.

Since stagnant diffusion layer of the mixture: $N_B = N_C = N_D = 0$

$$N_A = -D_{Am} C_T \frac{dX_A}{dz} + X_A (N_A) \quad \dots \dots \dots (2)$$

$$\frac{N_A}{C_T} \left(\frac{1 - X_A}{D_{Am}} \right) = - \frac{dX_A}{dz} \quad \dots \dots \dots (3)$$

Now consider binary system for diffusion of **A** in **B**.

$$N_A = -D_{AB} C_T \frac{dX_A}{dz} + X_A (N_A + N_B)$$

Since stagnant diffusion layer: $N_B = 0$

$$N_A = -D_{AB} C_T \frac{dX_A}{dz} + X_A (N_A)$$

$$N_A(1 - X_A) = -D_{AB} C_T \frac{dX_A}{dz}$$

$$\frac{N_A}{C_T} \left(\frac{1 - X_A}{D_{AB}} \right) = - \frac{dX_A}{dz}$$

$$\text{But: } (1 - X_A) = X_B \quad \Rightarrow \quad -dX_A = dX_B$$

$$\frac{N_A}{C_T} \left(\frac{X_B}{D_{AB}} \right) = \frac{dX_B}{dz} \dots \dots \dots (4)$$

Similarly for diffusion of A in C.

$$N_A = -D_{AC} C_T \frac{dX_A}{dz} + X_A (N_A + N_C)$$

Since stagnant diffusion layer: $N_C = 0$

$$N_A = -D_{AC} C_T \frac{dX_A}{dz} + X_A (N_A)$$

$$N_A (1 - X_A) = -D_{AC} C_T \frac{dX_A}{dz}$$

$$\frac{N_A}{C_T} \left(\frac{1 - X_A}{D_{AC}} \right) = - \frac{dX_A}{dz}$$

$$\text{But: } (1 - X_A) = X_C \quad \Rightarrow \quad -dX_A = dX_C$$

$$\frac{N_A}{C_T} \left(\frac{X_C}{D_{AC}} \right) = \frac{dX_C}{dz} \dots \dots \dots (5)$$

Similarly for diffusion of A in D.

$$N_A = -D_{AD} C_T \frac{dX_A}{dz} + X_A (N_A + N_D)$$

Since stagnant diffusion layer: $N_D = 0$

$$N_A = -D_{AD} C_T \frac{dX_A}{dz} + X_A (N_A)$$

$$N_A (1 - X_A) = -D_{AD} C_T \frac{dX_A}{dz}$$

$$\frac{N_A}{C_T} \left(\frac{1 - X_A}{D_{AD}} \right) = - \frac{dX_A}{dz}$$

$$\text{But: } (1 - X_A) = X_D \quad \Rightarrow \quad -dX_A = dX_D$$

$$\frac{N_A}{C_T} \left(\frac{X_D}{D_{AD}} \right) = \frac{dX_D}{dz} \dots \dots \dots (6)$$

Now adding Equations (4), (5) and (6):

$$\frac{N_A}{C_T} \left(\frac{X_B}{D_{AB}} + \frac{X_C}{D_{AC}} + \frac{X_D}{D_{AD}} \right) = \frac{d(X_B + X_C + X_D)}{dz}$$

But: $(X_B + X_C + X_D) = 1 - X_A$

$$\frac{d(X_B + X_C + X_D)}{dz} = - \frac{dX_A}{dz}$$

$$\frac{N_A}{C_T} \left(\frac{X_B}{D_{AB}} + \frac{X_C}{D_{AC}} + \frac{X_D}{D_{AD}} \right) = \frac{N_A}{C_T} \frac{(1 - X_A)}{D_{Am}}$$

$$\frac{1 - X_A}{D_{Am}} = \frac{X_B}{D_{AB}} + \frac{X_C}{D_{AC}} + \frac{X_D}{D_{AD}}$$

For dilute mixture (low concentration of A), $X_A \rightarrow 0$

$$\frac{1}{D_{Am}} = \frac{X_B}{D_{AB}} + \frac{X_C}{D_{AC}} + \frac{X_D}{D_{AD}}$$

Example: Nitrogen is diffusing under steady condition through a mixture of 2% N₂, 20% C₂H₆, 30% C₂H₄ and 48% C₄H₁₀ at 298 K and 100 kPa. The partial pressure of nitrogen at two planes (1 mm) apart are 13.3 & 6.67 kPa, respectively. Calculate the rate of N₂ across the two planes. The diffusivity of N₂ through C₄H₁₀, C₂H₆ and C₂H₄ may be taken as 9.6*10⁻⁶ m²/s, 14.8*10⁻⁶ m²/s and 16.3*10⁻⁶ m²/s, respectively.

Solution:

Since stagnant diffusion:

$$N_A = \frac{D_{Am}}{RT} \frac{P_T}{\Delta z} \ln \left[\frac{P_T - P_{A2}}{P_T - P_{A1}} \right]$$

$$\frac{1 - y_A}{D_{Am}} = \frac{y_B}{D_{AB}} + \frac{y_C}{D_{AC}} + \frac{y_D}{D_{AD}}$$

$$\frac{1 - 0.02}{D_{Am}} = \frac{0.48}{9.6 * 10^{-6}} + \frac{0.2}{14.8 * 10^{-6}} + \frac{0.3}{16.3 * 10^{-6}}$$

$$D_{Am} = 1.22 * 10^{-5} \frac{m^2}{s}$$

$$N_A = \frac{1.22 * 10^{-5}}{8.314 * 298} \frac{100}{0.001} \ln \left[\frac{100 - 6.67}{100 - 13.3} \right] = 0.0492 \frac{kmol}{m^2 \cdot s}$$

Diffusivities of gases and vapours

Experimental values of diffusivities are given in Table 10.2 for a number of gases and vapours in air at 298K and atmospheric pressure. The table also includes values of the Schmidt number **Sc**, the ratio of the kinematic viscosity (μ/A) to the diffusivity (**D**) for very low concentrations of the diffusing gas or vapour. The importance of the Schmidt number in problems involving mass transfer is discussed later.

Table 10.2. Diffusivities (diffusion coefficients) of gases and vapours in air at 298 K and atmospheric pressure⁽⁴⁾

Substance	D ($\text{m}^2/\text{s} \times 10^6$)	$\mu/\rho D$	Substance	D ($\text{m}^2/\text{s} \times 10^6$)	$\mu/\rho D$
Ammonia	28.0	0.55	Valeric acid	6.7	2.31
Carbon dioxide	16.4	0.94	i-Caproic acid	6.0	2.58
Hydrogen	71.0	0.22	Diethyl amine	10.5	1.47
Oxygen	20.6	0.75	Butyl amine	10.1	1.53
Water	25.6	0.60	Aniline	7.2	2.14
Carbon disulphide	10.7	1.45	Chlorobenzene	7.3	2.12
Ethyl ether	9.3	1.66	Chlorotoluene	6.5	2.38
Methanol	15.9	0.97	Propyl bromide	10.5	1.47
Ethanol	11.9	1.30	Propyl iodide	9.6	1.61
Propanol	10.0	1.55	Benzene	8.8	1.76
Butanol	9.0	1.72	Toluene	8.4	1.84
Pentanol	7.0	2.21	Xylene	7.1	2.18
Hexanol	5.9	2.60	Ethyl benzene	7.7	2.01
Formic acid	15.9	0.97	Propyl benzene	5.9	2.62
Acetic acid	13.3	1.16	Diphenyl	6.8	2.28
Propionic acid	9.9	1.56	n-Octane	6.0	2.58
i-Butyric acid	8.1	1.91	Mesitylene	6.7	2.31

Experimental determination of diffusivities

Diffusivities of **vapours** are most conveniently determined by the method developed by **Winkelmann** in which liquid is allowed to evaporate in a vertical glass tube over the top of which a stream of vapour-free gas is passed, at a rate such that the vapour pressure is maintained almost at zero (Figure 10.2). If the apparatus is maintained at a steady temperature, there will be no eddy currents in the vertical tube and mass transfer will take place from the surface by molecular diffusion alone. The rate of evaporation can be followed by the rate of fall of the liquid surface, and since the concentration gradient is known, the diffusivity can then be calculated.

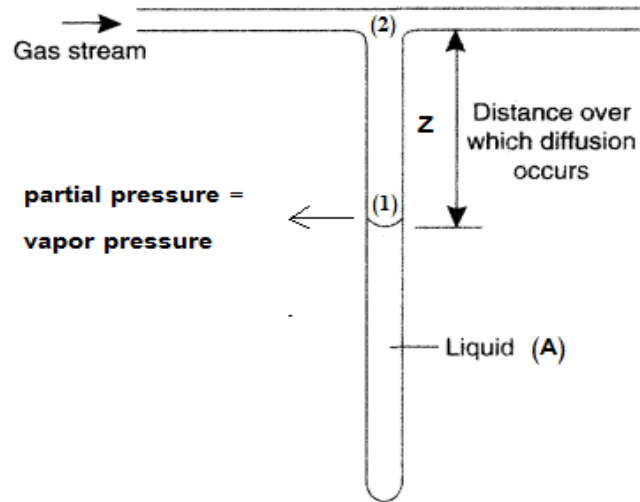


Figure 10.2. Determination of diffusivities of vapours

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right] \quad \dots \dots \dots (1)$$

$$N_A = \frac{D_{AB}}{RT} \frac{1}{z} \left[\frac{P_T}{P_{Bm}} \right] (P_{A_1} - P_{A_2})$$

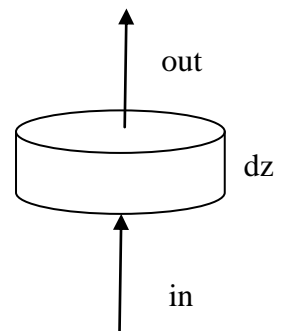
$$N_A = \frac{D_{AB}}{z} \left[\frac{C_T}{C_{Bm}} \right] (C_{A_1} - C_{A_2})$$

Material balance on component A over dz:

In = out (in kg)

$$(A * dz) \rho_L = N_A * A * M_{wt} * dt$$

$$N_A = \frac{\rho_L}{M_{wt}} \left(\frac{dz}{dt} \right) \quad \dots \dots \dots (2)$$



Substitute Eq.(1) in to Eq.(2). To get:

$$\frac{\rho_L}{M_{wt}} \left(\frac{dz}{dt} \right) = \frac{D_{AB}}{RT} \frac{P_T}{z} \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right]$$

$$\frac{D_{AB}}{RT} \frac{P_T}{\Delta z} \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right] \int_0^t dt = \frac{\rho_L}{M_{wt}} \int_{z_1}^{z_2} z dz$$

$$D_{AB} = \frac{\rho_L \cdot (z_2^2 - z_1^2) \cdot RT}{M_{wt} \cdot (2t) \cdot P_T \cdot \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right]}$$

Where:

ρ_L is the liquid density (kg/m³).

M_{wt} is the molecular weight of liquid.

$P_{A_2} = 0$ (always).

$$N_A = \frac{D_{AB}}{z} \left[\frac{C_T}{C_{Bm}} \right] (C_{A_1} - C_{A_2}) \dots \dots \dots (3)$$

Since $P_{A_2} = C_{A_2} = 0$

Let $C_{A_1} = C_A$

$$N_A = D_{AB} \frac{C_A}{z} \left[\frac{C_T}{C_{Bm}} \right] \dots \dots \dots (4)$$

Substitute Eq.(4) in to Eq.(2). To get:

$$\frac{\rho_L}{M_{wt}} \left(\frac{dz}{dt} \right) = D_{AB} \frac{C_A}{z} \left[\frac{C_T}{C_{Bm}} \right]$$

$$z_2^2 - z_1^2 = \frac{2 M_{wt} \cdot D_{AB} \cdot C_A}{\rho_L} \left[\frac{C_T}{C_{Bm}} \right] t$$

$$(z_2 - z_1)(z_2 + z_1) = \frac{2 M_{wt} \cdot D_{AB} \cdot C_A}{\rho_L} \left[\frac{C_T}{C_{Bm}} \right] t$$

$$\begin{aligned} \frac{t}{(z_2 - z_1)} &= \frac{\rho_L}{2 M_{wt} \cdot D_{AB} \cdot C_A} \left[\frac{C_{Bm}}{C_T} \right] (z_2 + z_1) \\ &+ \frac{\rho_L}{M_{wt} \cdot D_{AB} \cdot C_A} \left[\frac{C_{Bm}}{C_T} \right] z_1 \end{aligned}$$

If we draw $\frac{t}{(z_2 - z_1)}$ against $(z_2 - z_1)$ then:

$$\text{The slope (s)} = \frac{\rho_L}{2 M_{wt} \cdot D_{AB} \cdot C_A} \left[\frac{C_{Bm}}{C_T} \right]$$

$$D_{AB} = \frac{\rho_L}{2 M_{wt} \cdot \text{slope} \cdot C_A} \left[\frac{C_{Bm}}{C_T} \right]$$

Example 10.2: The diffusivity of the vapour of a volatile liquid in air can be conveniently determined by Winkelmann's method in which liquid is contained in a narrow diameter vertical tube, maintained at a constant temperature, and an air stream is passed over the top of the tube sufficiently rapidly to ensure that the partial pressure of the vapour there remains approximately zero. On the assumption that the vapour is transferred from the surface of the liquid to the air stream by molecular diffusion alone, calculate the diffusivity of carbon tetrachloride vapour in air at 321 K and atmospheric pressure from the experimental data given in Table 10.3.

Table 10.3. Experimental data for diffusivity calculation

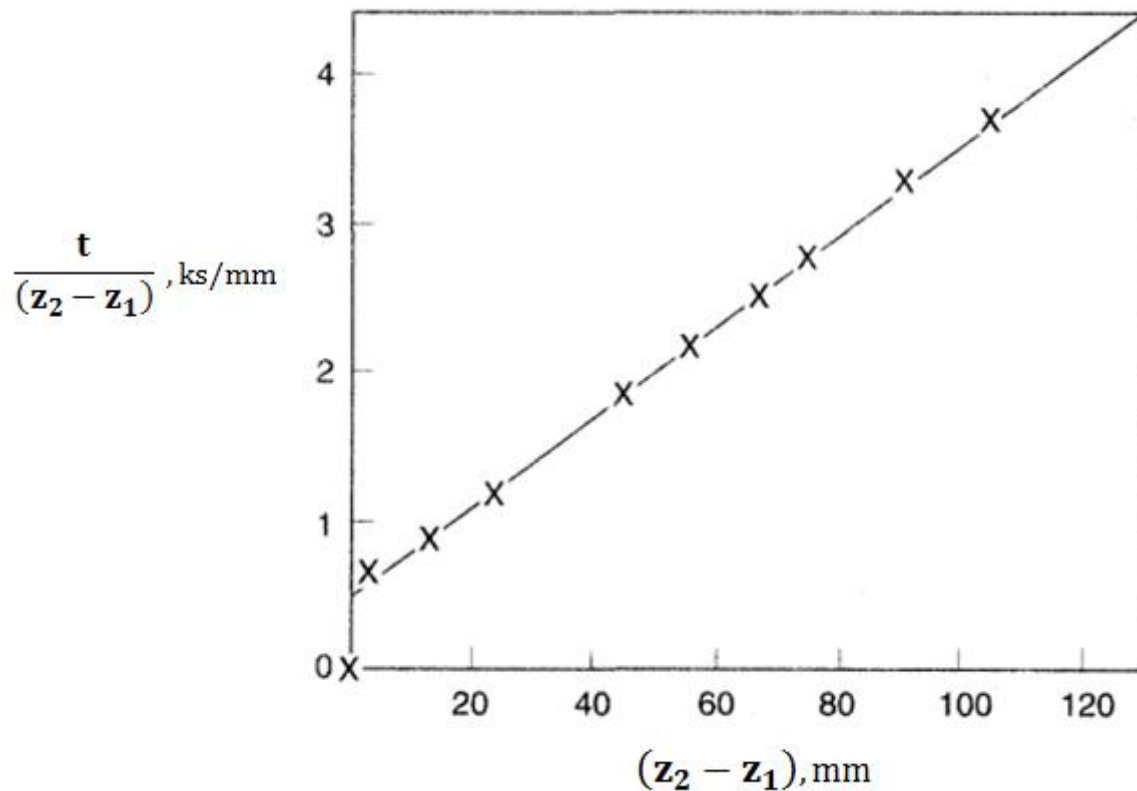
Time from commencement of experiment			Liquid level (mm)	Time from commencement of experiment			Liquid level (mm)
(h)	min)	(ks)		(h)	min)	(ks)	
0	0	0.0	0.0	32	38	117.5	54.7
0	26	1.6	2.5	46	50	168.6	67.0
3	5	11.1	12.9	55	25	199.7	73.8
7	36	27.4	23.2	80	22	289.3	90.3
22	16	80.2	43.9	106	25	383.1	104.8

The vapour pressure of carbon tetrachloride at 321 K is 37.6 kN/m² and the density of the liquid is 1540 kg/m³. The kilogram molecular volume may be taken as 22.4 m³.

Solution:

$$\frac{t}{(z_2 - z_1)} = \frac{\rho_L}{2 M_{wt} \cdot D_{AB} \cdot C_A} \left[\frac{C_{Bm}}{C_T} \right] (z_2 - z_1) + \frac{\rho_L}{M_{wt} \cdot D_{AB} \cdot C_A} \left[\frac{C_{Bm}}{C_T} \right] z_1$$

we draw $\frac{t}{(z_2 - z_1)}$ against $(z_2 - z_1)$



$$D_{AB} = \frac{\rho_L}{2 M_{wt} \cdot \text{slope} \cdot C_A} \left[\frac{C_{Bm}}{C_T} \right]$$

$$\text{The slope (s)} = 0.031 \frac{\text{ks}}{\text{mm}^2} = 3.1 \times 10^7 \frac{\text{s}}{\text{m}^2}$$

$$C_T = \left(\frac{1}{22.4} \right) \left(\frac{273}{321} \right) = 0.038 \frac{\text{kmol}}{\text{m}^3}$$

$$\text{or, } C_T = \frac{P_T}{R T} = \frac{101.3}{8.314 \times 321} = 0.038 \frac{\text{kmol}}{\text{m}^3}$$

$$M_{wt} = 154 \text{ kg/kmol}, \quad \rho_L = 1540 \text{ kg/m}^3$$

$$C_A = \text{mole fraction} \times C_T$$

$$C_A = \frac{P_A}{P_T} C_T = \frac{37.6}{101.3} (0.038) = 0.0141 \frac{\text{kmol}}{\text{m}^3}$$

$$C_{B1} = C_T - C_{A1} = 0.038 - 0.0141 = 0.0239$$

$$C_{B2} = C_T - 0 = 0.038$$

$$C_{Bm} = \frac{C_{B_2} - C_{B_1}}{\ln \frac{C_{B_2}}{C_{B_1}}} = \frac{0.038 - 0.0239}{\ln \frac{0.038}{0.0239}} = 0.0303 \frac{\text{kmol}}{\text{m}^3}$$

$$D_{AB} = \frac{1540}{2 (154)(3.1 * 10^7) (0.0141)} \left[\frac{0.0303}{0.038} \right] = 9.12 * 10^{-6} \frac{\text{m}^2}{\text{s}}$$

Example: A small diameter tube closed at one end was filled with acetone to within 18 mm of the top and maintained at 290 K with a gentle stream of air blowing across the top. After 15000 sec, the liquid level was fallen to 27.5 mm, the vapour pressure of acetone was 21.95 kPa and atmospheric pressure was 99.75 kPa. Calculate the diffusivity of acetone in air. Given: the density of acetone is 790 kg/m³ and the molecular weight of acetone is 58 kg/kmol.

Solution:

$$D_{AB} = \frac{\rho_L \cdot (z_2^2 - z_1^2) \cdot RT}{M_{wt} \cdot (2t) \cdot P_T \cdot \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right]}$$

$$D_{AB} = \frac{(790)(0.0275^2 - 0.018^2)(8.314)(290)}{(58) \cdot (15000) (99.75) \cdot \ln \left[\frac{99.75 - 0}{99.75 - 21.95} \right]} = 1.9 * 10^{-5} \frac{\text{m}^2}{\text{s}}$$

Prediction of diffusivities

The diffusivity (D_{AB}) for the transfer of one component (A) in another component (B) can be calculated empirically:

1. Empirical correlation for calculation of gas diffusivity:

The diffusivity (D_{AB}) for the transfer of one gas in another is not known and experimental determination is not practicable. It is necessary to use one of the many predictive procedures. The equation developed by **Fuller et al. (1966)** is easy to apply and gives reliable estimates:

$$D_{AB} = \frac{1.013 \times 10^{-7} T^{1.75} \sqrt{\frac{1}{M_A} + \frac{1}{M_B}}}{P \left[(\sum_a v_i)^{\frac{1}{3}} + (\sum_b v_i)^{\frac{1}{3}} \right]^2}$$

Where:

D_{AB} : is the diffusivity of (A) in (B), m^2/s .

T: is the temperature, K.

p is the total pressure, bar.

M_A, M_B are the molecular masses of component (A) and (B).

$\sum_a v_i, \sum_b v_i$: are the summation of the special diffusion volume coefficients for components a and b, given in Table 8.5.

Table 8.5. Special atomic diffusion volumes (Fuller *et al.*, 1966)

Atomic and structural diffusion volume increments			
C	16.5	Cl	19.5*
H	1.98	S	17.0*
O	5.48	Aromatic or hetrocyclic rings	-20.0
N	5.69*		

Diffusion volumes of simple molecules

H ₂	7.07	CO	18.9
D ₂	6.70	CO ₂	26.9
He	2.88	N ₂ O	35.9
N ₂	17.9	NH ₃	14.9
O ₂	16.6	H ₂	12.7
Air	20.1	CCL ₂ F ₂	114.8*
Ne	5.59	SF ₆	69.7*
Ar	16.1	Cl ₂	37.7*
Kr	22.8	Br ₂	67.2*
Xe	37.9*	SO ₂	41.1*

*Value based on only a few data points

Example: Estimate the diffusivity of methanol in air at atmospheric pressure and 25°C.

Solution:

Diffusion volumes from Table 8.5; methanol:

Element	v_i		No. of	
C	16.50	×	1	= 16.50
H	1.98	×	4	= 7.92
O	5.48	×	1	= 5.48
			$\sum_a v_i$	<u>29.90</u>

Diffusion volume for air = 20.1.

1 standard atmosphere = 1.013 bar.

molecular mass CH_3OH = 32, air = 29.

$$D_v = \frac{1.013 \times 10^{-7} \times 298^{1.75} (1/32 + 1/29)^{1/2}}{1.013[(29.90)^{1/3} + (20.1)^{1/3}]^2}$$

$$= \underline{\underline{16.2 \times 10^{-6} \text{ m}^2/\text{s}}}$$

Experimental value, $15.9 \times 10^{-6} \text{ m}^2/\text{s}$.

