

Chapter (10)

Solid-Liquid Extraction

((Leaching))

Leaching: is the separation of a solute from solid mixture by dissolving it in a liquid phase.

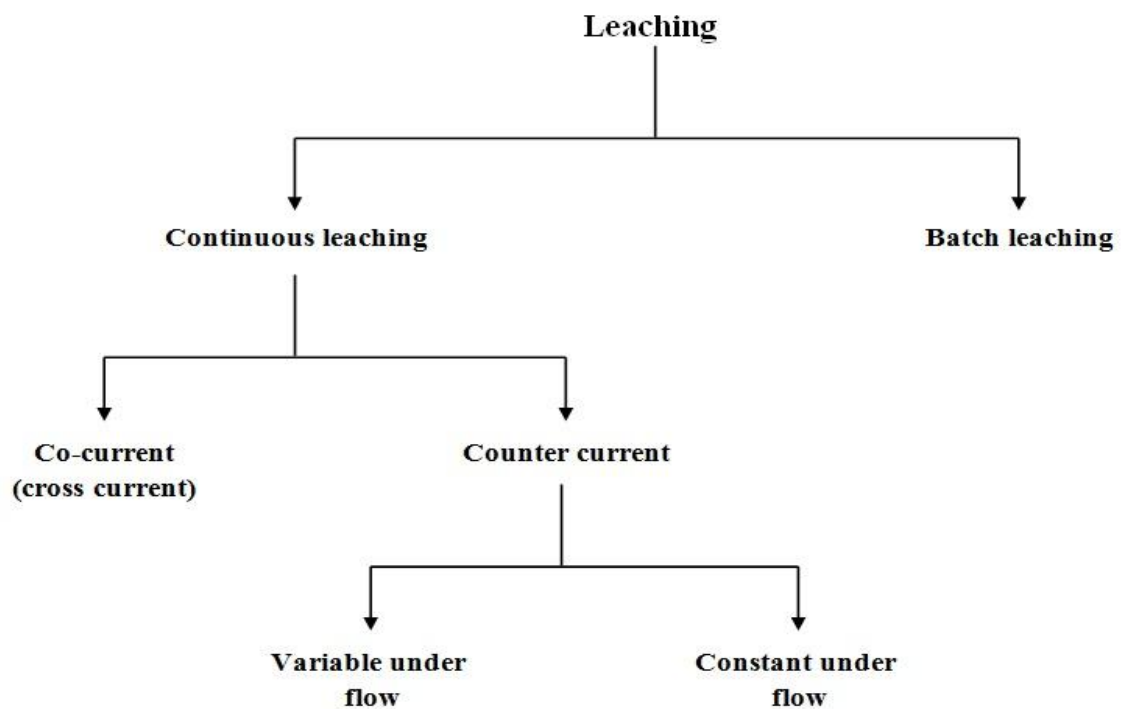
Leaching occurs in two steps:

1. Contacting solvent and solid to effect a transfer of a solute (leaching).
2. The separation of the solution from the remaining solid (washing).

Factors influencing the rate of extraction:

There are four important factors to be considered:

1. Particle size.
2. Solvent.
3. Temperature.
4. Agitation of the fluid.



Batch Leaching

Mass transfer rates within the porous residue are difficult to assess because it is impossible to define the shape of the channels through which transfer must take place. It is possible, however, to obtain an approximate indication of the rate of transfer from the particles to the bulk of the liquid. Using the concept of a thin film as providing the resistance to transfer, the equation for mass transfer may be written as:

(A) Leaching:

$$N_A = \frac{D_{AL}}{b} (C_S - C) = K_L (C_S - C)$$

$$\bar{N}_A = K_L \cdot A (C_S - C)$$

$$\bar{N}_A = \frac{\text{kmol}}{s} = \frac{dM}{dt}$$

$$\frac{dM}{dt} = K_L \cdot A (C_S - C)$$

$$M = C V$$

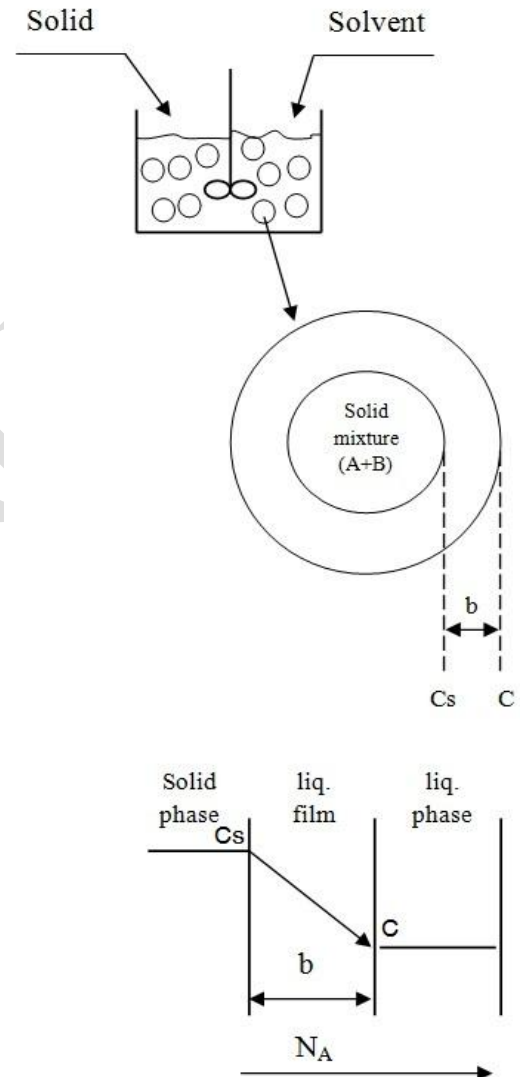
$$dM = V dC \quad (\text{Constant volume})$$

$$\int_0^C \frac{dC}{C_S - C} = \int_0^t \frac{K_L \cdot A}{V} dt$$

$$\ln \left[\frac{C_S - C}{C_S} \right] = \frac{-K_L \cdot A}{V} t$$

$$\frac{C_S - C}{C_S} = e^{\frac{-K_L \cdot A}{V} t}$$

$$C = C_S \left[1 - e^{\frac{-K_L \cdot A}{V} t} \right]$$



Where:

D_{AL} : is the diffusion coefficient in the liquid phase.

b : is the effective thickness of the liquid film surrounding the particles.

C : is the concentration of the solute in the bulk of the solution at time (t).

C_S : is the concentration of the saturated solution in contact with the particles.

M : is the mass of solute transferred in time (t).

V : is the volume of the solution.

K_L : is the mass transfer coefficient in the liquid phase.

(B) Number of Washing:

$$\theta_N = \frac{1}{(1 + \beta)^N}$$

$$\beta = \frac{b}{a}$$

Where:

θ_N : is the fraction of solute remain with the residue from the original.

β : Solvent decanted per solvent remaining in the insoluble solid.

a : is the solvent remaining.

b : is the solvent decanted.

$$\theta_N = \frac{S_N}{S}$$

S_N : is the weight of solute remaining in the solid after washing.

S : is the original weight of the solute which was in the solid before washing.

Example (1): 500 kg of the inert solid containing 28 percent by mass of the water-soluble component (A), is agitated with 100 m³ of water for 600 sec. After each decanting 25% of the solution produced remain in the residue. Water is saturated with the solute at a concentration of 2.5 kg/m³. Find the concentration of the solute (A) in the solution after the leaching and number of washing such that the concentration of A in the solid residue is 0.01% by mass. In a pilot scale test using a vessel 1m³ in volume, a solute was leached from an inert solid and the water was 75 percent saturated in 10 s. Assuming conditions are equivalent to those in the pilot scale vessel.

Solution:

For the pilot scale vessel:

$$V = 1 \text{ m}^3, \quad t = 10 \text{ sec}, \quad C_s = 2.5 \text{ kg/m}^3 \quad \text{and} \quad C = 0.75 C_s$$

$$C = C_s \left[1 - e^{\frac{-K_L \cdot A}{V} t} \right] \implies 0.75 C_s = C_s \left[1 - e^{\frac{-K_L \cdot A}{(1)} (10)} \right]$$

$$K_L \cdot A = 0.139$$

For the full scale vessel:

$$V = 100 \text{ m}^3, \quad t = ?,$$

$$C = C_S \left[1 - e^{\frac{-K_L \cdot A}{V} t} \right] \implies C = 2.5 \left[1 - e^{\frac{-0.139}{(100)} (600)} \right]$$

$$C = 1.4 \text{ kg/m}^3$$

The initial amount of the solute in the solid = (500) (0.28) = 140 kg

$$\text{The maximum conc. of the solute that can be removed} = \frac{\text{mass}}{\text{volume}} = \frac{140}{100} = 1.4 \text{ kg/m}^3$$

That's mean that the leaching process is efficient.