2. Empirical correlation for calculation of liquid diffusivity:

The equation developed by Wilke and Chang (1955), given below, can be used to predict liquid diffusivity diffusivities of dilute solutions of non-electrolytes. The Wilke-Chang equation gives satisfactory predictions for the diffusivity of organic compounds in water but not for water in organic solvents.

$$D_{AB} = \frac{1.173 \times 10^{-16} (\phi_B M_B)^{0.5} T}{\mu V_A^{0.6}}$$

Where:

D_{AB} : is the diffusivity of solute (A) in solvent (B), m^2/s .

T: is the absolute temperature, K.

M_B : is the molecular weight of the solvent.

μ : is viscosity of the solution, $(\text{N s}/\text{m}^2)$.

V_A : is the molar volume of the solute at its boiling point, m^3/kmol . Values for simple molecules are given in Table 10.4. For more complex molecules, V_A is calculated by

summation of the atomic volume and other contributions given in **Table 8.6**. It may be noted that for water a value of $0.0756 \text{ m}^3/\text{kmol}$ should be used.

\emptyset_B : is the association factor for the solvent (2.26 for water, 1.9 for methanol, 1.5 for ethanol and 1.0 for unassociated solvents such as hydrocarbons and ethers).

Table 8.6. Structural contributions to molar volumes, m^3/kmol (Gambil, 1958)

Molecular volumes							
Air	0.0299	CO ₂	0.0340	H ₂ S	0.0329	NO	0.0236
Br ₂	0.0532	COS	0.0515	I ₂	0.0715	N ₂ O	0.0364
Cl ₂	0.0484	H ₂	0.0143	N ₂	0.0312	O ₂	0.0256
CO	0.0307	H ₂ O	0.0189	NH ₃	0.0258	SO ₂	0.0448
Atomic volumes							
As	0.0305	F	0.0087	P	0.0270	Sn	0.0423
Bi	0.0480	Ge	0.0345	Pb	0.0480	Ti	0.0357
Br	0.0270	H	0.0037	S	0.0256	V	0.0320
C	0.0148	Hg	0.0190	Sb	0.0342	Zn	0.0204
Cr	0.0274	I	0.037	Si	0.0320		
Cl, terminal, as in RCl	0.0216	in higher esters, ethers		0.0110			
medial, as in R—CHCl—R	0.0246	in acids		0.0120			
Nitrogen, double-bonded	0.0156	in union with S, P, N		0.0083			
triple bonded, as in nitriles	0.0162	three-membered ring		−0.0060			
in primary amines, RNH ₂	0.0105	four-membered ring		−0.0085			
in secondary amines, R ₂ NH	0.012	five-membered ring		−0.0115			
in tertiary amines, R ₃ N	0.0108	six-membered ring as in benzene, cyclohexane, pyridine		−0.0150			
Oxygen, except as noted below	0.0074						
in methyl esters	0.0091	Naphthalene ring		−0.0300			
in methyl ethers	0.0099	Anthracene ring		−0.0475			

Example: Estimate the diffusivity of phenol in ethanol at 20°C (293 K).

Solution:

Viscosity of ethanol at 20°C, 1.2 mNs/m².

Molecular mass, 46.

Molar volume of phenol c1ccccc1O from Table 8.6:

Atom	Vol.		No. of		
C	0.0148	×	6	=	0.0888
H	0.0037	×	6	=	0.0222
O	0.0074	×	1	=	0.0074
ring	-0.015	×	1	=	-0.015
					<u>0.1034 m³/k mol</u>

$$D_L = \frac{1.173 \times 10^{-13} (1.5 \times 46)^{0.5} 293}{1.2 \times 0.1034^{0.6}} = \underline{\underline{9.28 \times 10^{-10} \text{ m}^2/\text{s}}}$$

Experimental value, $8 \times 10^{-10} \text{ m}^2/\text{s}$

* The **Wilke-Chang correlation** is shown graphically in Figure 8.2. This figure can be used to determine the association constant for a solvent from experimental values for D_{AB} (D_L) in the solvent. The Wilke-Chang equation gives satisfactory predictions for the diffusivity of organic compounds in water but not for water in organic solvents.

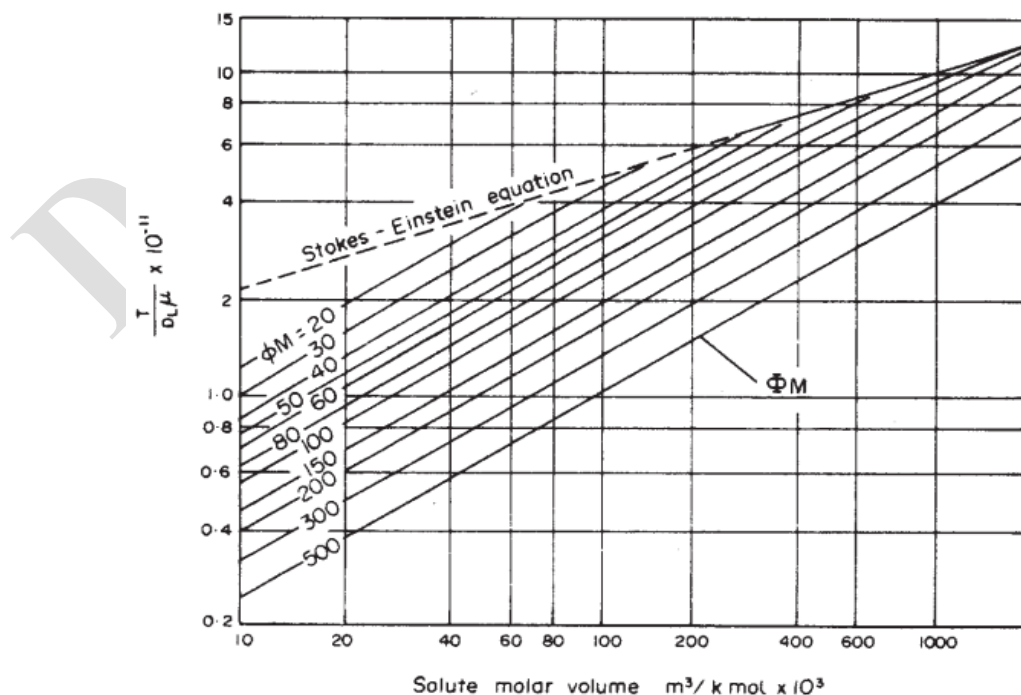


Figure 8.2. The Wilke-Chang correlation

It will be noted from the diffusivity equation is inversely proportional to the pressure and varies with the absolute temperature raised to the power of 1.5. In order to correct the diffusivity coefficient given in Table (10.2) we use the equation below:

$$\frac{D_{AB}(1)}{D_{AB}(2)} = \left(\frac{T_1}{T_2}\right)^{1.5} \left(\frac{P_2}{P_1}\right)$$

Where:

$D_{AB}(1)$ is the diffusivity at T_1 and P_1 given in Table (10.2) at 298 K and 101.3 kPa.

$D_{AB}(2)$ is the diffusivity at T_2 and P_2 .

Unsteady-state Diffusion

Considering an element of gas of cross-sectional area A and of thickness δy in the direction of mass transfer in which the concentrations C_A and C_B of the components **A** and **B** are a function of both position y and time t (See Figure below), then if the mass transfer flux is composed of two components, one attributable to diffusion according to Fick's Law and the other to a bulk flow velocity u_f , the fluxes of **A** and **B** at a distance y from the origin may be taken as N_A and N_B , respectively. These will increase to $N_A + (dN_A/dy) \delta y$ and $N_B + (dN_B/dy) \delta y$ at a distance $y + \delta y$ from the origin.

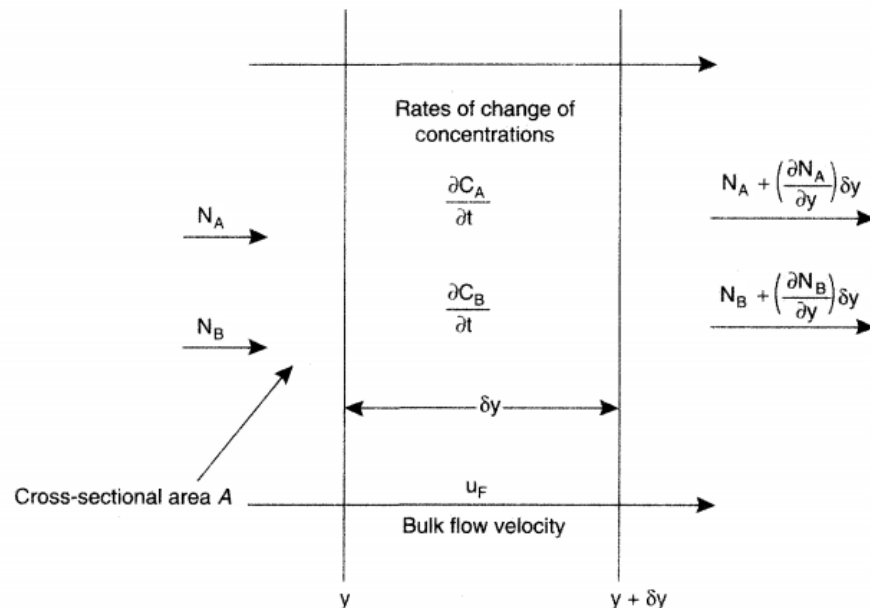


Figure 10.4: Unsteady state mass transfer

Thus, for A:

$$\begin{array}{ccc} \text{moles IN/unit time} & - & \text{moles OUT/unit time} & = \text{rate of change of concn.} \\ \text{(at } y) & & \text{(at } y + \delta y) & \times \text{element volume} \\ \left\{ -D \frac{\partial C_A}{\partial y} + u_F C_A \right\} A - \left\{ -D \frac{\partial C_A}{\partial y} + u_F C_A + \frac{\partial}{\partial y} \left[-D \frac{\partial C_A}{\partial y} + u_F C_A \right] \delta y \right\} A & = & \frac{\partial C_A}{\partial t} (\delta y \cdot A) \end{array}$$

Simplifying:
$$\frac{\partial C_A}{\partial t} = D \frac{\partial^2 C_A}{\partial y^2} - \frac{\partial(u_F C_A)}{\partial y}$$

For component B:
$$\frac{\partial C_B}{\partial t} = D \frac{\partial^2 C_B}{\partial y^2} - \frac{\partial(u_F C_B)}{\partial y}$$

and adding:
$$\frac{\partial(C_A + C_B)}{\partial t} = D \frac{\partial^2(C_A + C_B)}{\partial y^2} - \frac{\partial}{\partial y}[(C_A + C_B)u_F]$$

Since, for an ideal gas, $C_A + C_B = C_T = \text{constant}$:

$$0 = 0 - \frac{\partial}{\partial y}(u_F C_T)$$

and:
$$\frac{\partial u_F}{\partial y} = 0$$

where u_F is therefore independent of y .

$$\frac{\partial C_A}{\partial t} = D \frac{\partial^2 C_A}{\partial y^2} - u_F \frac{\partial C_A}{\partial y}$$

For equimolecular counterdiffusion, $u_F = 0$:

$$\frac{\partial C_A}{\partial t} = D \frac{\partial^2 C_A}{\partial y^2}$$

Fick's second law for unsteady state diffusion in one direction

This equation can be solved for a number of important boundary conditions. For the more general three-dimensional case where concentration gradients are changing in the x , y and z directions, these changes must be added to give:

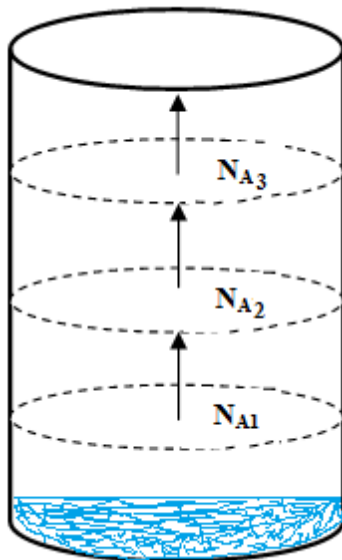
$$\frac{\partial C_A}{\partial t} = D \left[\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right]$$

Fick's second law for unsteady state diffusion in three direction

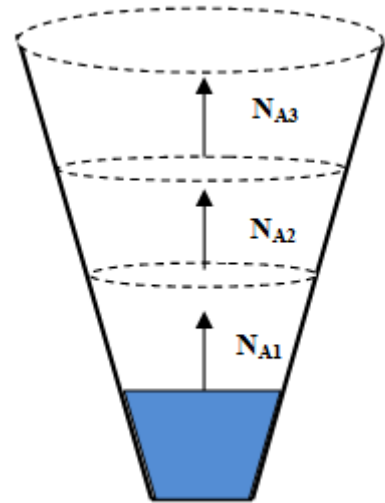
Diffusion through a varying cross-section area

The mole rate ($\bar{N}_A, \frac{\text{kmol}}{\text{s}}$) through a system of a varying cross section area is constant, while the mole flux ($N_A, \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$) is variable. The mass transfer through a cone and sphere can be consider as a mass transfer through a system of varying cross section area. On the other hand, the transfer through a cylinder can be consider as a mass transfer through a system of constant cross section area.

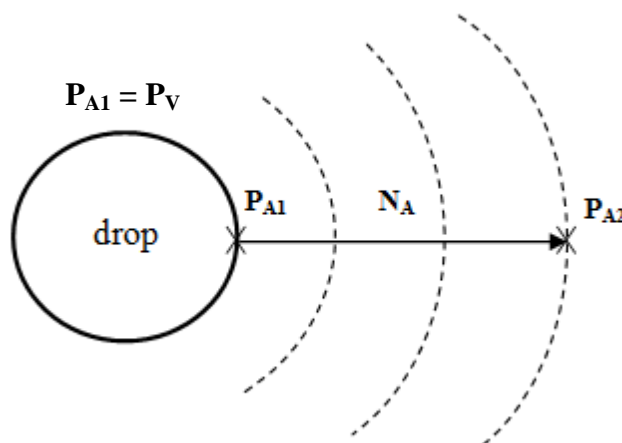
$$N_A = \frac{\text{mole rate}}{\text{surface area}} = \frac{\bar{N}_A}{A} = \frac{\frac{\text{kmol}}{\text{s}}}{\text{m}^2} = \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$



$$N_{A1} = N_{A2} = N_{A3}$$



$$N_{A1} > N_{A2} > N_{A3}$$



Diffusion through a spherical body

$$N_A = -D_{AB} \frac{dC_A}{dr} + \frac{C_A}{C_T} (N_A + N_B) \quad \dots \dots \dots (1)$$

$$\frac{\bar{N}_A}{A} = -D_{AB} \frac{dC_A}{dr} + \frac{C_A}{C_T} \left(\frac{\bar{N}_A}{A} + \frac{\bar{N}_B}{A} \right) \quad \dots \dots \dots (2)$$

$$\bar{N}_A = -D_{AB} A \frac{dC_A}{dr} + \frac{C_A}{C_T} (\bar{N}_A + \bar{N}_B) \quad \dots \dots \dots (3)$$

But: The surface area of sphere = $A = 4\pi r^2$

$$\bar{N}_A = -4\pi r^2 D_{AB} \frac{dC_A}{dr} + \frac{C_A}{C_T} (\bar{N}_A + \bar{N}_B) \quad \dots \dots \dots (4)$$

Case (I): Diffusion through a stagnant layer ($\bar{N}_B = 0$):

$$\bar{N}_A = -4\pi r^2 D_{AB} \frac{dC_A}{dr} + \frac{C_A}{C_T} (\bar{N}_A + 0)$$

$$\bar{N}_A (1 - C_A) = -4\pi r^2 D_{AB} C_T \frac{dC_A}{dr}$$

$$\bar{N}_A \int_{r_0}^{r_1} \frac{dr}{r^2} = 4\pi D_{AB} C_T \ln \left[\frac{C_T - C_{A2}}{C_T - C_{A1}} \right]$$

$$\bar{N}_A = \frac{4\pi D_{AB} C_T}{\frac{1}{r_0} - \frac{1}{r_1}} \ln \left[\frac{C_T - C_{A2}}{C_T - C_{A1}} \right] \quad \dots \dots \dots (1)$$

The most important things is to calculate the mass transfer rate for the sphere surface where the surface area is constant ($4\pi r_0^2$):

$$N_A \cdot A = \frac{4\pi D_{AB} C_T}{\frac{1}{r_0} - \frac{1}{r_1}} \ln \left[\frac{C_T - C_{A2}}{C_T - C_{A1}} \right]$$

$$N_A (4\pi r_0^2) = \frac{4\pi D_{AB} C_T}{\frac{1}{r_0} - \frac{1}{r_1}} \ln \left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}} \right]$$

$$N_A = \frac{D_{AB} C_T}{r_0^2 \left(\frac{1}{r_0} - \frac{1}{r_1} \right)} \ln \left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}} \right] \dots \dots \dots (2)$$

➔ Mole flux from the sphere surface

* When the mass transfer from surface to a large distance compare to the sphere surface (r_0):

$$r_1 \rightarrow \infty \quad \text{and} \quad C_{A_2} = 0$$

$$N_A = \frac{D_{AB} C_T}{r_0^2 \left(\frac{1}{r_0} - \frac{1}{\infty} \right)} \ln \left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}} \right]$$

$$N_A = \frac{D_{AB} C_T}{r_0} \ln \left[\frac{C_T - C_{A_2}}{C_T - C_{A_1}} \right] \dots \dots \dots (3)$$

In partial pressure form:

$$N_A = \frac{D_{AB} P_T}{r_0 \cdot RT} \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right] \dots \dots \dots (4)$$

Example: A sphere of naphthalene having a radius of 2 mm is suspended in a large volume of still air at 318 K and 101.3 kPa. The surface temperature of naphthalene can be assumed to be 318 K and its vapour pressure at this temperature is 0.555 mmHg. The diffusivity of naphthalene in air at 318 K is $6.92 \times 10^{-6} \text{ m}^2/\text{s}$. Calculate the rate of naphthalene evaporation from surface.

Solution:

The sphere is suspended in a large volume of still air means:

$$r_1 \rightarrow \infty \quad \text{and} \quad P_{A_2} = 0$$

$$N_A = \frac{D_{AB} P_T}{r_0 \cdot RT} \ln \left[\frac{P_T - P_{A_2}}{P_T - P_{A_1}} \right]$$

$$P_{A_1} = \left(\frac{0.555}{760} \right) * 101.3 = 0.07397 \text{ kPa}$$

$$r_0 = 2 * 10^{-3}$$

$$N_A = \frac{(6.92 * 10^{-6}) (101.3)}{(2 * 10^{-3}) (8.314) (318)} \ln \left[\frac{101.3 - 0}{101.3 - 0.07397} \right] = 9.68 * 10^{-8} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

Case (II): Equimolecular Counter Diffusion ($\bar{N}_B = -\bar{N}_A$):

$$N_A = -D_{AB} \frac{dC_A}{dr} + \frac{C_A}{C_T} (N_A + N_B) \quad \dots \dots \dots (1)$$

$$\frac{\bar{N}_A}{A} = -D_{AB} \frac{dC_A}{dr} + \frac{C_A}{C_T} \left(\frac{\bar{N}_A}{A} - \frac{\bar{N}_B}{A} \right) \quad \dots \dots \dots (2)$$

$$\bar{N}_A = -4\pi r^2 D_{AB} \frac{dC_A}{dr} \quad \dots \dots \dots (3)$$

$$\bar{N}_A \int_{r_0}^{r_1} \frac{dr}{r^2} = -4\pi D_{AB} \int_{C_{A1}}^{C_{A2}} dC_A \quad \dots \dots \dots (4)$$

$$\bar{N}_A \left[\frac{1}{r_0} - \frac{1}{r_1} \right] = 4\pi D_{AB} (C_{A1} - C_{A2}) \quad \dots \dots \dots (5)$$

$$\bar{N}_A = \frac{4\pi D_{AB}}{\left[\frac{1}{r_0} - \frac{1}{r_1} \right]} (C_{A1} - C_{A2})$$

For the mass transfer from surface ($A = 4\pi r_0^2$):

$$N_A = \frac{D_{AB}}{r_0^2 \left[\frac{1}{r_0} - \frac{1}{r_1} \right]} (C_{A1} - C_{A2})$$

In the case of r_1 is very large $\Rightarrow \frac{1}{r_1} = 0$

$$N_A = \frac{D_{AB}}{r_0} (C_{A1} - C_{A2})$$

In the form of partial pressure:

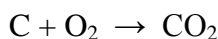
$$N_A = \frac{D_{AB}}{r_0 \cdot RT} (P_{A1} - P_{A2})$$

Case (III): Unequimolecular Counter Diffusion ($\bar{N}_B = -n \bar{N}_A$):

$$N_A = \frac{D_{AB}}{RT} \frac{P_T}{r_0} \frac{1}{(1-n)} \ln \left[\frac{P_T - (1-n) P_{A_2}}{P_T - (1-n) P_{A_1}} \right]$$

Example: Calculate the rate of burning of carbon particle 2.56 cm radius in an atmosphere of pure oxygen at 1000 K and 1 atm. Assuming a very large blanking layer of CO₂ has formed a round the particle. At the carbon surface $P_{CO_2} = 1$ atm and $P_{O_2} = 0$. At very large radius $P_{CO_2} = 0$ and $P_{O_2} = 1$ atm. Given the diffusivity of oxygen in carbon dioxide = 1.032 cm²/s.

Solution:



The diffusion is equimolecular counter diffusion:

$$N_A = \frac{D_{AB}}{RT \cdot r_0^2 \left[\frac{1}{r_0} - \frac{1}{r_1} \right]} (P_{A_1} - P_{A_2})$$

In the case of r_1 is very large ($r_1 \rightarrow \infty$) $\Rightarrow \frac{1}{r_1} = 0$

$$N_A = \frac{D_{AB}}{RT \cdot r_0} (P_{A_1} - P_{A_2}) = \frac{1.032 * 10^{-4}}{(8.314)(1000)(2.56 * 10^{-2})} (101.3 - 0)$$

$$N_A = 4.95 * 10^{-5} \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

Mass transfer theories

1. The Two-Film Theory

The two-film theory of **Whitman (1923)** was the first serious attempt to represent conditions occurring when material is transferred in *a steady state process* from one fluid stream to another. In this approach, it is assumed that a laminar layer exists in each of the two fluids. Outside the laminar layer, turbulent eddies supplement the action caused by the random movement of the molecules, and the resistance to transfer becomes progressively smaller as shown in Figure below. The thicknesses of the two films are z_1 and z_2 . Equilibrium is assumed to exist at the interface and therefore the relative positions of the points C and D are determined by the equilibrium relation between the phases.

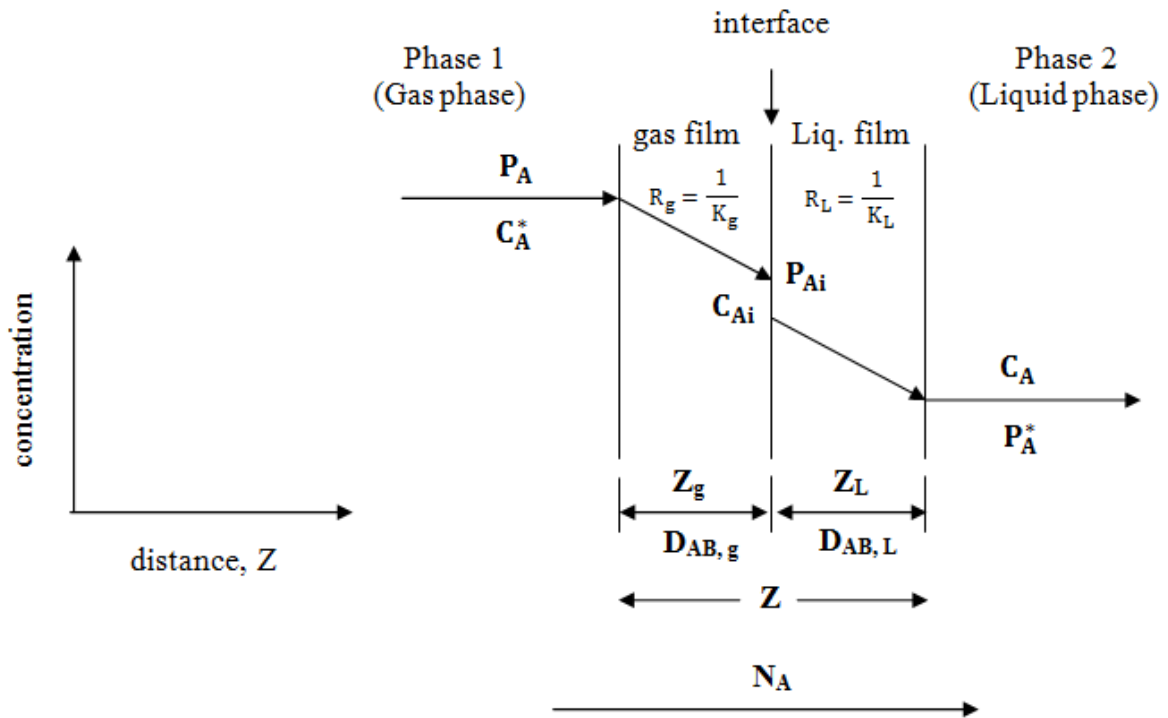


Figure 1: Two - Film Theory

The rate of mass transfer per unit area in terms of the two-film theory for equimolecular counterdiffusion is given for the first phase as:

$$N_A = -D_{AB} \frac{dC_A}{dz}$$

$$N_A = D_{AB} \frac{(C_A^* - C_A)}{Z}$$

In the form of partial pressure:

$$N_A = \frac{D_{AB}}{RT} \frac{(P_A - P_A^*)}{Z}$$

$$N_A = D_{AB} \frac{(C_A^* - C_A)}{Z} = \frac{D_{AB}}{RT} \frac{(P_A - P_A^*)}{Z}$$

Where: $Z = Z_g + Z_L$

The rate of mass transfer per unit area from the gas film:

$$N_{Ag} = \frac{(D_{AB})_g}{Z_g \cdot RT} (P_A - P_{Ai})$$

The rate of mass transfer per unit area from the liquid film:

$$N_{AL} = \frac{(D_{AB})_L}{Z_L} (C_{Ai} - C_A)$$

The relation between the partial pressure (P_A) and concentration (C_A):

Raoult's law:

$$P_A = P_A^0 X_A$$

$$\frac{P_A}{P_T} = \frac{P_A^0}{P_T} X_A$$

$$y_A = K X_A$$

Henry's law:

$$P_A = H C_A$$

Where: H is the Henry's constant.

$P_A^* = H C_A$	or	$y_A^* = H x_A$
$C_A = H P_A^*$	or	$y_A = H x_A^*$
$P_{Ai} = H C_{Ai}$	or	$y_{Ai} = H x_{Ai}$

2. The Penetration Theory

The penetration theory was suggested in **1935 by Higbie** who was investigating whether or not a resistance to transfer existed at the interface when a pure gas was absorbed in a liquid. In his experiments, a slug-like bubble of carbon dioxide was allowed rise through a vertical column of water in a 3 mm diameter glass tube. As the bubble rose, the displaced liquid ran back as a thin film between the bubble and the tube, *Higbie assumed that each element of surface in this liquid was exposed to the gas for the time taken for the gas bubble to pass it; that is for the time given by the quotient of the bubble length and its velocity*. It was further supposed that during this short period, which varied between 0.01 and 0.1 s in the experiments, absorption took place as the result of **unsteady state molecular diffusion** into the liquid, and, for the purposes of calculation, the liquid was regarded as infinite in depth because the time of exposure was so short.

The way in which the concentration gradient builds up as a result of exposing a liquid - initially pure - to the action of a soluble gas is shown in Figure 10.6. The percentage saturation of the liquid is plotted against the distance from the surface for a number of exposure times in arbitrary units. Initially only the surface layer contains solute and the concentration changes abruptly from 100 percent to 0 percent at the surface. For progressively longer exposure times the concentration profile develops as shown, until after an infinite time the whole of the liquid becomes saturated. The shape of the profiles is such that at any time the effective depth of liquid which contains an appreciable concentration of solute can be specified. If this depth of penetration is less than the total depth of liquid, no significant error is introduced by assuming that the total depth is infinite.

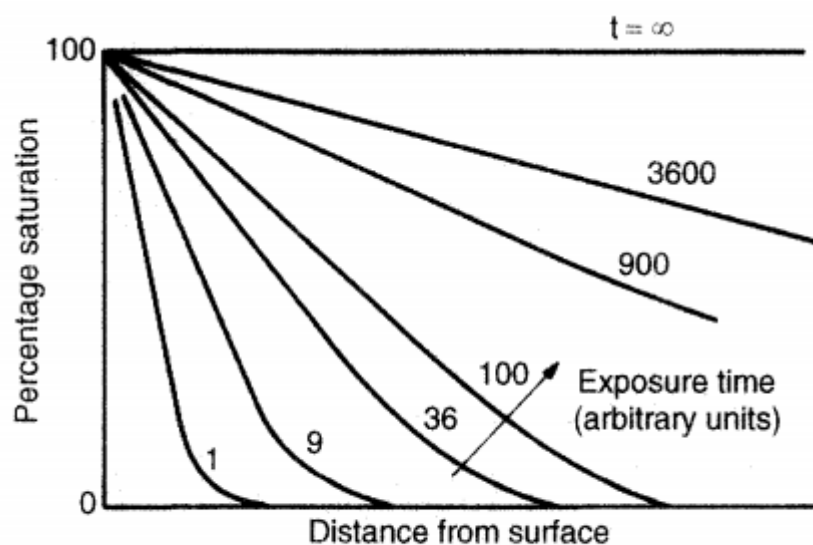


Figure 10.6. Penetration of solute into a solvent

The work of Higbie laid the basis of the penetration theory in which it is assumed that the eddies in the fluid bring an element of fluid to the interface where it is exposed to the second phase for a definite interval of time, after which the surface element is mixed with the bulk again. Thus, fluid whose initial composition corresponds with that of the bulk fluid remote from the interface is suddenly exposed to the second phase. It is assumed that equilibrium is immediately attained by the surface layers, that a process of unsteady state molecular diffusion then occurs and that the element is **remixed after a fixed interval of time**. In the calculation, the depth of the liquid element is assumed to be infinite and this is justifiable if the time of exposure is sufficiently short for penetration to be confined to the surface layers. Throughout, the existence of velocity gradients within the fluids is ignored and the fluid at all depths is assumed to be moving at the same rate as the interface.

The diffusion of solute **A** away from the interface (**y-direction**) is thus given by:

$$\frac{\partial C_A}{\partial t} = D \frac{\partial^2 C_A}{\partial y^2}$$

The following boundary conditions apply for the penetration theory:

$t = 0$	$0 < y < \infty$	$C_A = C_{Ao}$
$t > 0$	$y = 0$	$C_A = C_{Ai}$
$t > 0$	$y = \infty$	$C_A = C_{Ao}$

The mass transfer rate per unit area of surface is then given by:

$$(N_A)_{t,y=0} = -D \left(\frac{\partial C_A}{\partial y} \right)_{y=0}$$

$$(N_A)_{t,y=0} = (C_{Ai} - C_{Ao}) \sqrt{\frac{D}{\pi t}}$$

Note:

$$N_A \propto D_{AB} \quad \text{film theory}$$

$$N_A \propto \sqrt{D_{AB}} \quad \text{peretration theory}$$

3. The Random Surface Renewal Theory

Danckwerts (1951) suggested that each element of surface would not be exposed for the same time, but that a random distribution of ages would exist. It was assumed that the probability of any element of surface becoming destroyed and mixed with the bulk of the fluid was independent of the age of the element.

Thus, the overall rate of mass transfer per unit area when the surface is renewed in a random manner is:

$$N_A = (C_{Ai} - C_{Ao})\sqrt{Ds}$$

Where:

S is the rate of production of fresh surface per unit total area of surface.

4. The Film - Penetration Theory

A theory which incorporates some of the principles of both the two-film theory and the penetration theory has been proposed by *Toor and Marchello (1958)*. The whole of the resistance to transfer is regarded as lying within a laminar film at the interface, as in the two-film theory, but the mass transfer is regarded as an unsteady state process. It is assumed that fresh surface is formed at intervals from fluid which is brought from the bulk of the fluid to the interface by the action of the eddy currents mass transfer then takes place as in the penetration theory, except that the resistance is confined to the finite film, and material which traverses the film is immediately completely mixed with the bulk of the fluid. For short times of exposure, when none of the diffusing material has reached the far side of the layer, the process is identical to that postulated in the penetration theory. For prolonged periods of exposure when a steady concentration gradient has developed, conditions are similar to those considered in the two-film theory.

The diffusion of solute **A** away from the interface (**y-direction**) is thus given by:

$$\frac{\partial C_A}{\partial t} = D \frac{\partial^2 C_A}{\partial y^2}$$

The following boundary conditions apply for the penetration theory:

$$t = 0 \quad \rightarrow \quad C_A = C_{A0}$$

$$y = 0 \quad \rightarrow \quad C_A = C_A^*$$

$$y = L \quad \rightarrow \quad C_A = C_{A0}$$

The mass transfer rate across the interface per unit area is therefore given by:

$$(N_A)_t = (C_A^* - C_{A0}) \frac{D}{L} \left(1 + 2 \sum_{n=1}^{\infty} e^{-(n^2 \pi^2 D t) / L^2} \right)$$

When:

$\frac{Dt}{L^2}$ is small the penetration theory applicable.

$\frac{Dt}{L^2}$ is large the two film theory applicable.

The concentration profiles near an interface on the basis of:

- (a) the film theory (steady-state)
- (b) the penetration-theory (unsteady-state)
- (c) the film-penetration theory (unsteady-state)

are shown in Figure 10.7.

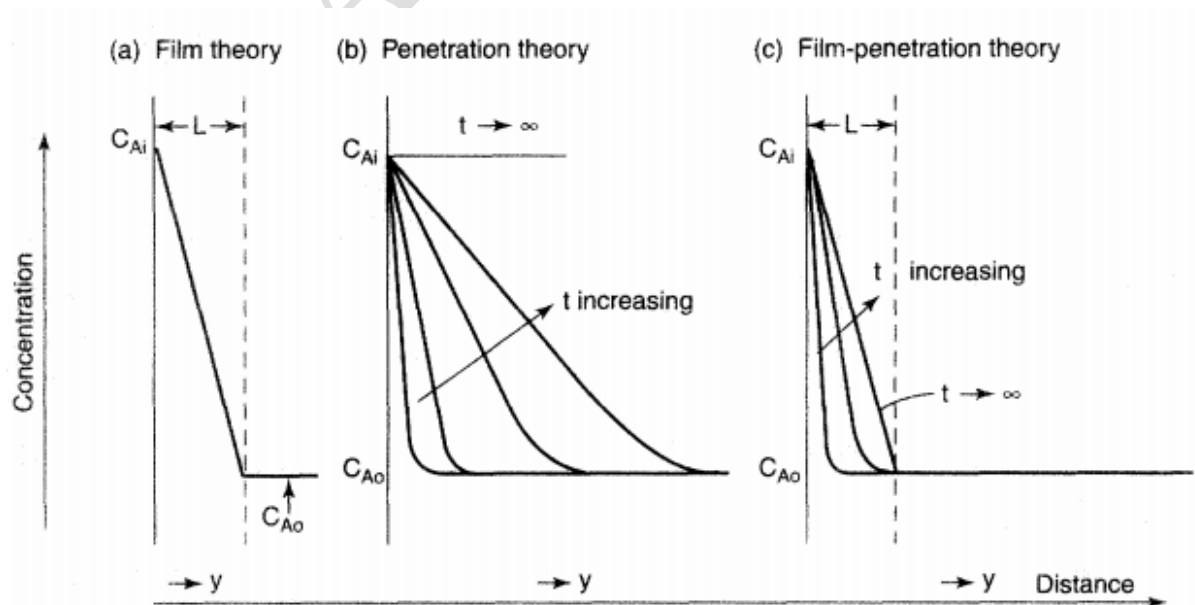


Figure 10.7. Concentration profiles near an interface

Mass transfer coefficients

Consider the two-film theory as shown in Figure (1):

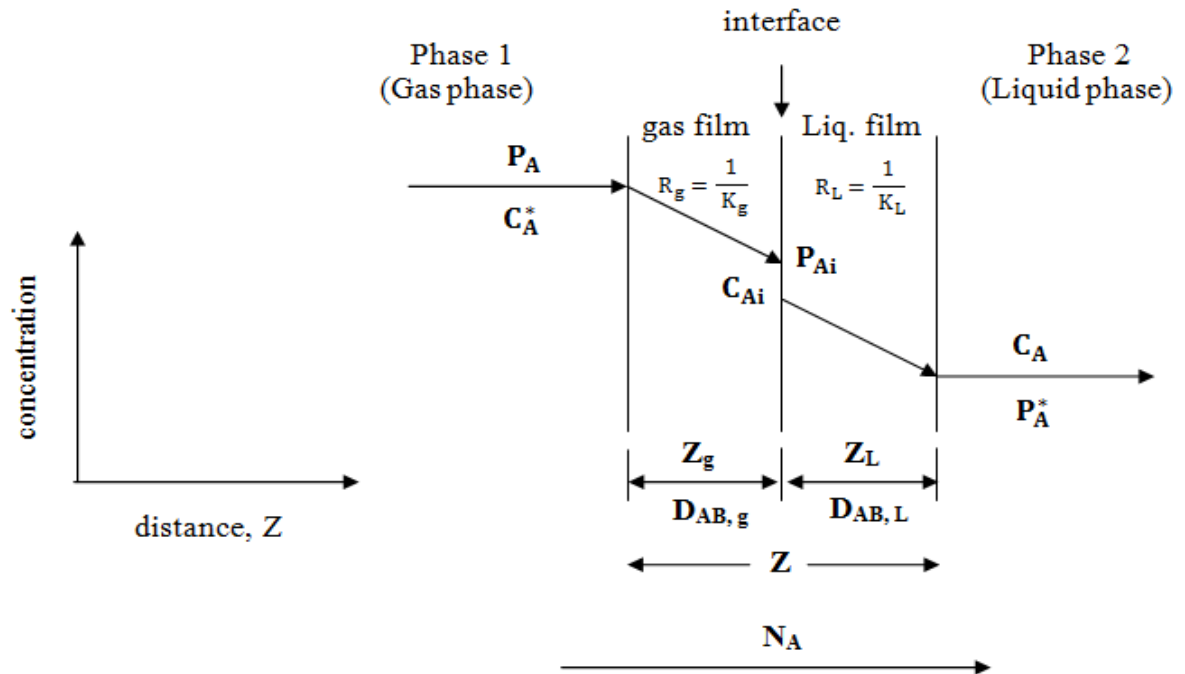


Figure 1: Two - Film Theory

The rate of mass transfer per unit area from the gas film:

$$N_{Ag} = \frac{(D_{AB})_g}{Z_g \cdot RT} (P_A - P_{Ai})$$

The rate of mass transfer per unit area from the liquid film:

$$N_{AL} = \frac{(D_{AB})_L}{Z_L} (C_{Ai} - C_A)$$

Where:

$$(D_{AB})_g = (D_{AB})_L$$

$$N_{Ag} = N_{AL}$$

Since the film thickness Z_g and Z_L are difficult to define or estimate, then we rewrite the above equations as follow:

$$N_A = k_g (P_A - P_{A_i})$$

$$N_A = k_L (C_{A_i} - C_A)$$

But: P_{A_i} and C_{A_i} are difficult to measure, therefore we define the overall mass transfer coefficient:

$$N_A = K_{OG} (P_A - P_A^*)$$

$$N_A = K_{OL} (C_A^* - C_A)$$

Where:

k_L is the individual liquid film mass transfer coefficient.

k_g is the individual gas film mass transfer coefficient.

K_{OL} is the overall mass transfer coefficient based on liquid phase.

K_{OG} is the overall mass transfer coefficient based on gas phase.

P_{A_i} is the partial pressure of the gas (A) at the interface.

C_{A_i} is the concentration of the liquid (A) at the interface.

P_A^* is the partial pressure of the gas phase which is in equilibrium with the liquid phase C_A .

C_A^* is the concentration of the liquid phase which is in equilibrium with the gas phase P_A .

The Relationships between the various mass transfer coefficients

$$N_A = k_g (P_A - P_{A_i}) \quad \dots \dots \dots (1)$$

$$N_A = k_L (C_{A_i} - C_A) \quad \dots \dots \dots (2)$$

$$N_A = K_{OG} (P_A - P_A^*) \quad \dots \dots \dots (3)$$

$$N_A = K_{OL} (C_A^* - C_A) \quad \dots \dots \dots (4)$$

$$\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L} \quad \dots \dots \dots (5)$$

$$\frac{1}{K_{OL}} = \frac{1}{H k_g} + \frac{1}{k_L} \quad \dots \dots \dots (6)$$

Q: Prove that $\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L}$

From Eq.(3) above:

$$\frac{1}{K_{OG}} = \frac{P_A - P_A^*}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_A^* + P_{A_i} - P_{A_i}}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_{A_i}}{N_A} + \frac{P_{A_i} - P_A^*}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_{A_i}}{N_A} + \frac{H C_{A_i} - H C_A}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{P_A - P_{A_i}}{N_A} + \frac{H (C_{A_i} - C_A)}{N_A}$$

$$\frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L}$$

Q: Prove that $\frac{1}{K_{OL}} = \frac{1}{H k_g} + \frac{1}{k_L}$

From Eq.(4) above:

$$\frac{1}{K_{OL}} = \frac{C_A^* - C_A}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{C_A^* - C_A + C_{A_i} - C_{A_i}}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{C_A^* - C_{A_i}}{N_A} + \frac{C_{A_i} - C_A}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{\frac{P_A}{H} - \frac{P_{A_i}}{H}}{N_A} + \frac{C_{A_i} - C_A}{N_A}$$

$$\frac{1}{K_{OL}} = \frac{1}{H} \left(\frac{P_A - P_{A_i}}{N_A} + \frac{C_{A_i} - C_A}{N_A} \right)$$

$$\frac{1}{K_{OL}} = \frac{1}{H k_g} + \frac{1}{k_L}$$

Notes:

1. The inverse to mass transfer coefficient $\left(\frac{1}{K}\right)$ is termed as a resistance to mass transfer.
2. The term (gas film control) refers to the resistance lie in the gas film.

Thus: $\frac{1}{K_{OG}} = \frac{1}{k_g} + \cancel{\frac{H}{k_L}}^0$ [when the solute is very soluble in liquid solvent]

3. The term (liquid film control) refers to the resistance lie in the liquid film.

Thus: $\frac{1}{K_{OL}} = \cancel{\frac{1}{H k_g}}^0 + \frac{1}{k_L}$

4. The units of mass transfer coefficients are as follows:

$$N_A = k_L (C_{A1} - C_{A2}) \quad \rightarrow \quad k_L = \frac{m}{s}$$

$$N_A = \bar{k}_g (P_{A1} - P_{A2}) \quad \rightarrow \quad \bar{k}_g = \frac{\text{kmol}}{m^2 \cdot s \cdot \text{kPa}}$$

$$N_A = \bar{\bar{k}}_g (X_{A1} - X_{A2}) \quad \rightarrow \quad \bar{\bar{k}}_g = \frac{\text{kmol}}{m^2 \cdot s}$$

$$k_L * \frac{1}{RT} = \bar{k}_g$$

$$k_L * \frac{P_T}{RT} = \bar{\bar{k}}_g$$

$$\bar{k}_g * P_T = \bar{\bar{k}}_g$$

$$\text{kJ} = \text{kN} \cdot \text{m} = \text{kg} \cdot \frac{m^2}{s^2}, \quad \text{kPa} = \frac{\text{kN}}{m^2}$$

$$\begin{aligned} \frac{m}{s} * \frac{1}{\frac{\text{kJ}}{\text{kmol} \cdot K} * K} &= \frac{m}{s} * \frac{\text{kmol}}{\text{kJ}} = \frac{m}{s} * \frac{\text{kmol}}{\text{kN} \cdot \text{m}} = \frac{\text{kmol}}{S \cdot \text{kN}} = \frac{\text{kmol}}{S \cdot \frac{\text{kN}}{m^2} * m^2} \\ &= \frac{\text{kmol}}{m^2 \cdot \text{kPa} \cdot S} = \bar{k}_g \end{aligned}$$

$$\begin{aligned} \frac{m}{s} * \frac{\text{kPa}}{\frac{\text{kJ}}{\text{kmol} \cdot K} * K} &= \frac{m}{s} * \frac{\text{kPa} \cdot \text{kmol}}{\text{kJ}} = \frac{m}{s} * \frac{\text{kN}}{m^2} * \frac{\text{kmol}}{\text{kN} \cdot \text{m}} = \frac{\text{kmol}}{m^2 \cdot s} \\ &= \bar{\bar{k}}_g \end{aligned}$$

Example: For a system in which component (A) is transferring from the liquid to the gas phase, the equilibrium is given by $y_A^* = 0.75 x_A$. At one point in the apparatus the liquid contain 90 mol% of (A) and gas contain 45 mol% of (A). The individual gas film mass transfer coefficient at this point in the apparatus of $0.02716 \text{ kmol/m}^2\cdot\text{s}$, and 70% of the overall resistance to mass transfer is known to be encountered in the gas film: determine:

1. The molar flux of (A).
2. The interfacial concentration of (A).
3. The overall mass transfer coefficient for liquid and gas phases.

Solution:

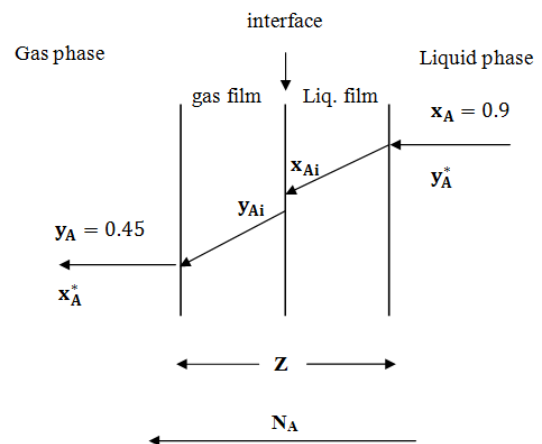
$$\frac{1}{k_g} = 0.7 \left(\frac{1}{K_{OG}} \right)$$

$$\frac{1}{0.02716} = 0.7 \left(\frac{1}{K_{OG}} \right)$$

$$K_{OG} = 0.019 \frac{\text{kmol}}{\text{m}^2\cdot\text{s}}$$

$$N_A = K_{OG} (y_A^* - y_A)$$

$$y_A^* = 0.75 x_A = (0.75)(0.9) = 0.675$$



$$1. \quad N_A = (0.019) (0.675 - 0.45) = 4.274 * 10^{-3} \frac{\text{kmol}}{\text{m}^2\cdot\text{s}}$$

$$2. \quad N_A = k_g (y_{Ai} - y_A)$$

$$4.274 * 10^{-3} = (0.02716)(y_{Ai} - 0.45)$$

$$y_{Ai} = 0.607$$

$$3. \quad \frac{1}{K_{OG}} = \frac{1}{k_g} + \frac{H}{k_L}$$

$$\frac{1}{0.019} = \frac{1}{0.02716} + \frac{0.75}{k_L}$$

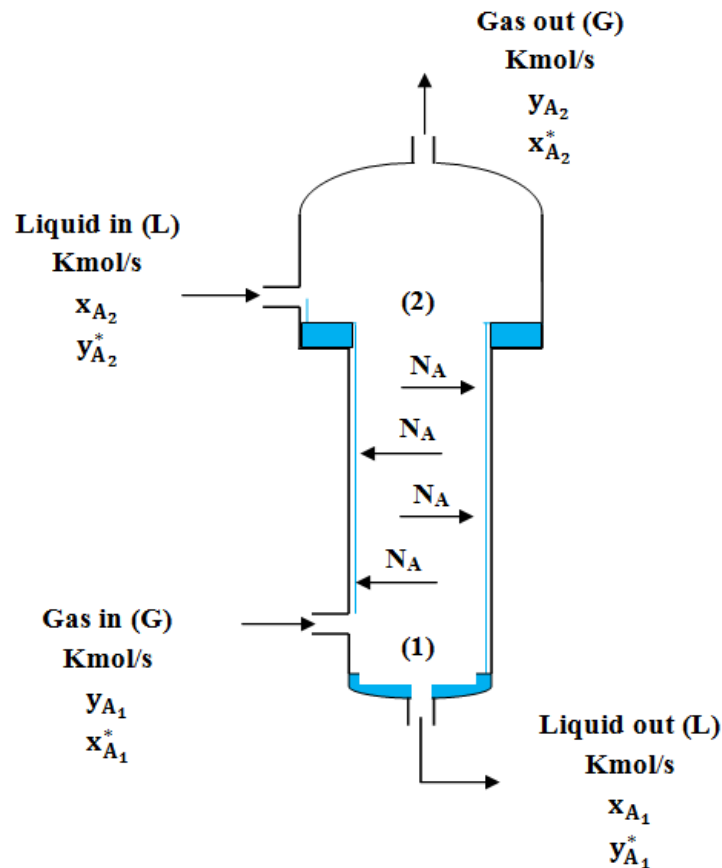
$$k_L = 0.0476$$

$$\frac{1}{K_{OL}} = \frac{1}{H k_g} + \frac{1}{k_L}$$

$$\frac{1}{K_{OL}} = \frac{1}{(0.75)(0.02716)} + \frac{1}{0.0476} \rightarrow K_{OL} = 0.0142 \frac{\text{kmol}}{\text{m}^2\cdot\text{s}}$$

The wetted wall column

The wetted wall column is a small experimental device used to determine the average mass transfer coefficient (K_{OG}). Usually it is (1 - 1.5 inch) in diameter and one meter long.



Let the mass transfer being from gas phase to liquid phase:

$$\bar{W}_A = G (y_{A1} - y_{A2}) = L (x_{A1} - x_{A2})$$

Where:

\bar{W}_A : is the total mole rate diffusion along the column.

At one point in the apparatus the diffusion is as follows:

$$\bar{N}_A = K_{OG} \cdot A \cdot (P_A - P_A^*) \quad \text{or} \quad \bar{N}_A = K_{OG} \cdot A \cdot (y_A - y_A^*)$$

$$\sum \bar{N}_{A_i} = \bar{W}_A = \sum K_{OG_i} \cdot A \cdot (\Delta y_A)_i$$

$$\bar{W}_A = K_{OG} \cdot A \cdot \Delta P_{Lm} \quad \dots\dots\dots (\text{kmol/s})$$

$$W_A = K_{OG} \cdot \Delta P_{Lm} \quad \dots\dots\dots (\text{kmol/m}^2 \cdot \text{s})$$

Where:

A is the transfer area (πdL) in (m^2).

ΔP_{Lm} is the logarithmic mean of partial pressure.

$$\bar{W}_A = K_{OG} \cdot A \cdot \Delta P_{Lm} \quad \dots\dots\dots (1)$$

Now to calculate (K_{OG}) from Eq.(1) we must calculate (\bar{W}_A) and (ΔP_{Am}):

$$1. \quad \Delta P_{Lm} = \frac{\Delta P_1 - \Delta P_2}{\ln \left[\frac{\Delta P_1}{\Delta P_2} \right]}$$

Where:

$$\Delta P_1 = P_{A_1} - P_{A_1}^* \quad , \quad P_{A_1}^* = H C_{A_1}$$

$$\Delta P_2 = P_{A_2} - P_{A_2}^* \quad , \quad P_{A_2}^* = H C_{A_2}$$

2. From overall material balance:

$$\bar{W}_A = G (y_{A_1} - y_{A_2}) = L (x_{A_1} - x_{A_2})$$

Where:

y_A and x_A : are the mole fraction of component (A) in gas and liquid phase, respectively.

G : is the mole flow rate of gas (kmol/s).

L : is the mole flow rate of liquid (kmol/s).

We can rewrite the Eq.(1) in the mole fraction form instead of partial pressure as follows:

$$\bar{W}_A = K_{OG} \cdot A \cdot \Delta y_{Lm} \quad \dots\dots\dots (2)$$

Where:

$$\Delta y_{Lm} = \frac{\Delta y_1 - \Delta y_2}{\ln \left[\frac{\Delta y_1}{\Delta y_2} \right]}$$

$$\Delta y_1 = y_{A_1} - y_{A_1}^* \quad \text{and} \quad y_{A_1}^* = H x_{A_1}$$

$$\Delta y_2 = y_{A_2} - y_{A_2}^* \quad \text{and} \quad y_{A_2}^* = H x_{A_2}$$

Example: A wetted wall column is used to absorb NH_3 by water from 6 vol.% in air. The gas flow rate is 1.2 kmol/min at 1 atm and 20°C . Calculate the overall mass transfer coefficient. The data given are:

- The water flow rate to gas flow rate ratio is 1.4.
- The outlet gas concentration is 1.5 vol.%.
- The column height is 100 cm.
- The column diameter is 2 cm.
- Henry's constant is 1.3.

Solution:

$$\bar{W}_A = G (y_{A1} - y_{A2}) = 1.2 (0.06 - 0.015) = 0.054 \frac{\text{kmol}}{\text{min}}$$

$$\bar{W}_A = 9 \times 10^{-4} \frac{\text{kmol}}{\text{s}}$$

$$A = \pi dL = (3.14) (2 \times 10^{-2}) (1)$$

$$A = 0.06283 \text{ m}^2$$

$$\Delta y_{\text{Lm}} = \frac{\Delta y_1 - \Delta y_2}{\ln \left[\frac{\Delta y_1}{\Delta y_2} \right]}$$

$$\Delta y_1 = y_{A1} - y_{A1}^*$$

$$\Delta y_2 = y_{A2} - y_{A2}^*$$

To find (x_{A1})

$$G (y_{A1} - y_{A2}) = L (x_{A1} - x_{A2})$$

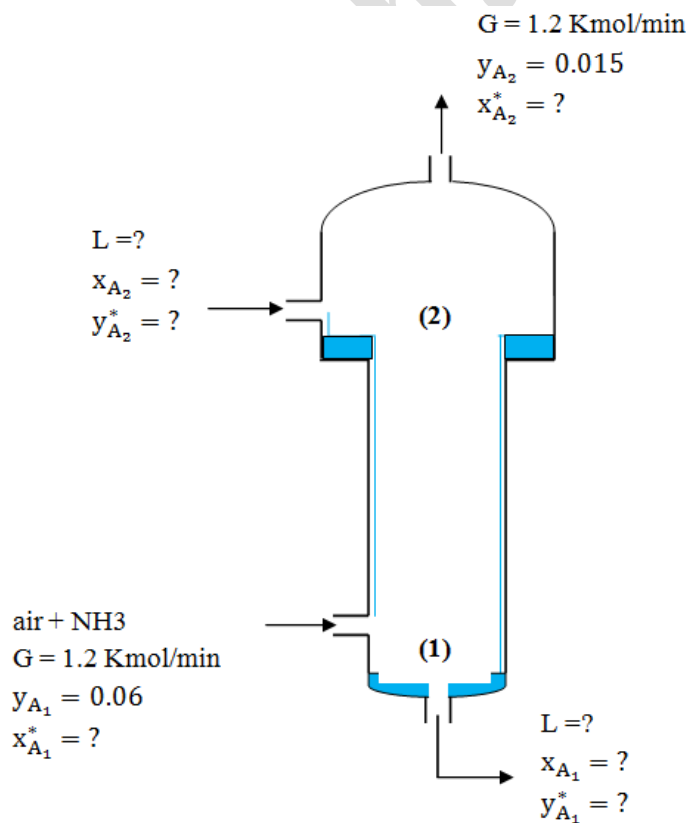
$$x_{A1} = \frac{G}{L} (y_{A1} - y_{A2}) + x_{A2}$$

$$x_{A1} = \frac{1}{1.4} (0.06 - 0.015) + 0$$

$$x_{A1} = 0.0321$$

$$y_{A1}^* = Hx_{A1} = 1.3 \times 0.0321 = 0.04173$$

$$y_{A2}^* = Hx_{A2} = 1.3 \times 0 = 0$$



$$\Delta y_1 = y_{A_1} - y_{A_1}^* = 0.06 - 0.04173 = 0.01827$$

$$\Delta y_2 = y_{A_2} - y_{A_2}^* = 0.015 - 0 = 0.015$$

$$\Delta y_{Lm} = \frac{0.01827 - 0.015}{\ln \left[\frac{0.01827}{0.015} \right]} = 0.0165$$

$$\bar{W}_A = K_{OG} \cdot A \cdot \Delta y_{Lm}$$

$$K_{OG} = \frac{\bar{W}_A}{A \cdot \Delta y_{Lm}} = \frac{9 * 10^{-4}}{(0.06283)(0.0165)} = 0.868 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

Mass Transfer Coefficient From Empirical Correlation

Several workers have measured the rate of transfer from a liquid flowing down the inside wall of a tube to a gas passing counter currently upwards. GILLILAND and SHERWOOD vaporized a number of liquids including water, toluene, aniline and propyl, amyl and butyl alcohols into an air stream flowing up the tube in order to measure the individual mass transfer coefficient (k_g). GILLILAND and SHERWOOD used the empirical relation below to measure the individual mass transfer coefficient (k_g) and this relation in forms of dimensionless groups:

$$Sh = a \cdot (Re)^b \cdot (Sc)^c$$

Where:

Sh: Sherwood number , $Sh = \frac{k_g d}{D}$

Re: Renold's number , $Re = \frac{\rho u d}{\mu}$

Sc: Schmidt number , $Sc = \frac{\mu}{\rho D}$

k_g : individual mass transfer coefficient (m/s).

d: diameter (m).

D: diffusivity (m^2/s).

ρ : density (kg/m^3).

μ : viscosity ($N.s/m^2$).

- When the constant (a), (b) and (c) in the above equation are unknown, then we can take them as below:

$$Sh = 0.023 (Re)^{0.83} (Sc)^{0.44}$$

Example: Calculate the gas film coefficient for the absorption of sulphur dioxide from a dilute mixture with air in a wetted wall column using Gilliland and Sherwood correlation for wetted wall column for the following data:

- Gas velocity = 2.5 m/s.
- Gas temperature = 293 K.
- Gas viscosity = 1.78×10^{-5} N.s/m².
- Gas density = 1.22 kg/m³.
- Gas diffusivity = 1.22×10^{-5} m²/s.
- Inside column diameter = 25 mm.

Solution:

$$Sh = 0.023 (Re)^{0.83} (Sc)^{0.44}$$

$$Sh = \frac{k_g d}{D} = k_g \left(\frac{25 \times 10^{-3}}{1.2 \times 10^{-5}} \right) = 2049 k_g$$

$$Re = \frac{\rho u d}{\mu} = \frac{(1.22)(2.2)(25 \times 10^{-3})}{1.78 \times 10^{-5}} = 3370$$

$$Sc = \frac{\mu}{\rho D} = \frac{1.78 \times 10^{-5}}{(1.22)(1.2 \times 10^{-5})} = 1.1959$$

$$2049 k_g = 0.023 (3370)^{0.83} (1.1959)^{0.44}$$

$$k_g = 0.01129 \frac{m}{s}$$

Chapter (12) in Volume (2)

Absorption of Gases

((Gas – Liquid Separation))

In absorption (also called gas absorption, gas scrubbing, and gas washing), a gas mixture is contacted with a liquid (the absorbent or solvent) to selectively dissolve one or more components by mass transfer from the gas to the liquid. The components transferred to the liquid are referred to as solute or absorbate.

Absorption is used to separate gas mixture; remove impurities, contaminants, pollutants, or catalyst poisons from gas; or recovery valuable chemicals. Thus, the species of interest in the gas mixture may be all components, only the component(s) not transferred, or only the component(s) transferred. The opposite of absorption is *stripping* (also called *desorption*), wherein a liquid mixture is contacted with gas to selectively remove components by mass transfer from the liquid to the gas phase.

There are two types of absorption processes:

1. Physical process (e.g. absorption of acetone from acetone – air mixture by water).
2. Chemical process, sometimes called chemi-sorption (e.g. absorption of nitrogen oxides by water to produce nitric acid).

Equipment:

Absorption and stripping are conducted in tray towers (plate column), packed column, spray tower, bubble column, and centrifugal contactors. The first two types of these equipment will be considered in our course for this year.

1. Tray tower:

A tray tower is a vertical, cylindrical pressure vessel in which gas and liquid, which flow counter currently, are contacted on a series of metal trays or plates. Liquid flows across any tray over an outlet weir, and into a down comer, which takes the liquid by gravity to the tray below. The gas flows upward through opening in each tray, bubbling through the liquid

on the other tray. A schematic diagram for the flow patterns inside the tray column is shown below.

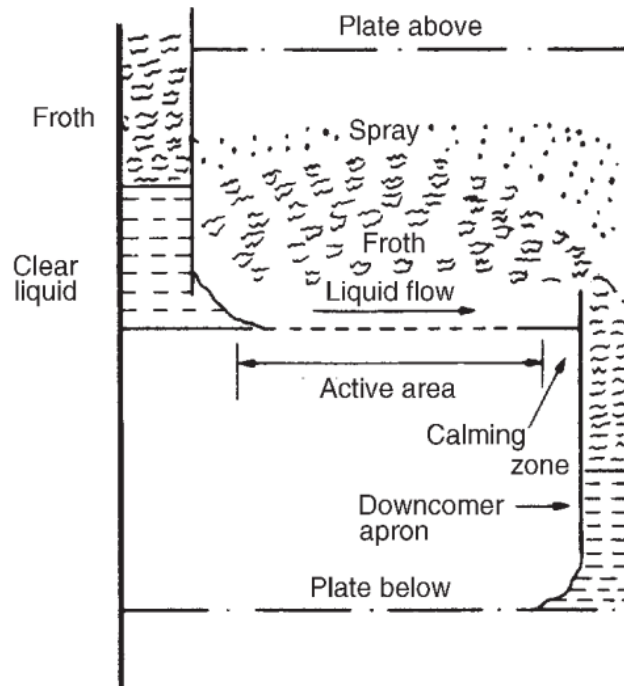


Figure : Typical cross-flow plate (sieve)

2. Packed tower:

The packed column is a vertical, cylindrical pressure vessel containing one or more section of packing material over who's the liquid flows down wards by gravity as a film or as droplets between packing elements. Gas flows upwards through the wetted packing contacting the liquid. The sections of packing are contained between a lower gas – injection support plate, which holds the packing, and an upper grid or mish hold – down plate, which prevent packing movement. A liquid distributor, placed above the hold – down plate, ensures uniform distribution of liquid as it enters the packing section.

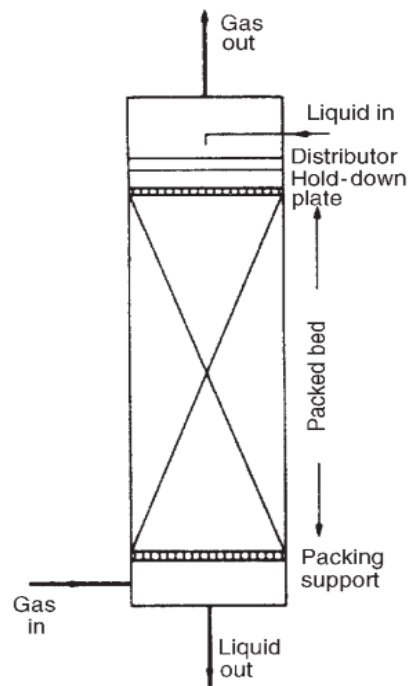


Figure: Packing absorber column.

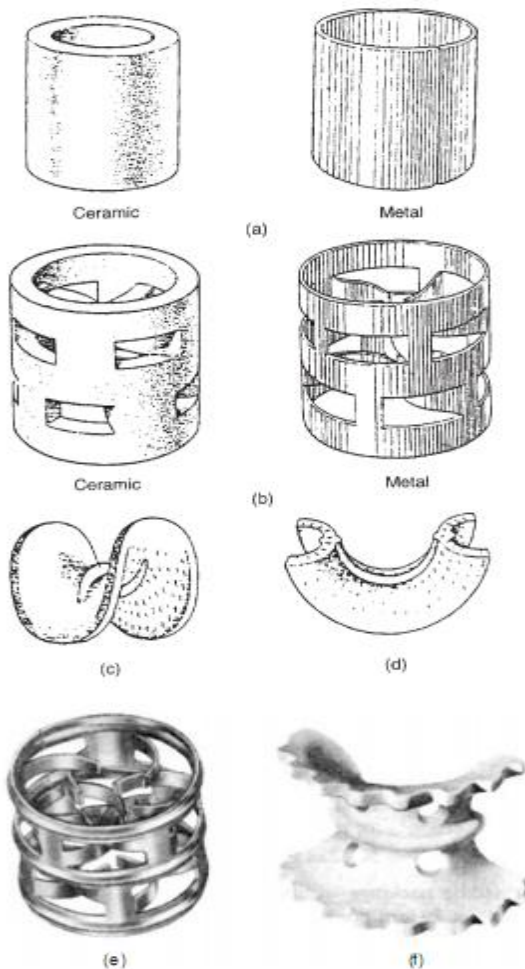


Figure: Types of packing (a) Raschig rings (b) Pall rings (c) Berl saddle ceramic (d) Intalox saddle ceramic (e) Metal Hypac (f) Ceramic, super Intalox.

General Design Consideration:

Design or analysis of an absorber (or stripper) requires consideration of a number of factors, including:

1. Entering gas (liquid) flow rate, composition, temperature, and pressure.
2. Design degree of recovery (R) of one or more solutes.
3. Choice absorbent (solvent) agent.
4. Operating pressure and temperature and allowable pressure drop.
5. Minimum absorbent (solvent) agent flow rate and actual solvent flow rate as a multiple of the minimum rate needed to make the separation.
6. Number of equilibrium stages.
7. Heat effects and need for cooling (heating).
8. Type of absorber (stripper) equipment.
9. Height of absorber (stripper) column.
10. Diameter of absorber (stripper) column.

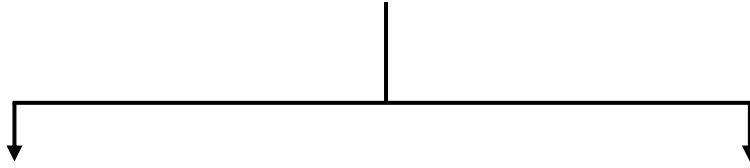
The ideal absorbent (solvent) should have:

- a. High solubility for the solute(s) to minimize the need for absorbent (solvent).
- b. A low volatility to reduce the loss of absorbent (solvent) and facilitate separation of absorbent (solvent) from solute(s).
- c. Be stable to maximize absorbent (solvent) life and reduce absorbent makeup requirement.
- d. Be non – corrosive to permit use of common material of construction.
- e. Have a low viscosity to provide low pressure drop and high mass and heat transfer rates.
- f. Be non – foaming when contacted with gas so as to make it unnecessary.
- g. Be non – toxic and non – flammable to facilitate its safe use.
- h. Be available, if possible.

The most widely absorbent (solvent) used are water, hydrocarbon oils, and aqueous solutions of acids and bases. While the most common stripping agents used are water vapor, air, inert gases, and hydrocarbon gases.

Equilibrium Relations Between Gas and Liquid Phases:

The equilibrium of any gas-liquid system can be expressed as:



Non-ideal system (Henry's law):

$$P_A = H x_A \quad \text{divided by } (P_T)$$

$$\frac{P_A}{P_T} = \frac{H}{P_T} x_A$$

$$y_A = m x_A$$

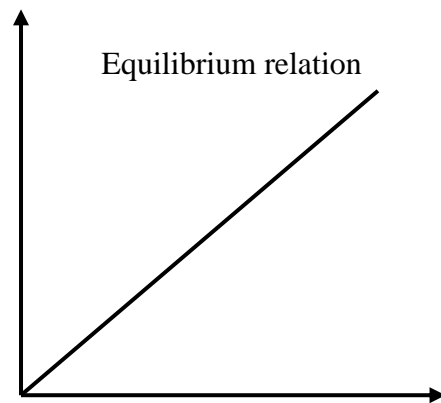
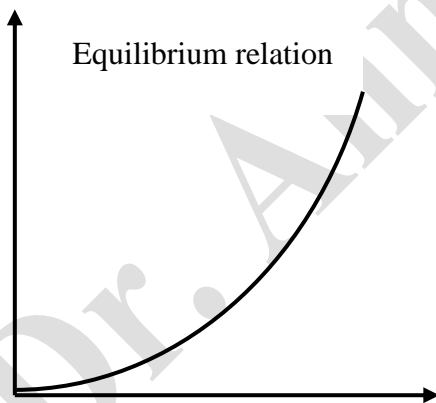
Ideal system (Raoult's law):

$$P_A = P_A^0 x_A \quad \text{divided by } (P_T)$$

$$\frac{P_A}{P_T} = \frac{P_A^0}{P_T} x_A$$

$$y_A = m x_A$$

* في عمليات الامتصاص من المهم جدا معرفة طبيعة علاقة التبادل (وهي العلاقة بين تركيز المذاب (A) في الغاز (Y_A) مع تركيز المذاب (A) في السائل (X_A) . فقد تكون علاقة التبادل بين (X_A) و (Y_A) علاقة خطية أو علاقة غير خطية اعتمادا على طبيعة المواد وتركيز المذاب.



Where:

X_A : is the mole ratio of solute in liquid phase (A/C).

Y_A : is the mole ratio of solute in gas phase (A/B).

Notes:

The equilibrium relation is the ratio between the *mole ratio* of solute in gas phase (Y_A) and the *mole ratio* of solute in liquid phase (X_A). The equilibrium relation may be linear or non linear.

* إذا كانت علاقة التبادل خطية فتعطى بالشكل التالي ($Y_A = m X_A$).

* أما إذا أعطيت علاقة التبادل بشكل بيانات كما في أدناه:

X_A	-	-	-	-	-	-
Y_A	-	-	-	-	-	-

ففي هذه الحالة لمعرفة طبيعة علاقة التبادل فيتم رسم البيانات أولاً فإذا كان الرسم بين X_A و Y_A هو خط مستقيم عند

ذلك سوف يتم اخذ الميل من الرسم فقط وتكوين علاقة التبادل ($Y_A = m X_A$).

أما إذا كان الرسم الناتج بين X_A و Y_A بشكل منحنى فعند ذلك سيكون الحل بالرسم.

* في بعض الأحيان تعطى علاقة التبادل بين الضغط الجزئي (P_A) والنسبة المولية (x_A) كما في علاقة راؤول أو

هنري ففي هذه الحالة يجب تحويلها الى علاقة بين (X_A, Y_A).

The relation between the mole fraction and mole ratio:

$$Y_A = \frac{y_A}{1 - y_A}$$

and

$$X_A = \frac{x_A}{1 - x_A}$$

Where:

x_A and y_A : are the mole fractions of solute (A) in liquid and gas phases, respectively.

X_A and Y_A : are the mole ratio of solute (A) in liquid and gas phases, respectively.

The relation between the mole fraction and weight fraction:

$$\text{wt. \%} = \frac{(\text{mol\%}) * (\text{M. wt})}{\sum[(\text{mol\%}) * (\text{M. wt})]}$$

$$\text{mol\%} = \frac{(\text{wt. \%}) / (\text{M. wt})}{\sum[(\text{wt. \%}) / (\text{M. wt})]}$$

Where:

wt. % : is the weight fraction.

mol% : is the mole fraction.

M. wt : is the molecular weight.

Symbols used in the absorption processes:

A solute (A) in a mixture (A, B) shall be absorbed in Liquid (C), the inert gas (B) is insoluble in solvent (C). The following symbols will be used:

G : is the mole rate of the gas mixture (A + B), kmol/s.

G_s : is the mole rate of the inert (insoluble) gas (B), kmol/s.

\bar{G} : is the mole flux of the gas mixture (A + B), kmol/m².s.

\bar{G}_s : is the mole flux of the inert (insoluble) gas (B), kmol/m².s.

L : is the mole rate of the liquid mixture (A + C), kmol/s.

L_s : is the mole rate of the liquid solvent only (C), kmol/s.

\bar{L} : is the mole flux of the liquid mixture (A + C), kmol/m².s.

\bar{L}_s : is the mole flux of the liquid solvent only (C), kmol/m².s.

x_A : is the mole fraction of solute (A) in liquid, (A / A+C).

y_A : is the mole fraction of solute (A) in gas, (A / A+B).

X_A : is the mole ratio of solute (A) in liquid, (A / C).

Y_A : is the mole ratio of solute (A) in gas, (A / B).

Calculation of Tower Height

The physical absorption process can be carried out in countercurrent flow process, which may be carried out in packed or tray column:

Packed Tower

$$Z = \text{HOG} * \text{NOG}$$

Where:

HOG: is the height of transfer unit (HTU) based on gas phase, and it can be calculated from the equation below:

$$\text{HOG} = \frac{\bar{G}_s}{K_o G \cdot a} \quad , \text{ in (meter)}$$

NOG: is the number of transfer unit (NTU) based on gas phase, and it can be calculated based on equilibrium data:

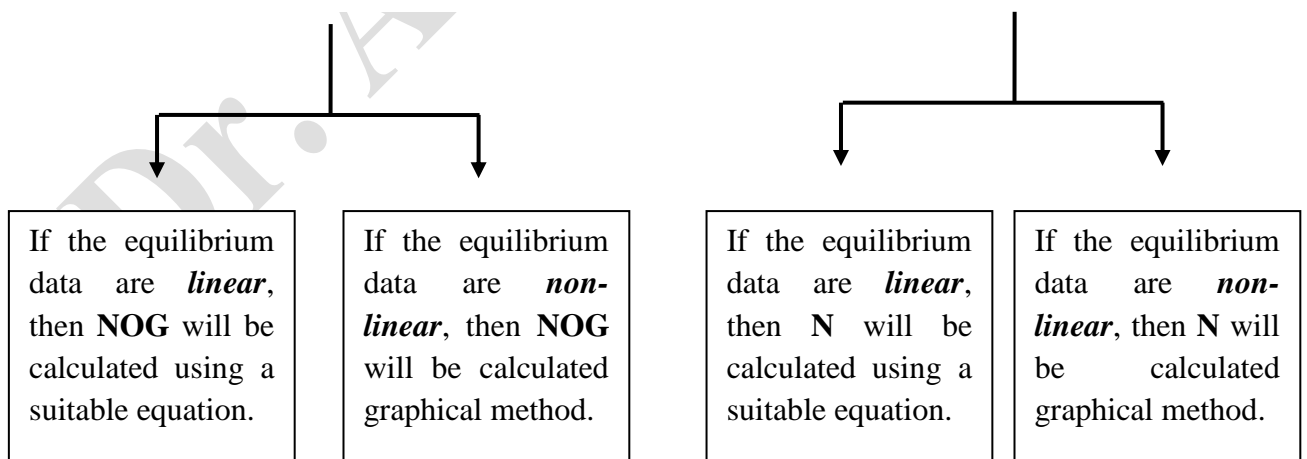
Tray Tower

$$Z = H * N$$

Where:

H : is the distance between two trays, and it is given (0.3 - 0.7 m)

N : is the number of trays, and it can be calculated based on equilibrium data:



1. Packed tower:

Absorption and stripping are frequently conducted in packed columns, particularly when:

- (1) the required column diameter is less than 0.6 m.
- (2) the pressure drop must be low, as for a vacuum service.
- (3) corrosion consideration favor the use of ceramic or polymeric material.
- (4) low liquid holdup is desirable.

The gas liquid contact in a packed bed column is continuous, not stage-wise, as in a plate column. The liquid flows down the column over the packing surface and the gas or vapour, counter-currently, up the column. In some gas-absorption columns co-current flow is used. The performance of a packed column is very dependent on the maintenance of good liquid and gas distribution throughout the packed bed, and this is an important consideration in packed-column design.

Calculations of the packing height based on gas phase:

Overall material balance on the solute (A) over an element (∂z) based on gas phase:

$$G_s dY = L_s dX = N_A \cdot A$$

$$N_A = G_s Y - G_s \left(Y + \frac{dY}{dz} \partial z \right) = (K_o G)(a \cdot S \partial z)(Y - Y^*)$$

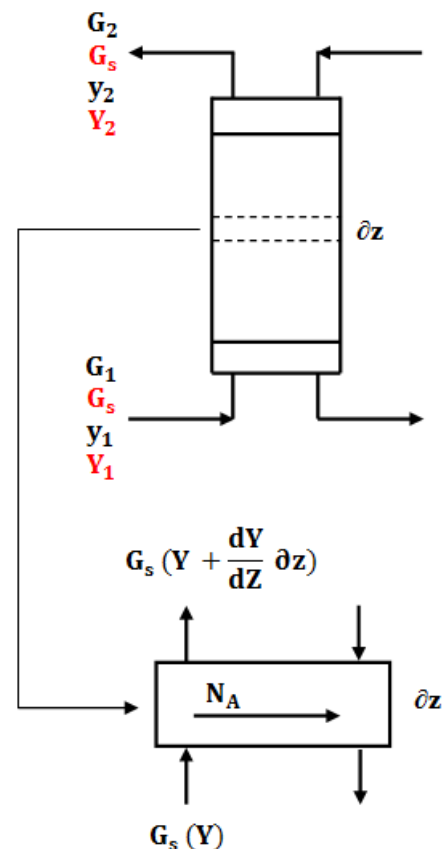
Where:

The interfacial area for transfer = $a dV = a \cdot S \partial z$

S : is the cross-sectional area of column (m^2).

a : is the surface area of interface per unit volume of column (m^2/m^3).

$$- G_s \left(\frac{dY}{dz} \partial z \right) = (K_o G)(a)(S \cdot \partial z)(Y - Y^*)$$



$$G_s \frac{dY}{dZ} = -(KoG \cdot a)(S \cdot dz)(Y - Y^*)$$

$$\int_0^Z dZ = \frac{-G_s}{(KoG \cdot a) \cdot S} \int_{Y_1}^{Y_2} \frac{dY}{(Y - Y^*)}$$

$$Z = \frac{(G_s/S)}{KoG \cdot a} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

$$Z = \frac{\bar{G}_s}{KoG \cdot a} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

$$Z = HOG * NOG = HTU * NTU$$

Where:

$HOG = \frac{\bar{G}_s}{KoG \cdot a}$: height of transfer unit (HTU) based on gas phase, with the units of (m).

$NOG = \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$: number of transfer unit (NTU) based on gas phase, without units.

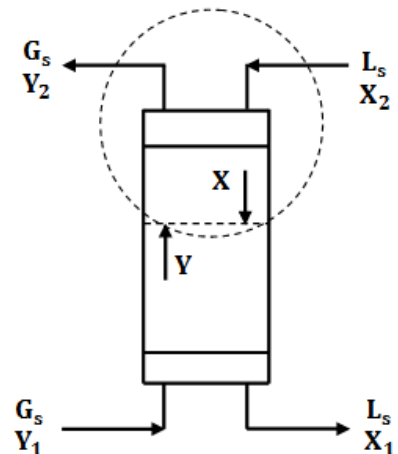
Equation of the operating line:

Solute material balance between one end of the column and any point will give:

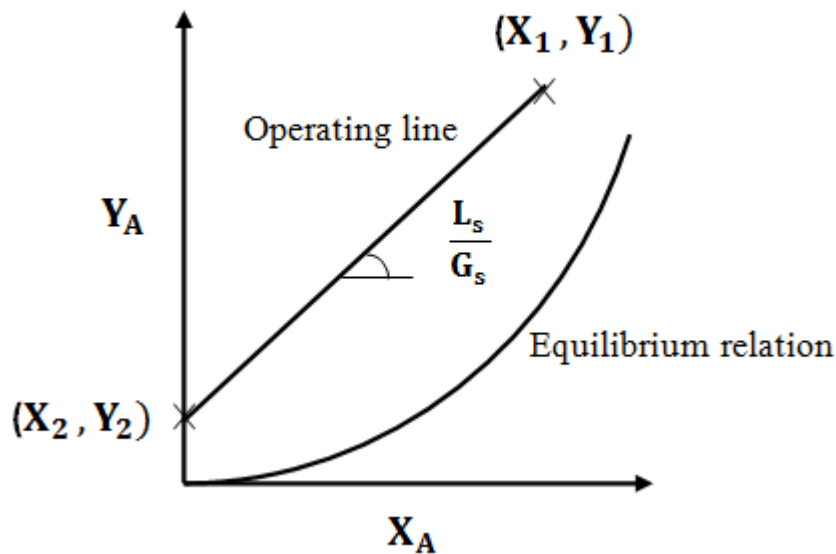
$$G_s (Y - Y_2) = L_s (X - X_2)$$

$$Y = \frac{L_s}{G_s} (X - X_2) + Y_2$$

* The equation of operating line is a relation between mole ratio of solute in gas phase (Y) and the mole ratio of solute in liquid phase (X).



* The operating line can be draw from two points (X_1, Y_1) and (X_2, Y_2) , or from its slope $(\frac{L_s}{G_s})$ and one of the two points.



Calculation of Number of Transfer Unit (NOG):

A. For Linear Equilibrium Relationship ($Y^* = m X$):

$$NOG = \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)} \dots \dots \dots (1)$$

$$Y^* = m X \dots \dots \dots (2)$$

$$G_s (Y - Y_2) = L_s (X - X_2) \dots \dots \dots (3)$$

$$\Rightarrow X = \frac{G_s}{L_s} (Y - Y_2) + X_2$$

For pure liquid solvent used then, $X_2 = 0$

$$X = \frac{G_s}{L_s} (Y - Y_2) \dots \dots \dots (4)$$

Substitution Eq.(4) into Eq.(2) to get:

$$Y^* = \frac{m G_s}{L_s} (Y - Y_2) \dots \dots \dots (5)$$

Substitution Eq.(5) into Eq.(1) to get:

$$NOG = \int_{Y_2}^{Y_1} \frac{dY}{\left(Y - \frac{m G_s}{L_s} (Y - Y_2)\right)}$$

$$\text{Let: } \frac{m G_s}{L_s} = \phi = \frac{\text{Slope of equilibrium line}}{\text{Slope of operating line}} = \frac{m}{L_s/G_s} < 1.0$$

$$NOG = \int_{Y_2}^{Y_1} \frac{dY}{Y - \phi Y + \phi Y_2}$$

$$NOG = \int_{Y_2}^{Y_1} \frac{dY}{(1 - \phi)Y + \phi Y_2}$$

$$NOG = \frac{1}{(1 - \phi)} \ln \left[\frac{(1 - \phi)Y_1 + \phi Y_2}{(1 - \phi)Y_2 + \phi Y_2} \right]$$

$$NOG = \frac{1}{(1 - \phi)}$$

$$NOG = \frac{1}{(1 - \phi)}$$

B. For Non-linear Equilibrium Relationship:

In this case the integration $[NOG = \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}]$ will be solved using graphical method or numerical method (Simpson rule) following steps below:

1. Draw the given equilibrium data.
2. Draw the operating line, from two points (X_1, Y_1) and (X_2, Y_2) or one point and slope of $(\frac{L_s}{G_s})$.
3. Create the table below by calculated (Y^*) from the plot as below:

Y Assume points between ($Y_1 - Y_2$)	Y* Calculated from plot	$\frac{1}{(Y - Y^*)}$
Y_1	- calculated	$\sqrt{= f_0}$
- (assumed)	- calculated	$\sqrt{= f_1}$
- (assumed)	- calculated	$\sqrt{= f_2}$
- (assumed)	- calculated	$\sqrt{= f_3}$
Y_2	- calculated	$\sqrt{= f_n}$

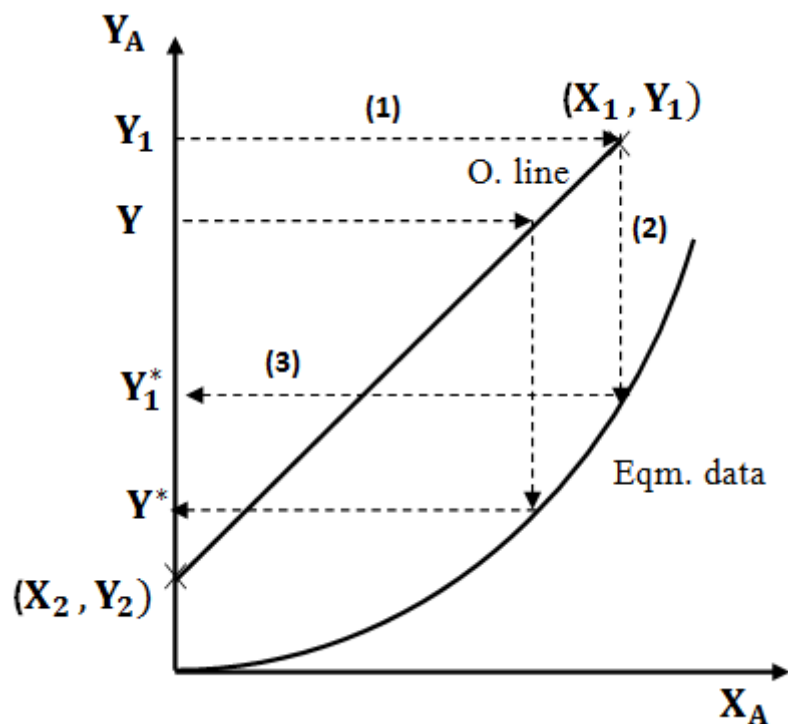
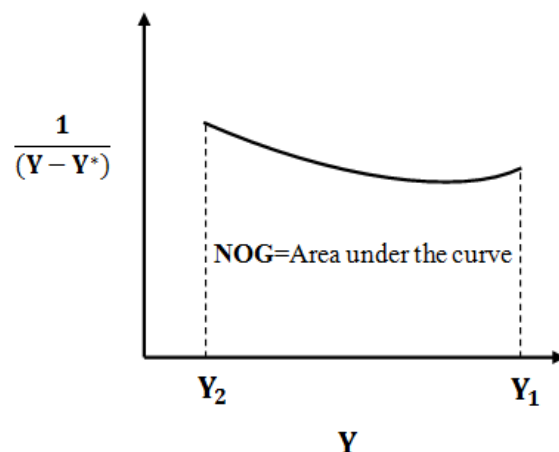


Figure: Calculation of (Y^*) for packed column.

4. To calculate **NOG** we draw $\left[\frac{1}{(Y-Y^*)}\right]$ Vs. $[Y]$ to find the area under the curve:

Where:

NOG = Area under the curve



Simpson rule for calculation of NOG:

NOG = Area under the curve

$$\text{NOG} = \frac{h}{3} \left[f_0 + f_n + 2 \sum f_{\text{even}} + 4 \sum f_{\text{odd}} \right]$$

Where:

$$h = \frac{Y_1 - Y_2}{n}, \quad n = 2, 4, 6, 8, \dots \text{etc.}$$

Notes:

* If the entering solute concentration is dilute ($Y < 5\%$), then:

$$Y_A = y_A, \quad X_A = x_A, \quad G_s = G, \quad L_s = L$$

* If the tower type is not mention in the problem we can take it as a packed tower.

Example (1):

Ammonia is to be removed from a 10 percent ammonia–air mixture by countercurrent scrubbing with water in a packed tower at 293 K so that 99 percent of the ammonia is removed when working at a total pressure of 101.3 kN/m². If the gas rate is 0.95 kg/m².s of tower cross-section and the liquid rate is 0.65 kg/m². s, find the necessary height of the tower if the absorption coefficient $K_{OG}a = 0.008 \text{ kmol/m}^3\text{.s. kPa.}$, The equilibrium data are: $Y^* = 0.8 X$.

Solution:

$$y_2 = (1 - \text{recovery}) y_1 = (1 - 0.99)(0.1) = 0.001$$

Convert mole fraction to mole ratio:

$$Y_1 = \frac{y_1}{1 - y_1} = \frac{0.1}{1 - 0.1} = 0.11$$

$$Y_2 = \frac{y_2}{1 - y_2} = \frac{0.001}{1 - 0.001} = 0.001$$

We can see that at low conc. (mole ratio = mole fraction):

$$\begin{aligned} \text{The gas mole flux, } \bar{G} &= \frac{\text{gas mass flux}}{\text{average gas molecular weight}} \\ &= \frac{0.95}{[(0.1)(17) + (0.9)(29)]} = 0.0341 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}} \end{aligned}$$

$$\begin{aligned} \text{The liquid mole flux, } \bar{L} &= \frac{\text{liquid mass flux}}{\text{average liquid molecular weight}} \\ &= \frac{0.65}{(18)} = 0.0361 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}} \end{aligned}$$

$$\text{the mole flux of the inert gas, } \bar{G}_s = \bar{G}(1 - y_1) = (0.0341)(1 - 0.1) = 0.0307 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$\text{the mole flux of the inert liquid, } \bar{L}_s = \bar{L}(1 - x_2) = (0.0361)(1 - 0) = 0.0361 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

Therefore, for pure solvent: $\bar{L}_s = \bar{L}$

$$\text{HOG} = \frac{\bar{G}_s}{\text{KoG. a. } P_T} = \frac{0.0307}{(0.0008)(101.3)} = 0.38 \text{ m}$$

Since the equilibrium is linear:

$$\phi = \frac{m \bar{G}_s}{\bar{L}_s} = \frac{(0.8)(0.0307)}{(0.0361)} = 0.68$$

$$\text{NOG} = \frac{1}{(1 - \phi)}$$

$$\text{NOG} = \frac{1}{(1 - 0.68)}$$

$$Z = \text{HOG} * \text{NOG} = (0.38)(11.19) = 4.25 \text{ m}$$

Example (2):

Ammonia is to be removed from a 10 percent ammonia–air mixture by countercurrent absorption with water in a packed tower at 293 K. The outlet gas concentration from the top of the tower is 0.1%. The absorption tower is working at a total pressure of 101.3 kN/m². If the inlet gas is 0.034 kmol/m².s and the liquid rate is 0.036 kmol/m². s, find the necessary height of the tower if the absorption coefficient $KoG.a = 0.081$ kmol/m³.s. The equilibrium data is given by the following data:

kmol NH ₃ /kmol water:	0.021	0.031	0.042	0.053	0.079	0.106	0.159
Partial pressure NH ₃ in gas phase (kN/m ²):	1.6	2.4	3.3	4.2	6.7	9.3	15.2

Solution:

First of all we have to convert the equilibrium data to mole ratio:

$$\text{mole fraction of NH}_3 \text{ in gas phase, } y_{\text{NH}_3} = \frac{P_A}{P_T} = \frac{1.6}{101.3} = 0.0158$$

$$\text{mole ratio of NH}_3 \text{ in gas phase, } Y_{\text{NH}_3} = \frac{y_{\text{NH}_3}}{1 - y_{\text{NH}_3}} = \frac{0.0158}{1 - 0.0158} = 0.0160$$

The equilibrium data becomes:

X_{NH_3}	0.021	0.031	0.042	0.053	0.079	0.106	0.159
Y_{NH_3}	0.0160	0.0243	0.0337	0.0433	0.0708	0.1011	0.1765

$$\text{HOG} = \frac{\bar{G}_s}{KoG.a} = \frac{0.034}{0.081} = 0.419 \text{ m}$$

$$\text{NOG} = \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

The equilibrium data may be not linear relation, so that the integration should be solved by plotting or by Simpson's rule as follows:

1. Draw the equilibrium data:
2. Draw the operating line from two points:
(X_1, Y_1) and (X_2, Y_2)

$$Y_1 = \frac{y_1}{1 - y_1} = \frac{0.1}{1 - 0.1} = 0.11$$

$$Y_2 = \frac{y_2}{1 - y_2} = \frac{0.001}{1 - 0.001} = 0.001$$

Overall ammonia material balance:

$$\bar{G}_s (Y_1 - Y_2) = \bar{L}_s (X_1 - X_2)$$

$$X_1 = \frac{\bar{G}_s}{\bar{L}_s} (Y_1 - Y_2) + X_2 = \frac{0.034}{0.036} (0.11 - 0.001) + 0$$

$$X_1 = 0.0935$$

Operating line:

$$(X_1, Y_1) = (0.0935, 0.11) = (9.35 \times 10^{-2}, 10 \times 10^{-2})$$

$$(X_2, Y_2) = (0, 0.001) = (0, 0.1 \times 10^{-2})$$

We will solve the integration by Simpson's rule:

$$h = \frac{Y_1 - Y_2}{n}, \quad \text{We choose } n = 4$$

$$h = \frac{0.11 - 0.001}{4} = 0.02725$$

Calculate Y^* from the plot as follows:

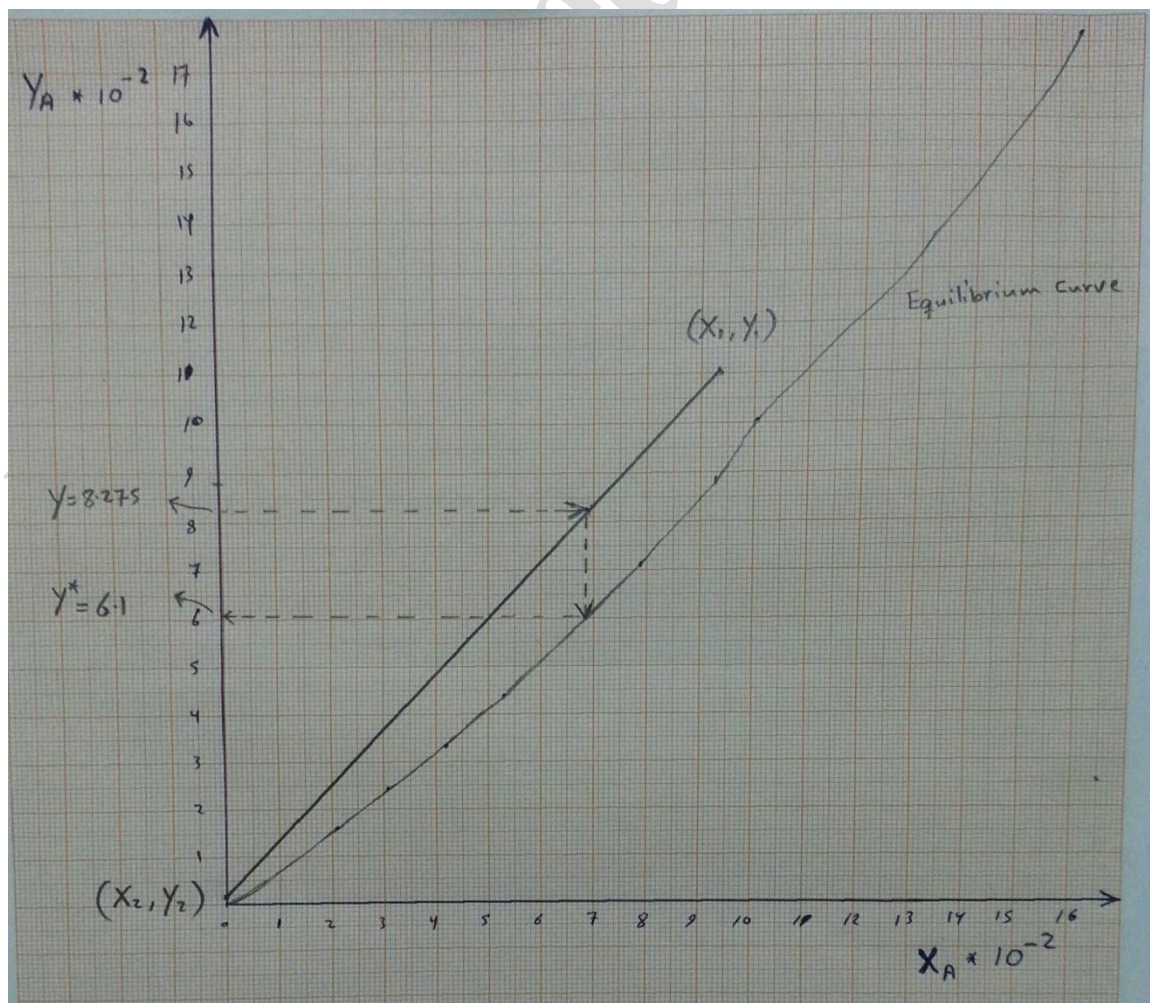
Y	Y*	$\frac{1}{(Y - Y^*)}$
Assume points between (Y ₁ - Y ₂)	Calculated from plot	
0.11	0.088	45.45 = f ₀
0.08275	0.061	45.98 = f ₁
0.05550	0.0375	55.56 = f ₂
0.02825	0.0175	93.02 = f ₃
0.001	0.00	1000 = f _n

$$NOG = \frac{h}{3} \left[f_0 + f_n + 2 \sum f_{\text{even}} + 4 \sum f_{\text{odd}} \right]$$

$$NOG = \frac{0.02725}{3} [45.45 + 1000 + 2(55.56) + 4[(45.98) + (93.02)]]$$

$$NOG = 15.56$$

$$Z = HOG * NOG = (0.419) (15.56) = 6.52 \text{ m}$$



Calculation of Minimum Liquid Flow Rate:

The minimum liquid (solvent) flow rate is calculated when the exit solvent concentration from the absorber (X_1) is *in equilibrium* with the entering gas concentration to the absorber (Y_1). However, this calculations based on the equilibrium relationship natural:

A. If the equilibrium relationship is linear ($Y^* = m X$):

The exit solvent concentration from the absorber (X_1) is calculated from the equilibrium relationship as below:

$$Y_1 = m X_1$$

$$\rightarrow \boxed{X_1 = \frac{Y_1}{m}} \dots \dots \dots (1)$$

Overall solute material balance on the absorber column:

$$G_s (Y_1 - Y_2) = L_s (X_1 - X_2)$$

$$\frac{L_s}{G_s} = \frac{Y_1 - Y_2}{X_1 - X_2}$$

For pure solvent ($X_2 = 0$):

$$\boxed{\frac{L_s}{G_s} = \frac{Y_1 - Y_2}{X_1}} \dots \dots \dots (2)$$

To calculate minimum liquid flow rate $\left[\left(\frac{L_s}{G_s} \right)_{\min} \right]$ we substitute Eq. (1) into Eq. (2):

$$\left(\frac{L_s}{G_s} \right)_{\min} = \frac{Y_1 - Y_2}{\frac{Y_1}{m}} = m \frac{Y_1 - Y_2}{Y_1} = m \left(1 - \frac{Y_2}{Y_1} \right)$$

$$\boxed{\left(\frac{L_s}{G_s} \right)_{\min} = m \left(1 - \frac{Y_2}{Y_1} \right)}$$

Where:

$$\boxed{\left(\frac{L_s}{G_s} \right)_{\text{actual}} = (1.1 - 1.5) \left(\frac{L_s}{G_s} \right)_{\min}}$$

B. If the equilibrium relationship is non-linear:

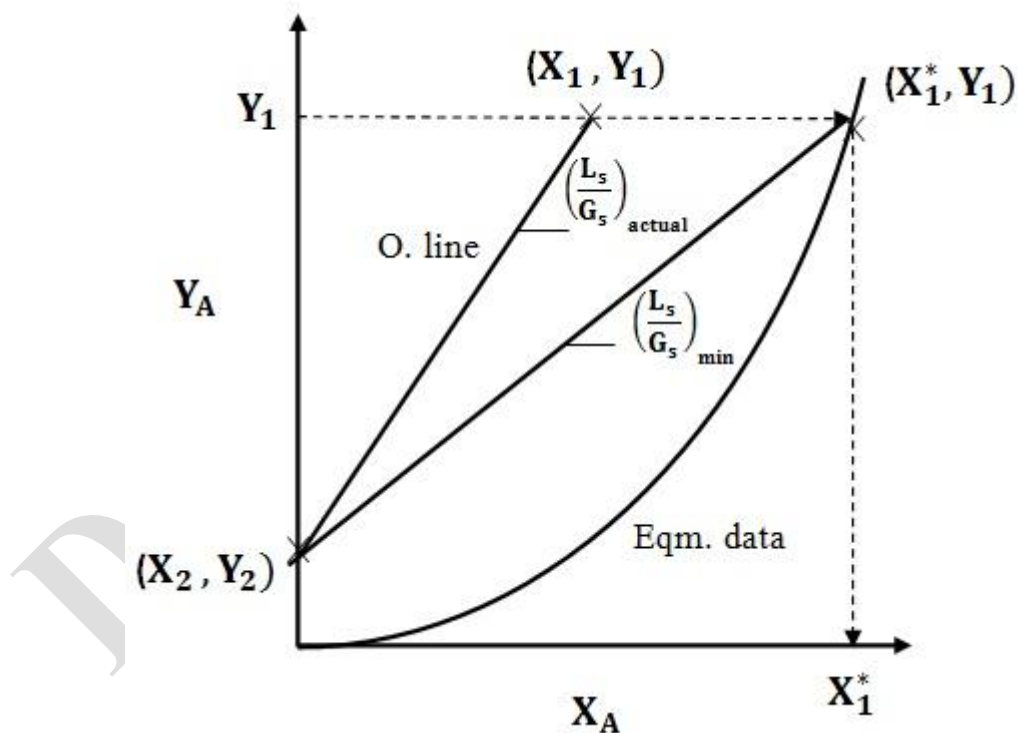
The exit solvent concentration from the absorber (X_1) is calculated from the equilibrium relationship as below:

$$\left(\frac{L_s}{G_s}\right)_{\min} = \frac{Y_1 - Y_2}{X_1^* - X_2}$$

For pure solvent ($X_2 = 0$):

$$\left(\frac{L_s}{G_s}\right)_{\min} = \frac{Y_1 - Y_2}{X_1^*}$$

Where: X_1^* the exit liquid concentration which is in equilibrium with (Y_1) is calculated from the plot as show bellow:



Example (3):

A solute gas is absorbed from a dilute gas-air mixture by counter current scrubbing with a solvent in a packed tower. The equilibrium relation is $Y = m X$. Show that the number of transfer units (**NOG**) required is given by the following equation:

$$\text{NOG} = \frac{1}{\ln \left(\frac{1 - Y_2}{1 - Y_1} \right)}$$

If (99%) of the solute is to be recovered using a liquid rate of 1.75 times the minimum and the height of transfer unit is (1 m). What the height of packing will be required.

Solution:

$$Z = \text{HOG} * \text{NOG}$$

For linear equilibrium relationship:

$$\left(\frac{L_s}{G_s} \right)_{\min} = m \left(1 - \frac{Y_2}{Y_1} \right)$$

$$Y_2 = (1 - \text{Recovery}) Y_1 = (1 - 0.99) Y_1 = 0.01 Y_1$$

$$Y_2 = 0.01 Y_1$$

$$\left(\frac{L_s}{G_s} \right)_{\min} = m \left(1 - \frac{0.01 Y_1}{Y_1} \right) = 0.99 m$$

$$\left(\frac{L_s}{G_s} \right)_{\text{actual}} = 1.75 \left(\frac{L_s}{G_s} \right)_{\min} = (1.75) (0.99 m) = 1.7325 m$$

$$\phi = \frac{m G_s}{L_s} = \frac{m}{1.7325 m} = 0.577$$

$$\text{NOG} = \frac{1}{\ln \left(\frac{1 - Y_2}{1 - Y_1} \right)}$$

$$\text{NOG} = \frac{1}{\ln \left(\frac{1 - 0.01}{1 - 0.99} \right)}$$

$$\text{NOG} = 8.88$$

$$Z = \text{HOG} * \text{NOG} = (1) (8.8) = 8.8 \text{ m}$$

Calculations of the packing height based on liquid phase:

Overall material balance on the solute (A) over an element (∂z) based on liquid phase:

$$\mathbf{G}_S \, d\mathbf{Y} = \mathbf{L}_S \, d\mathbf{X} = \mathbf{N}_A. \, A$$

$$\mathbf{N}_A = \mathbf{L}_s \left(\mathbf{X} + \frac{d\mathbf{X}}{dz} \partial z \right) - \mathbf{L}_s \mathbf{X} = (\mathbf{K} \mathbf{o} \mathbf{L})(\mathbf{a} \ \mathbf{S} \ \partial z)(\mathbf{X}^* - \mathbf{X})$$

Where:

The interfacial area for transfer = $a \, dV = a \, S \, \partial z$

S: is the cross-sectional area of column (m^2).

a: is the surface area of interface per unit volume of column (m^2/m^3).

$$\mathbf{L}_s \left(\frac{d\mathbf{X}}{dz} \partial \mathbf{z} \right) = (\mathbf{K} \mathbf{o} \mathbf{L} . \mathbf{a}) (\mathbf{S} . \partial \mathbf{z}) (\mathbf{X}^* - \mathbf{X})$$

$$L_s \frac{dX}{dz} = (K_o L_a)(S \cdot \partial z)(X^* - X)$$

$$\int_0^Z dZ = \frac{L_s}{(KoL \cdot a) \cdot S} \int_{x_2}^{x_1} \frac{dX}{(X^* - X)}$$

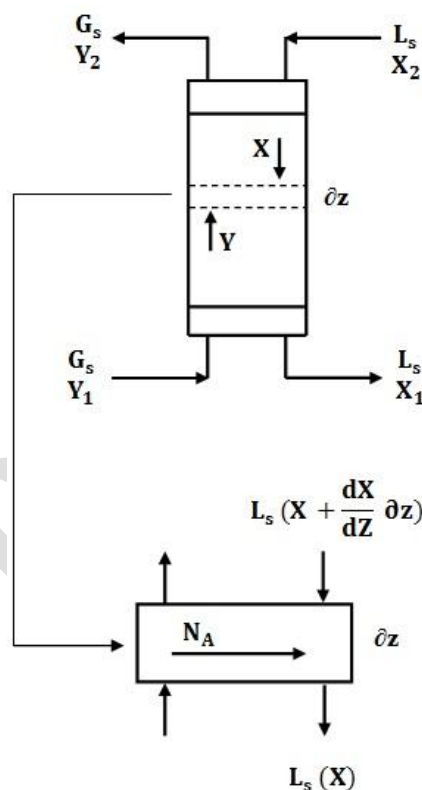
$$Z = \frac{(L_s/S)}{KoL \cdot a} \int_{X_2}^{X_1} \frac{dX}{(X^* - X)}$$

$$Z = \frac{\bar{L}_s}{KoL.a} \int_{x_2}^{x_1} \frac{dX}{(X^* - X)}$$

$$\mathbf{Z} = \mathbf{HOL} * \mathbf{NOL} = \mathbf{HTU} * \mathbf{NTU}$$

Where:

$$\mathbf{HOL} = \frac{\bar{L}_s}{K_{OL} a} : \text{height of transfer unit (HTU) based on liquid phase, with the units of (m).}$$



$$\text{NOL} = \int_{X_2}^{X_1} \frac{dX}{(X^* - X)} : \text{number of transfer unit (NTU) based on liquid phase, without units.}$$

Calculation of Number of Transfer Unit (NOL):

A. For Linear Equilibrium Relationship ($Y = m X^*$):

$$\text{NOL} = \int_{X_2}^{X_1} \frac{dX}{(X^* - X)} \dots \dots \dots (1)$$

$$X^* = \frac{Y}{m} \dots \dots \dots (2)$$

$$G_s (Y - Y_2) = L_s (X - X_2) \dots \dots \dots (3)$$

$$\Rightarrow Y = \frac{L_s}{G_s} (X - X_2) + Y_2$$

For pure liquid solvent used then, $X_2 = 0$

$$Y = \frac{L_s}{G_s} X + Y_2 \dots \dots \dots (4)$$

Substitution Eq.(4) into Eq.(2) to get:

$$X^* = \frac{L_s}{m G_s} X + \frac{Y_2}{m} = \frac{1}{\phi} X + \frac{Y_2}{m} \dots \dots \dots (5)$$

Substitution Eq.(5) into Eq.(1) to get:

$$\boxed{\text{NOL} = \frac{1}{1 - \phi} \ln \left(\frac{X_1 - X_2}{X_1 - X_2^*} \right)}$$

$$\text{NOL} = \phi \text{ NOG}$$

$$\text{Where: } \phi = \frac{m G_s}{L_s}$$

$$\text{NOL} = \frac{1}{1 - \phi} \ln \left(\frac{X_1 - X_2}{X_1 - X_2^*} \right)$$

B. For Non-linear Equilibrium Relationship:

In this case the integration $[\text{NOL} = \int_{X_2}^{X_1} \frac{dX}{(X^* - X)}]$ will be solved using graphical method or numerical method (Simpson rule) following steps below:

1. Draw the given equilibrium data.
2. Draw the operating line, from two points (X_1, Y_1) and (X_2, Y_2) or one point and slope of $(\frac{L_s}{G_s})$.
3. Create the table below by calculated (X^*) from the plot as below:

X Assume points between $(X_1 - X_2)$	X^* Calculated from plot	$\frac{1}{(X^* - X)}$
X_1	- calculated	$\sqrt{= f_0}$
- (assumed)	- calculated	$\sqrt{= f_1}$
- (assumed)	- calculated	$\sqrt{= f_2}$
- (assumed)	- calculated	$\sqrt{= f_3}$
X_2	- calculated	$\sqrt{= f_n}$

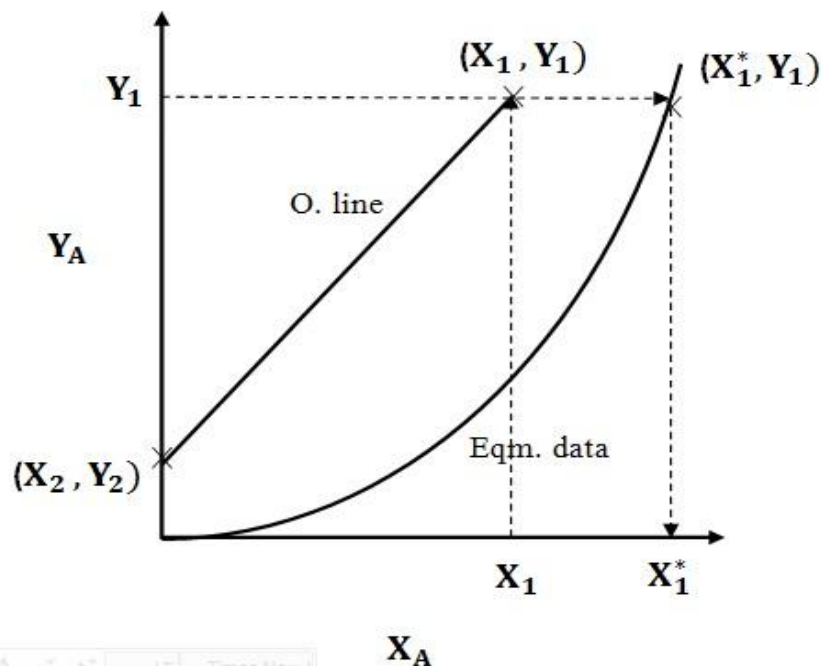
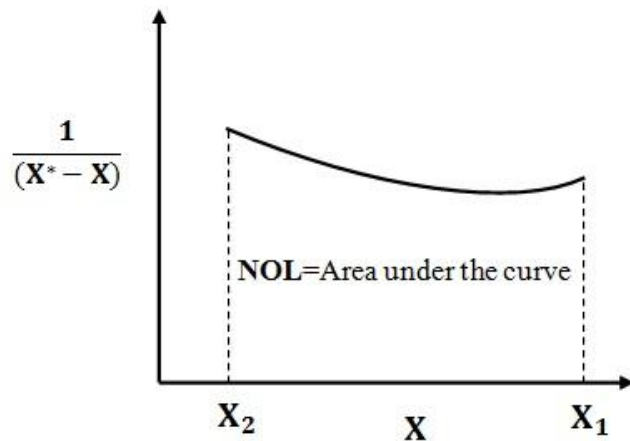


Figure: Calculation of (X^*) for packed column.

4. To calculate **NOL** we draw $\left[\frac{1}{(X^*-X)}\right]$ Vs. $[X]$ to find the area under the curve:

Where:

NOL = Area under the curve



Simpson rule for calculation of NOL:

NOL = Area under the curve

$$\text{NOL} = \frac{h}{3} \left[f_0 + f_n + 2 \sum f_{\text{even}} + 4 \sum f_{\text{odd}} \right]$$

E-operator

يستفاد من هذا الموضوع الرياضي لاشتقاق العلاقات الرياضية التي من خلالها يتم حساب عدد المراحل (الصواني) في الأبراج ذات الصواني وكما يلي:

$$Y_{n+1} = E Y_n$$

$$Y_{n+2} = E^2 Y_n$$

$$Y_{n-1} = E^{-1} Y_n$$

$$Y_{n-2} = E^{-2} Y_n$$

عند عمل موازنة مادة على البرج ذو الصواني سوف يظهر تركيز المذاب على الصواني بالشكل التالي:

$[Y_n, Y_{n-1}, X_{n+1}, X_n]$ ولكي نوحّد هذه المتغيرات على مرحلة واحدة (n) فسوف نستخدم المعامل (E-operator)

والذي بدوره سوف يساعدنا على تحويل المعادلة بدلالة متغير واحد (Y_n). ثم لإيجاد الحل لتلك المعادلة (أي إيجاد جذور

المعادلة) يتم استبدال (E) بمتغير جديد هو (ρ) ومن ثم نقوم بالتحليل لإيجاد جذور المعادلة فإذا كانت الجذور مختلفة

فسكون الحل كالتالي:

$$Y_n = c_1 \rho_1^n + c_2 \rho_2^n$$

Where:

ρ_1, ρ_2 : are roots of equation.

Y_n : is the concentration (mole ratio) of solute on tray (n).

n : is the number of trays.

c_1, c_2 : are equation constants.

To find the equation constants we will use the boundary conditions at:

$$n = 0$$

$$n = 1$$

Then we will have two equations, we can solve them simultaneously to find c_1 and c_2 .

2. Tray or plate tower:

The plate column is a common type of absorption equipment for large installations. Bubble-cap columns or sieve trays are sometimes used for gas absorption, particularly when the load is more than can be handled in a packed tower of about 1 m diameter and when there is any probability of deposition of solids which would quickly choke a packing. Plate towers are particularly useful when the liquid rate is sufficient to flood a packed tower. Phase equilibrium is assumed to be achieved at each tray between the vapor and liquid streams leaving the tray. That is, each tray is treated as equilibrium stage. Assume that the only component transferred from one phase to the other is solute A.

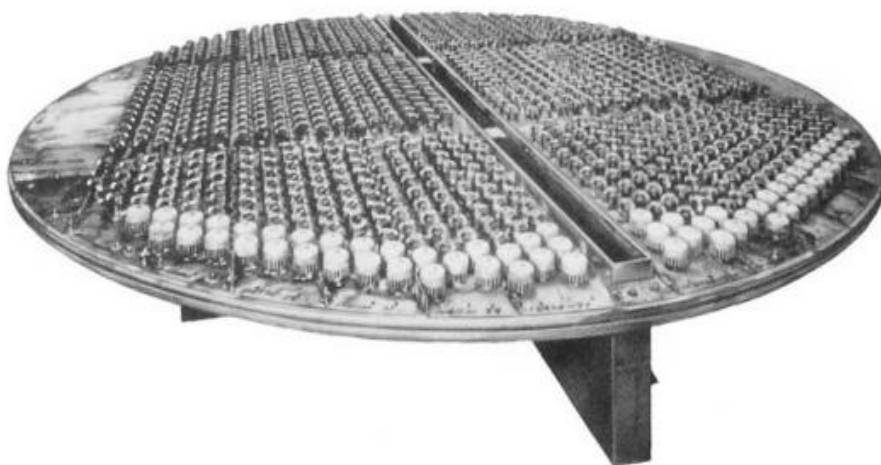


Figure 11.51: Bubble-cap tray.

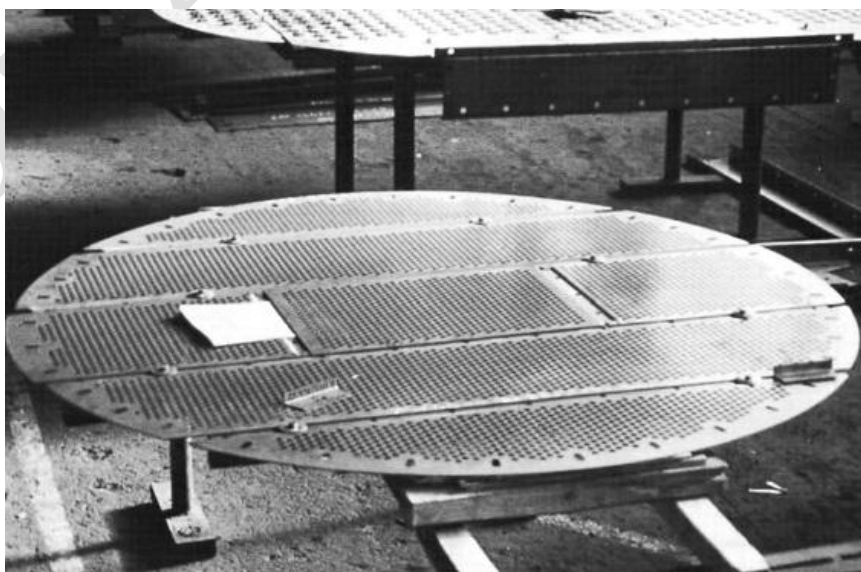


Figure 11.52: A perforated or sieve tray.

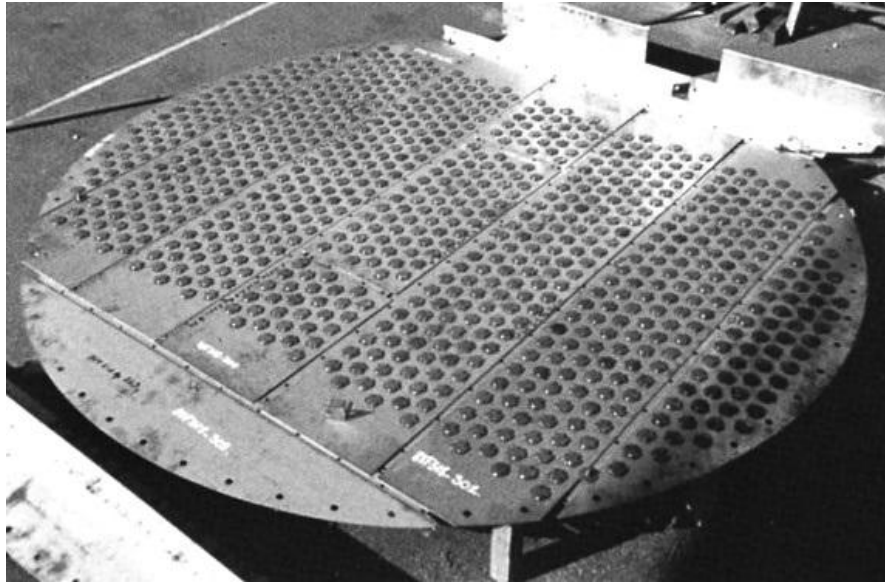


Figure 11.53: A bubble tray.

The height of tray tower can be obtained by using the following equation:

$$Z = H * N$$

Where:

H : is the distance between two trays, and it is given (0.3 - 0.7 m) and usually used (0.5 m).

N : is the number of trays, and it can be calculated based on equilibrium data.

Calculation of Number of theoretical Trays (N):

A. For Linear Equilibrium Relationship ($Y = m X$):

Solute material balance over tray (n):

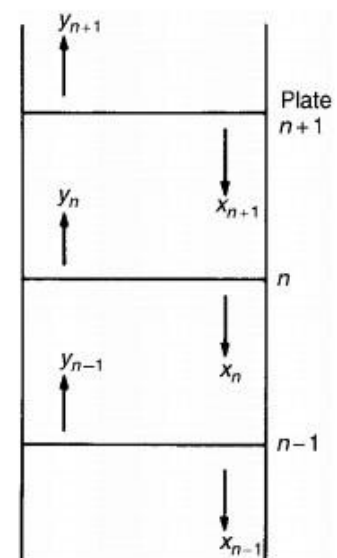
$$G_s Y_{n-1} + L_s X_{n+1} = G_s Y_n + L_s X_n \quad \dots\dots\dots(1)$$

The equilibrium relation is:

$$Y = m X \quad \dots\dots\dots(2)$$

Substitute Eq.(2) in to Eq.(1) to get:

$$G_s Y_{n-1} + \frac{L_s}{m} Y_{n+1} = G_s Y_n + \frac{L_s}{m} Y_n$$



$$G_s Y_{n-1} + \frac{L_s}{m} Y_{n+1} = \left(G_s + \frac{L_s}{m} \right) Y_n$$

$$Y_{n+1} - \left(\frac{m G_s}{L_s} + 1 \right) Y_n + \frac{m G_s}{L_s} Y_{n-1} = 0$$

Where:

$$\frac{m G_s}{L_s} = \phi$$

$$Y_{n+1} - (1 + \phi) Y_n + \phi Y_{n-1} = 0$$

By using E-operator:

$$E Y_n - (1 + \phi) Y_n + \phi E^{-1} Y_n = 0 \quad \dots \dots \dots (3) \quad \text{multiply by (E)}$$

$$(E^2 - (1 + \phi) E + \phi) Y_n = 0$$

Change (E) symbol by (ρ):

$$\rho^2 - (1 + \phi) \rho + \phi = 0$$

$$(\rho - 1)(\rho - \phi) = 0$$

The equation roots are:

$$\rho_1 = 1 \quad \text{and} \quad \rho_2 = \phi$$

The general solution is:

$$Y_n = c_1 \rho_1^n + c_2 \rho_2^n$$

Substitute the equation roots in to the general solution to get:

$$Y_n = c_1 + c_2 \phi^n$$

$$n = \frac{\ln \left[\frac{Y_n - c_1}{c_2} \right]}{\ln \phi}$$

To find the total number of trays, we substitute (n) by (N) to get:

$$N = \frac{\ln \left[\frac{Y_N - c_1}{c_2} \right]}{\ln \phi}$$

To find the equation constants C_1 and C_2 we substitute the boundary conditions:

B. C. 1: at $n = 0$ \rightarrow $Y_n = Y_0$

B. C. 2: at $n = 1$ \rightarrow $Y_n = Y_1$ \rightarrow $Y_1 = m X_1$

$$Y_n = c_1 + c_2 \phi^n$$

B. C. 1: $Y_0 = c_1 + c_2 \phi^0$ \rightarrow $Y_0 = c_1 + c_2$ (1)

B. C. 2: $Y_1 = c_1 + c_2 \phi^1$ \rightarrow $m X_1 = c_1 + c_2 \phi$ (2)

From Eq.(1) and Eq.(2) we get:

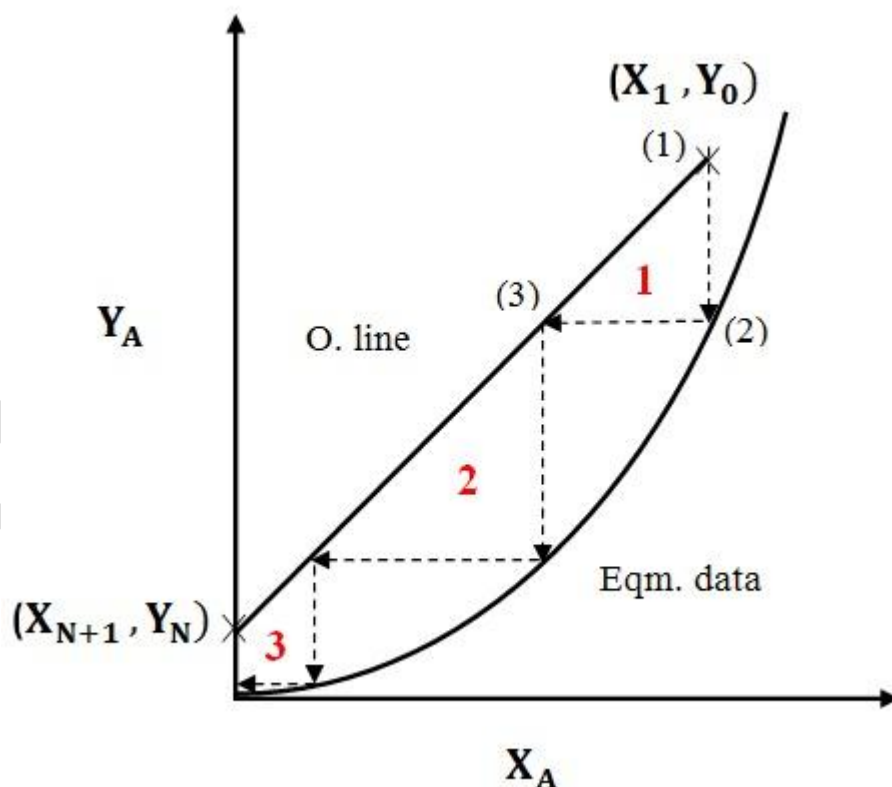
$$c_2 = \frac{Y_0 - m X_1}{1 - \phi}$$

$$c_1 = Y_0 - c_2$$

B. For Non-linear Equilibrium Relationship (Graphical method):

In this case the number of theoretical plates will be calculated using graphical method following steps below:

1. Complete the material balance to calculate all the unknowns (all compositions and flow rates of the inlet and the outlet streams must be known).
2. Draw the equilibrium curve (or line) either from given data or from the equilibrium equation: $Y = m X$.
3. Draw the operating line, from two points (X_1, Y_0) and (X_{N+1}, Y_N) or one point and slope of $(\frac{L_s}{G_s})$ according to the condition of the process.
4. Draw a vertical line from **point 1** which represents the point (X_1, Y_0) {as shown in the figure} to **point 2** which will intersect the equilibrium line (Curve). Then draw a horizontal line from **point 2** to **point 3**, intersecting the operating line. The triangular formed will represent the plate number one.
5. Continue drawing the vertical lines and horizontal lines as in step 4 (shown in the fig.) until we reach to the point (X_{N+1}, Y_N) or pass it.
6. Count the triangles constructed, this number represents the number of theoretical plates.



Column efficiency:

The number of ideal stages required for a desired separation may be calculated by one of the methods discussed previously, although in practice more trays are required than ideal stages. There are two types of efficiency usually used:

1. Overall column efficiency (E_c):

$$\text{overall column efficiency } (E_c) = \frac{\text{The theoritical number plates}}{\text{The actual number plates}} = \frac{N_{th}}{N_{act}}$$

$$E_c = \frac{N_{th}}{N_{act}}$$

Where: $N_{act} > N_{th}$

and: $N_{act} = N_{th}$ if $E_c = 100\%$

$$Z = N_{act} * \text{tray spacing}$$

Where: $Z_{act} > Z_{th}$

2. Plate efficiency (E_m):

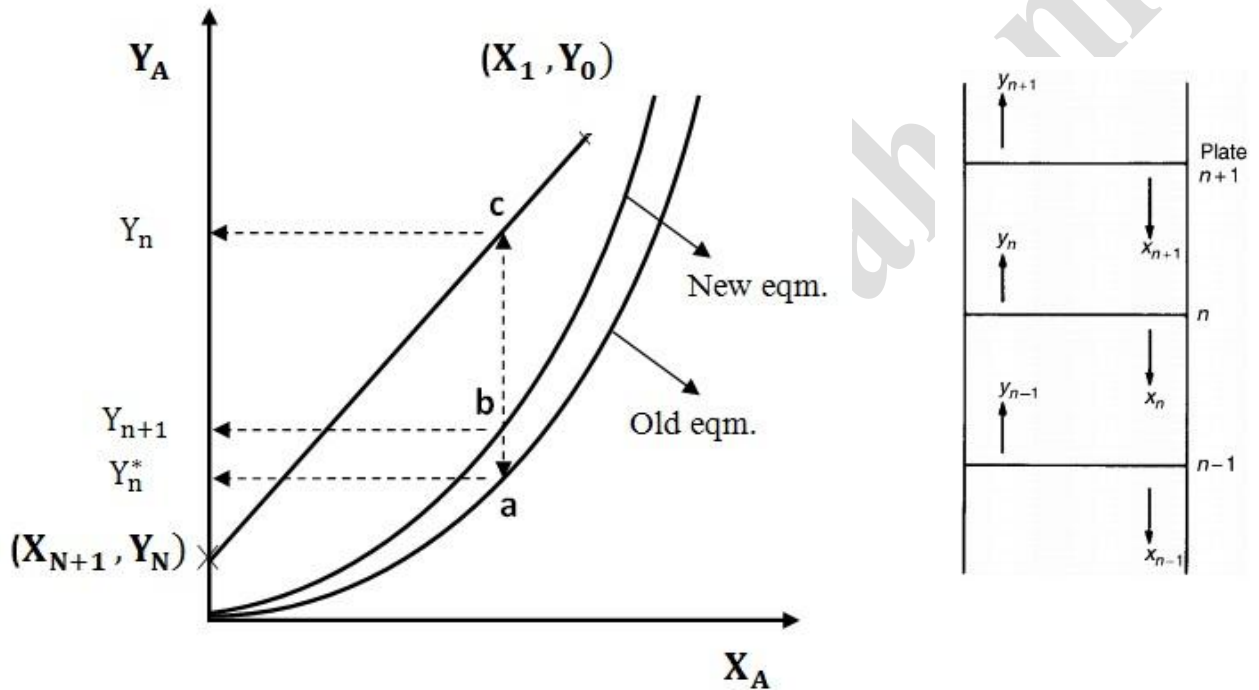
The proportion of liquid and vapour, and the physical properties of the mixtures on the trays, will vary up the column, and conditions on individual trays must be examined, as suggested by *Murphree* (1925). For a single ideal tray, the vapour leaving is in equilibrium with the liquid leaving, and the ratio of the actual change in composition achieved to that which would occur if equilibrium between Y_n and X_n were attained is known as the **Murphree plate efficiency (E_m)**. The plate efficiency can be expressed in terms of gas and liquid as given below:

a. Plate efficiency based on gas phase (E_{mv}):

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n - Y_n^*} = \frac{\overline{bc}}{\overline{ac}}$$

Where:

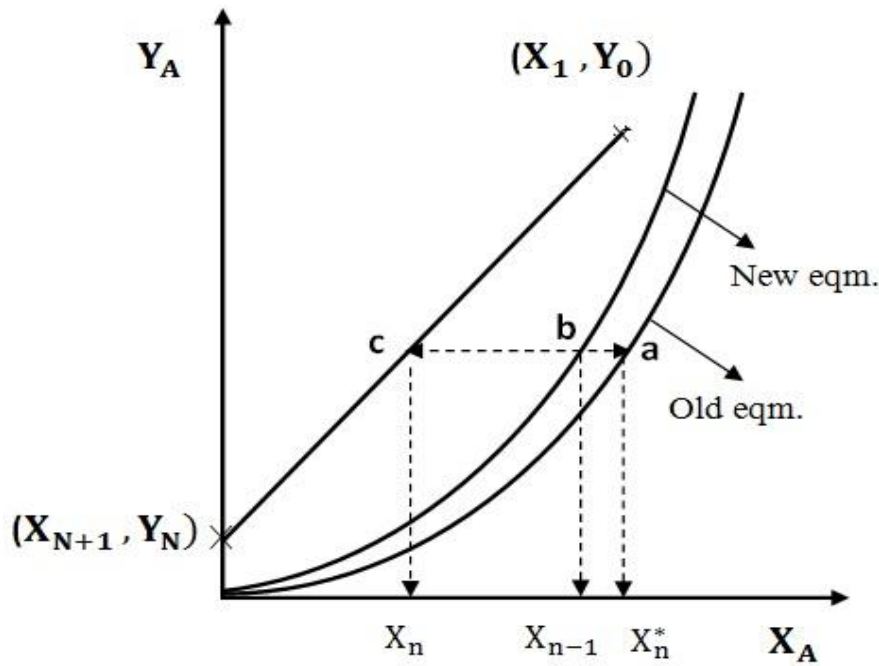
Y_n^* : is the composition of the gas that would be in equilibrium with the liquid of composition X_n actually leaving the plate.

**b. Plate efficiency based on liquid phase (E_{ml}):**

$$E_{ml} = \frac{X_n - X_{n-1}}{X_n - X_n^*} = \frac{\overline{bc}}{\overline{ac}}$$

Where:

X_n^* : is the composition of the liquid that would be in equilibrium with the gas of composition Y_n actually leaving the plate.



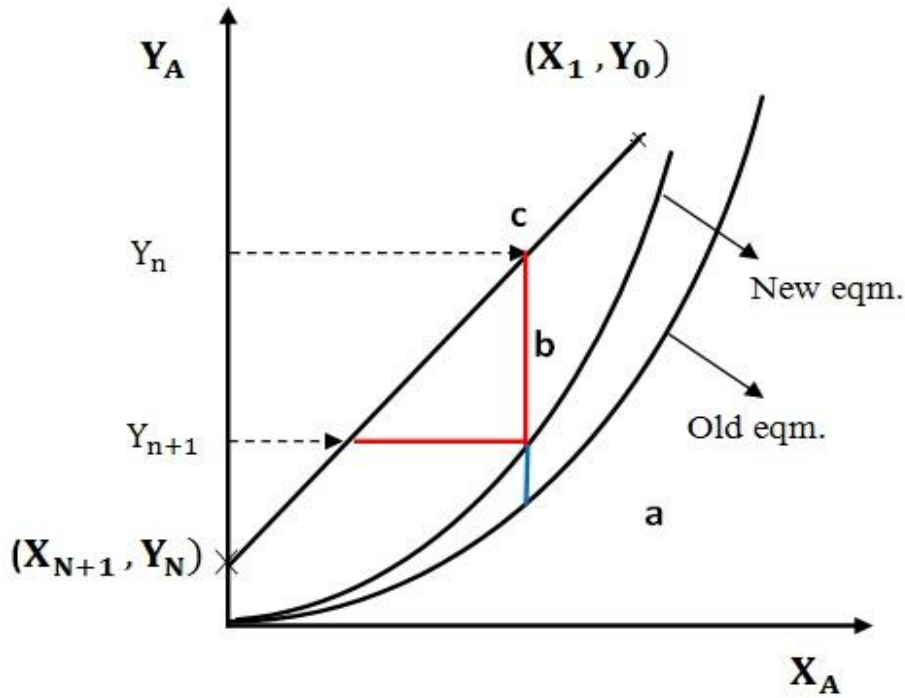
لايجاد عدد الصواني الحقيقي (N_{act}) فيما لو اعطيت كفاءة الصينية المعتمدة على البخار (E_{mv}) او السائل (E_{ml}) نتبع الخطوات التالية:

1. يتم القياس بالمسطرة المسافة بين منحنى التعادل (a) وخط التشغيل (c) وتمثل هذه المسافة (ac).
2. نجد المسافة (bc) باستخدام معادلة كفاءة الصينية (E_{mv}) او (E_{ml}).
3. تعاد هذه العملية لخمس نقاط لكي يتم ايجاد منجني التعادل الجديد.
4. عدد الصواني الحقيقي (N_{act}) يتم ايجاده بالتسقيط بين خط التشغيل ومنحنى التعادل الجديد، في حين عدد الصواني النظري (N_{th}) يتم ايجاده بالتسقيط بين خط التشغيل ومنحنى التعادل القديم.

ملاحظات مهمة جدا:

1. اذا اعطي في السؤال معلومات التعادل وكذلك تركيز المذاب على صينيتين متجلورتين فلكي يتم حساب كفاءة الصينية نتبع الطريقة التالية:

نرسم معلومات التعادل وكذلك خط التشغيل ثم نحدد تركيز المذاب المعطى على الصينيتين المتجاورتين على محور (Y-axis) (اذا كان التركيز المعطى بالنسبة للغاز). بعد ذلك نسقط هذه التراكيز على خط التشغيل ونرسم المثلث كما موضح ادناه فالنقطة التي تمثل راس المثلث هي نقطة على منحنى التعادل الجديد وعند ذاك نطبق علاقة كفاءة الصينية بقياس المسافات.



2. إذا لم تعطى معلومات التعادل في السؤال واعطيت كفاءة الصينية (E_{mv}) فلكي يتم حساب ارتفاع برج الامتصاص نستخدم علاقة كفاءة الصينية التالية:

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n - Y_n^*}$$

وبما أن معلومات التعادل لم تعطى في السؤال هذا يعني السائل المستخدم شديد الامتصاص أي أن ميل علاقة التعادل = صفر، أي لا يوجد منحنى تعادل وهذا يعني أن (Y_n^{*} = 0) فيتم التعويض بالمعادلة السابقة وتبسيط المعادلة باستخدام (E-operator) لإيجاد العلاقة بين (Y_n) و (n) وبالتعويض عن التركيز النهائي نجد عدد الصواني الحقيقية ثم بعد ذلك نجد ارتفاع البرج.

3. إذا لم تعطى كفاءة الصينية في السؤال فيتم فرضها على إنها (E_{mv}).

Calculation of the Height Equivalent of a Theoretical Plate (HETP):

The height of a theoretical plate (HETP), also called the height of an equivalent equilibrium stage, is the **height of packing** that will give the same separation as an equilibrium stage. The relationship between transfer units (HOG) and the height of an equivalent theoretical plate (HETP) is given by:

$$\text{HETP} = \text{tray spacing} = \text{HOG} * \frac{\ln \phi}{1 - \phi}$$

Where: $\phi = \frac{mG_s}{L_s}$

$$Z = N * \text{HETP}$$

Where: N = number of plates

* اذا لم يعطى في السؤال المسافة بين الصواني وكان المطلوب حساب ارتفاع البرج ذو الصواني ولم تعطى المسافة بين الصواني وهناك معلومات متوفرة يمكن من خلالها حساب (HOG) فيتم حساب (HETP) لاستخدامه في حساب ارتفاع البرج.

Relation between overall and film transfer units:

$$Z_{OG} = HOG * NOG = \frac{\bar{G}_s}{KoG \cdot a} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y^*)}$$

$$Z_{OL} = HOL * NOL = \frac{\bar{L}_s}{KoL \cdot a} \int_{X_2}^{X_1} \frac{dX}{(X^* - X)}$$

$$Z_g = Hg * Ng = \frac{\bar{G}_s}{Kg \cdot a} \int_{Y_2}^{Y_1} \frac{dY}{(Y - Y_i)}$$

$$Z_L = H_L * N_L = \frac{\bar{L}_s}{K_L \cdot a} \int_{X_2}^{X_1} \frac{dY}{(X_i - X)}$$

Where: $Z = Z_{OG} = Z_{OL} = Z_g = Z_L$

Example (1): Calculate the height of plate column with tray spacing of 0.51 m and plate efficiency based on gas phase 40% to reduce the concentration of NH_3 from 5.5 mol% to 0.1 mol% in an NH_3 -Air mixture using fresh water. The gas and liquid flow rates are 300 and 400 $\text{Kg/m}^2\cdot\text{hr}$, respectively, and the equilibrium relationship is such that the vapor pressure of NH_3 over the liquid is negligible.

Solution:

For plate tower: $Z = N * \text{tray spacing}$

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n - Y_n^*}$$

Since the vapor pressure of NH_3 over the liquid is negligible, then: **$m = 0$**

$$E_{mv} = \frac{Y_n - Y_{n+1}}{Y_n} \rightarrow 0.4 = \frac{Y_n - Y_{n+1}}{Y_n}$$

$$Y_{n+1} - 0.6 Y_n = 0$$

By using E-operator:

$$(E - 0.6)Y_n = 0 \rightarrow \rho - 0.6 = 0$$

$$\rho = 0.6$$

$$Y_n = C (\rho)^n = C (0.6)^n$$

Using the boundry condition:

$$n = 0 \rightarrow Y_n = Y_0 = 0.055$$

$$0.055 = C (0.6)^0$$

$$\rightarrow C = 0.055$$

$$Y_n = 0.055 (0.6)^n$$

$$0.001 = 0.055 (0.6)^N$$

$$N = 7.83 \approx 8$$

$Z = N * \text{tray spacing}$

$$Z = 7.83 * 0.51 = 4 \text{ m}$$

Example (2): A mixture of ammonia and air is scrubbed in a plate column with fresh water. If the ammonia concentration is reduced from 5% to 0.5% .Given that: $Y = 2 X$.

- Calculate the No. of theoretical plate and the tower height. Given that: $L = 0.65 \text{ Kg/m}^2.\text{s}$ and $G = 0.4 \text{ Kg/m}^2.\text{s}$, $KOG.a = 0.0008 \text{ Kmole/m}^3.\text{s.kPa}$
- Calculate the No. of theoretical plate, given that: $\left(\frac{L}{G}\right) = 2 \left(\frac{L}{G}\right)_{\min}$.
- Calculate $\left(\frac{L}{G}\right)$ if the actual No. of plates = 12, and the column efficiency = 0.5.
- Calculate the theoretical and actual No. of plates, give that:
 $\left(\frac{L}{G}\right) = 1.5 \left(\frac{L}{G}\right)_{\min}$ and $E_{mv} = 0.7$
- Given the concentration of a gas in the two adjacent plates are 4% and 3.3%. Calculate E_{mv} and E_{ml} if $L = 0.65 \text{ Kg/m}^2.\text{s}$ and $G = 0.4 \text{ Kg/m}^2.\text{s}$.

Solution:

Since the inlet gas concentration is 5% then no need to convert the mole fraction to mole ratio:

$$\bar{L}_s = \frac{0.65}{18} = 0.0361 \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

$$\bar{G}_s = \frac{0.4}{29} = 0.01379 \frac{\text{kmol}}{\text{m}^2.\text{s}}$$

Overall solute material balance on the tower:

$$\bar{G}_s (Y_1 - Y_2) = \bar{L}_s (X_1 - X_2)$$

$$0.01379 (0.05 - 0.005) = 0.0361 (X_1 - 0)$$

$$X_1 = 0.01718$$

To find the number of theoretical plates:

1. Plot the operating line:

$$(X_1, Y_1) = (0.01718, 0.05)$$

$$(X_2, Y_2) = (0, 0.005)$$

2. Plot the equilibrium relation ($Y = 2 X$):

From the figure below we can find the theoretical No. of plates by stepping off:

$N = 5$ Plates

The height of plate tower is:

$$Z = N * HETP$$

$$HOG = \frac{\bar{G}_s}{KOG.a.P_T} = \frac{0.01379}{(0.0008)(101.3)} = 0.17 \text{ m}$$

$$\phi = \frac{mG_s}{L_s} = \frac{2(0.01379)}{0.0361} = 0.7639$$

$$HETP = HOG * \left(\frac{\ln \phi}{1 - \phi} \right) = (0.17) * \left(\frac{\ln 0.7639}{1 - 0.7639} \right) = 0.19 \text{ m}$$

$$Z = (5) (0.19) = 0.97 \text{ m}$$

$$\text{b. } \left(\frac{L}{G} \right)_{\text{act}} = 2 \left(\frac{L}{G} \right)_{\text{min}}$$

For linear equilibrium relation:

$$\left(\frac{L}{G} \right)_{\text{min}} = m \left(1 - \frac{Y_2}{Y_1} \right) = 2 \left(1 - \frac{0.005}{0.05} \right) = 1.8$$

$$\left(\frac{L}{G} \right)_{\text{act}} = 2 \left(\frac{L}{G} \right)_{\text{min}} = 2 (1.8) = 3.6$$

Overall solute material balance on the tower:

$$\bar{G}_s (Y_1 - Y_2) = \bar{L}_s (X_1 - X_2)$$

$$X_1 = \frac{\bar{G}_s}{\bar{L}_s} (Y_1 - Y_2) = \frac{1}{3.6} (0.05 - 0.005)$$

$$X_1 = 0.0125$$

To find the number of theoretical plates:

1. Plot the operating line:

$$(X_1, Y_1) = (0.0125, 0.05)$$

$$(X_2, Y_2) = (0, 0.005)$$

2. Plot the equilibrium relation ($Y = 2X$):

From the figure below we can find the theoretical No. of plates by stepping off:

$N = 3$ Plates

c. Theoretical No. of plates = Actual No. of plates * overall column efficiency

$$N_{th} = N_{act} * E_c$$

$$N_{th} = 12 * 0.5 = 6 \text{ plates}$$

المعلوم في هذا السؤال هو عدد الصواني (N) والمطلوب هو $(\frac{L}{G})$ أي ميل خط التشغيل فسوف نقوم بالمحاولة والخطأ برسم خط التشغيل من النقطة :

$$(X_2, Y_2) = (0, 0.005)$$

بحيث يحقق عدد الصواني (6) عند $(Y_1 = 0.05)$. عند ايجاد خط التشغيل الصحيح نقوم بحساب ميله والذي يساوي $(\frac{L}{G})$:

From plot:

$$\frac{\bar{L}_s}{\bar{G}_s} = 2.23$$

$$d. \left(\frac{L_s}{G_s} \right)_{\min} = \frac{Y_1 - Y_2}{X_1^* - X_2}$$

From plot at $Y_1 = 0.05 \rightarrow X_1^* = 0.025$

$$\left(\frac{L_s}{G_s} \right)_{\min} = \frac{0.05 - 0.005}{0.025 - 0} = 1.8$$

$$\left(\frac{L_s}{G_s} \right)_{\text{act}} = 1.5 \left(\frac{L_s}{G_s} \right)_{\min} = 1.5(1.8) = 2.7$$

Overall solute material balance on the tower:

$$\bar{G}_s (Y_1 - Y_2) = \bar{L}_s (X_1 - X_2)$$

$$X_1 = \frac{\bar{G}_s}{\bar{L}_s} (Y_1 - Y_2) = \frac{1}{2.7} (0.05 - 0.005)$$

$$X_1 = 0.0166$$

To find the number of theoretical plates:

1. Plot the operating line:

$$(X_1, Y_1) = (0.0166, 0.05)$$

$$(X_2, Y_2) = (0, 0.005)$$

2. Plot the equilibrium relation ($Y = 2 X$):

From the figure below we can find the theoretical No. of plates by stepping off:

$$N = 4 \text{ Plates}$$

To find the actual No. of plates:

Plot the new equilibrium curve at ($E_{mv} = 0.7$), and stepping off:

The actual No. of plates = 5

e.

$$\bar{L}_s = \frac{0.65}{18} = 0.0361 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

$$\bar{G}_s = \frac{0.4}{29} = 0.01379 \frac{\text{kmol}}{\text{m}^2 \cdot \text{s}}$$

Overall solute material balance on the tower:

$$\bar{G}_s (Y_1 - Y_2) = \bar{L}_s (X_1 - X_2)$$

$$0.01379 (0.05 - 0.005) = 0.0361 (X_1 - 0)$$

$$X_1 = 0.01718$$

To find the number of theoretical plates:

1. Plot the operating line:

$$(X_1, Y_1) = (0.01718, 0.05)$$

$$(X_2, Y_2) = (0, 0.005)$$

2. Plot the equilibrium relation ($Y = 2 X$):

3. From the plot we find (E_{mv}) and (E_{mL}) at:

$$Y_N = 4\% \quad \text{and} \quad Y_{N+1} = 3\%$$

$$E_{mv} = \frac{ab}{ac} = \frac{19 \text{ mm}}{25 \text{ mm}} = 0.67$$

$$E_{ml} = \frac{ab}{ac} = \frac{6 \text{ mm}}{12 \text{ mm}} = 0.5$$

Calculation of column diameter:

a. Packed tower:

$$U_g = \frac{G * M.wt}{\rho_g * S} = \frac{G * M.wt}{\rho_g * \frac{\pi}{4} D^2}$$

$$U_g = f(D)$$

$$U_g \ll U_L \quad \text{Loading}$$

$$U_g \gg U_L \quad \text{flooding}$$

$$U_L < U_g < U_f$$

$$U_g = [0.7 - 0.9] U_f$$

The diameter of the tray tower can be estimated following the procedure below:

1. Calculate F_{LV} from the following equation:

$$F_{LV} = \frac{L}{G} \sqrt{\frac{\rho_g}{\rho_L}}$$

Where:

L = Liquid mass flow rate, $\frac{\text{kg}}{\text{s}}$.

G = Gas mass flow rate, $\frac{\text{kg}}{\text{s}}$.

2. Using Figure 11.44 to find K_4 with the used pressure drop:

$$K_4 = \frac{13.1 \bar{G}^2 F_p \left(\frac{\mu_L}{\rho_L} \right)^{0.1}}{\rho_g (\rho_L - \rho_g)}$$

Where:

\bar{G} = gas mass flux ($\text{kg/m}^2 \cdot \text{s}$).

F_p = packing factor, characteristic of the size and type of packing, see Table 11.3, m^{-1} .

μ_L = liquid viscosity, Ns/m^2 .

ρ_L, ρ_g = liquid and gas densities, $\frac{kg}{m^3}$.

$$\bar{G} = \frac{G * Mwt}{S}$$

Where:

G = gas mole rate (kmol/s).

S = cross section area (m^2).

Mwt = gas molecular weight.

$$S = \frac{G * Mwt}{\bar{G}} = \frac{\pi}{4} D^2$$

$$D = \sqrt{\frac{4 * Mwt * G}{\pi \bar{G}}} \quad (\text{meter})$$

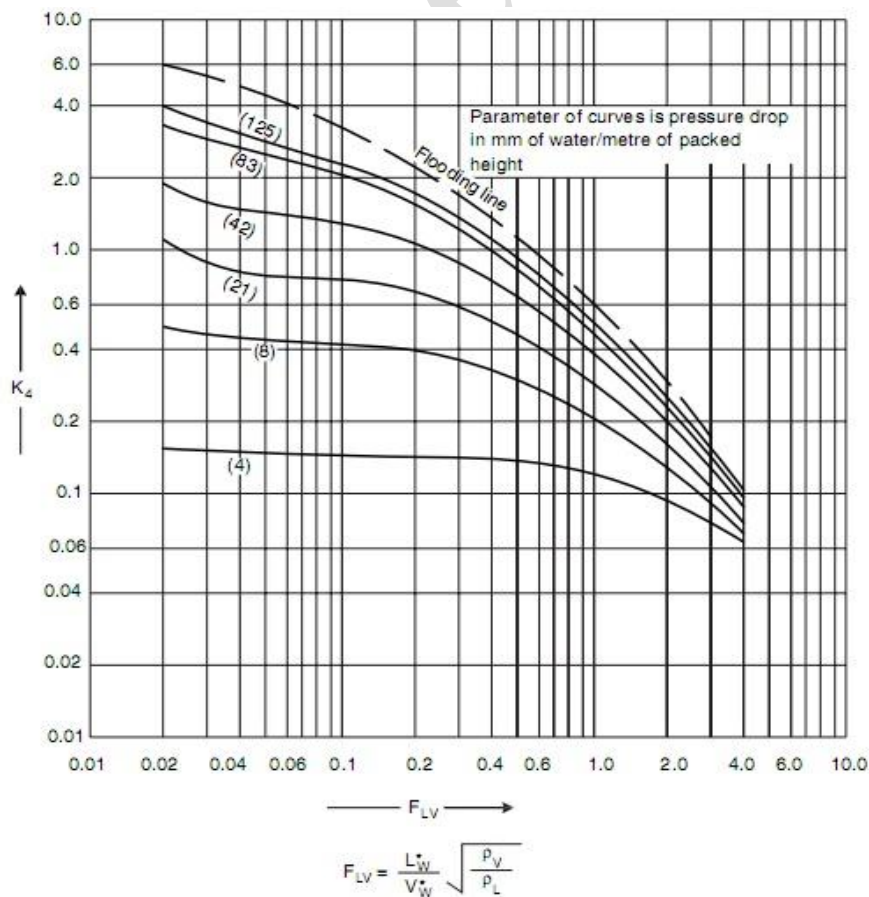


Figure 11.44. Generalised pressure drop correlation, adapted from a figure by the Norton Co, with permission

b. Tray tower:

The diameter of the tray tower can be estimated following the procedure below:

3. Calculate F_{LV} from the following equation:

$$F_{LV} = \frac{L}{G} \sqrt{\frac{\rho_g}{\rho_L}}$$

Where:

L = Liquid mass flow rate, $\frac{\text{kg}}{\text{s}}$.

G = Gas mass flow rate, $\frac{\text{kg}}{\text{s}}$.

4. Using Figure 11.27 to find K_1 with the used plate spacing:

The flooding velocity can be estimated from the correlation given by **Fair (1961)**:

$$U_f = K_1 \sqrt{\frac{\rho_L - \rho_g}{\rho_g}}$$

Where:

U_f = flooding velocity of vapour, $\frac{\text{m}}{\text{s}}$.

K_1 = a constant obtained from Figure 11.27 .

ρ_L, ρ_g = liquid and gas densities, $\frac{\text{kg}}{\text{m}^3}$.

5. Find the tower diameter:

$$U_{\text{act}} = [0.7 - 0.9] U_f$$

$$U_{\text{act}} = \frac{G * \text{M. wt}}{\rho_g * S} = \frac{G * \text{M. wt}}{\rho_g * \frac{\pi}{4} D^2}$$

$$D = \sqrt{\frac{4 G * \text{M. wt}}{\pi \rho_g * U_{\text{act}}}}$$

Where:

D = column diameter, m .

S = column cross section area, m^2 .

M. wt = gas molecular weight, $\frac{\text{kg}}{\text{kgmol}}$.

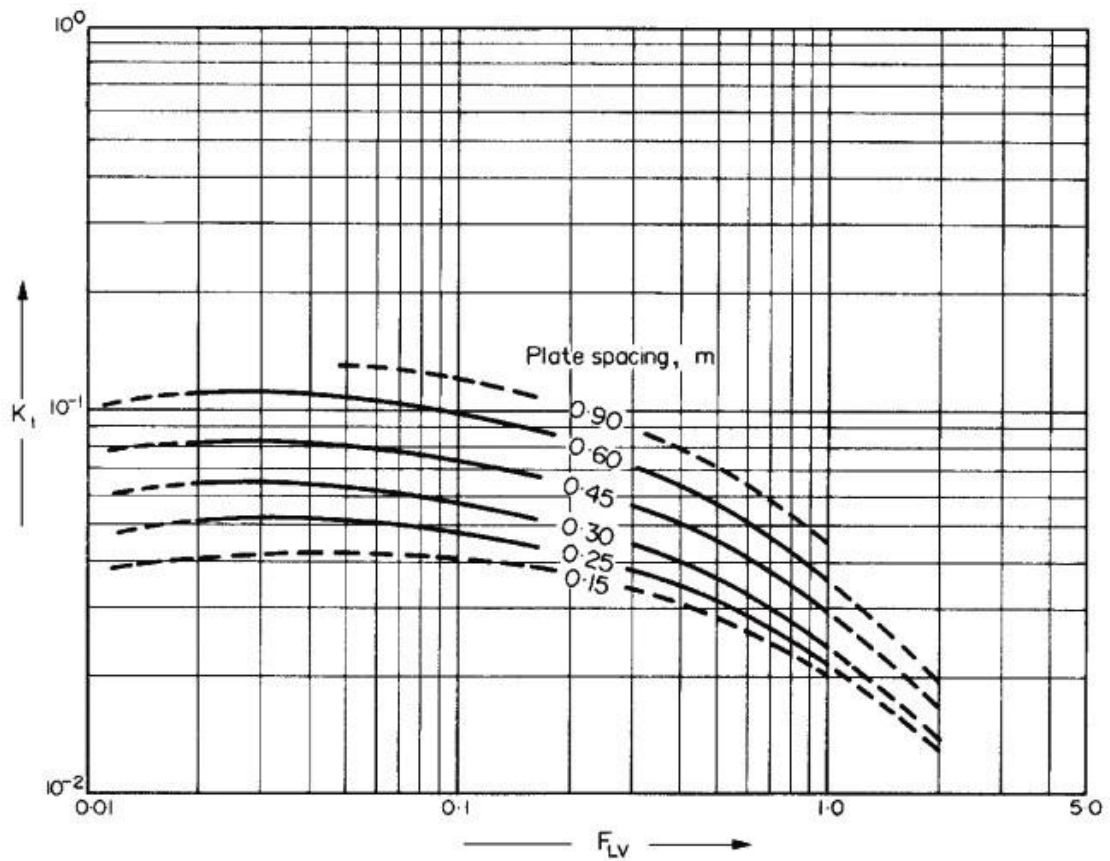


Figure 11.27. Flooding velocity, sieve plates

Choice of plates or packing:

The choice between a plate or packed column for a particular application can only be made with complete assurance by costing each design. However, this will not always be worthwhile, or necessary, and the choice can usually be made, on the basis of experience by considering main advantages and disadvantages of each type; which are listed below:

1. Plate columns can be designed to handle a wider range of liquid and gas flow-rates than packed columns.
2. Packed columns are not suitable for very low liquid rates.
3. The efficiency of a plate can be predicted with more certainty than the equivalent term for packing (HETP or HTU).
4. Plate columns can be designed with more assurance than packed columns. There is always some doubt that good liquid distribution can be maintained throughout a packed column under all operating conditions, particularly in large columns.
5. It is easier to make provision for cooling in a plate column; coils can be installed on the plates.
6. It is easier to make provision for the withdrawal of side-streams from plate columns.
7. If the liquid causes fouling, or contains solids, it is easier to make provision for cleaning in a plate column; manways can be installed on the plates. With small-diameter columns it may be cheaper to use packing and replace the packing when it becomes fouled.
8. For corrosive liquids a packed column will usually be cheaper than the equivalent plate column.
9. The liquid hold-up is appreciably lower in a packed column than a plate column. This can be important when the inventory of toxic or flammable liquids needs to be kept as small as possible for safety reasons.
10. Packed columns are more suitable for handling foaming systems.
11. The pressure drop per equilibrium stage (HETP) can be lower for packing than plates; and packing should be considered for vacuum columns.
12. Packing should always be considered for small diameter columns, say less than **0.6 m**, where plates would be difficult to install, and expensive.

Chapter (13) in Volume (2)

((Liquid–Liquid Extraction))

Dr. Ahmed Daham