Number of washing:

$$\theta_N = \frac{1}{(1 + \beta)^N}$$

$$\text{Final conc.} = 0.0001 = \frac{S_N}{B + S_N}, \quad \text{dry basis}$$

$$B = (500) (1 - 0.28) = 360 \text{ kg} \implies 0.0001 = \frac{S_N}{360 + S_N}$$

$$S_N = 0.036 \text{ kg}$$

$$S = (0.25) (100) (1.4) = 35 \text{ kg}$$

$$\theta_N = \frac{S_N}{S} = \frac{0.036}{35} = 1.028 \times 10^{-3}$$

$$\beta = \frac{0.75}{0.25} = 3$$

$$\theta_N = \frac{1}{(1 + \beta)^N}$$

$$N = \frac{\log \left[\frac{1}{\theta_N} \right]}{\log [1 + \beta]} = \frac{\log \left[\frac{1}{1.028 \times 10^{-3}} \right]}{\log [1 + 3]} = 4.96 \approx 5$$

Example (2): Repeat the previous example (1) but the time of leaching is 300 sec. Find the concentration of solute in the solid (dry basis)? after five washes where the decanting ratio is the same.

Solution:

$$C = C_S \left[1 - e^{\frac{-K_L \cdot A}{V} t} \right] \implies C = 2.5 \left[1 - e^{\frac{-0.139}{(100)} (300)} \right]$$

$$C = 0.847 \text{ kg/m}^3$$

$$\text{The amount of solute unleached} = (500) (0.28) - (0.847) (100) = 140 - 84.7 = 55.3 \text{ kg}$$

$$\theta_N = \frac{1}{(1 + \beta)^N} = \frac{1}{(1 + 3)^5} = 9.76 \times 10^{-4}$$

$$S = (0.847) (100) (0.25) = 21.175 \text{ kg}$$

$$\theta_N = \frac{S_N}{S} \implies 9.76 \times 10^{-4} = \frac{S_N}{21.175}$$

$$S_N = (21.175) (9.76 \times 10^{-4}) = 2.06 \times 10^{-2} \text{ kg}$$

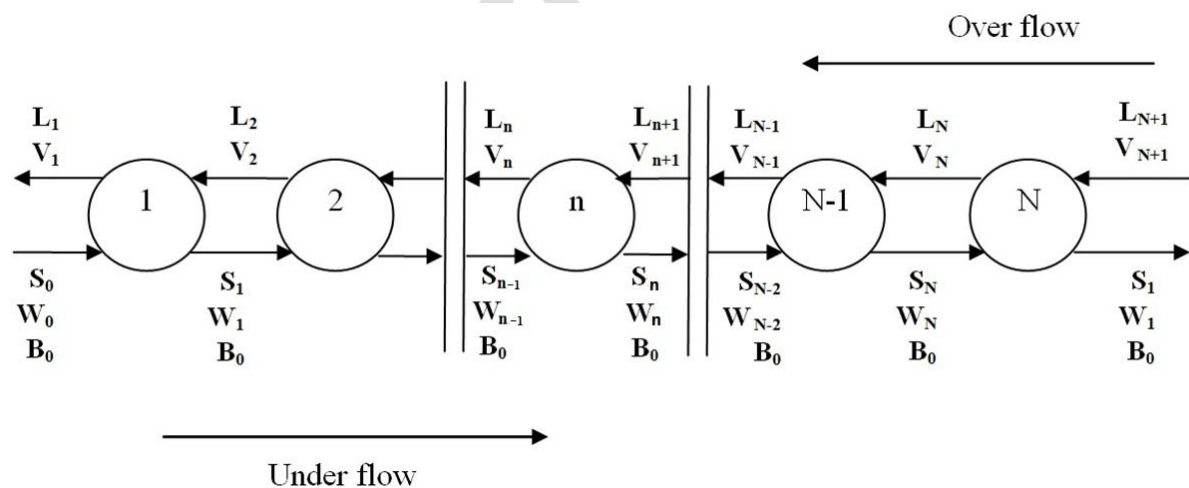
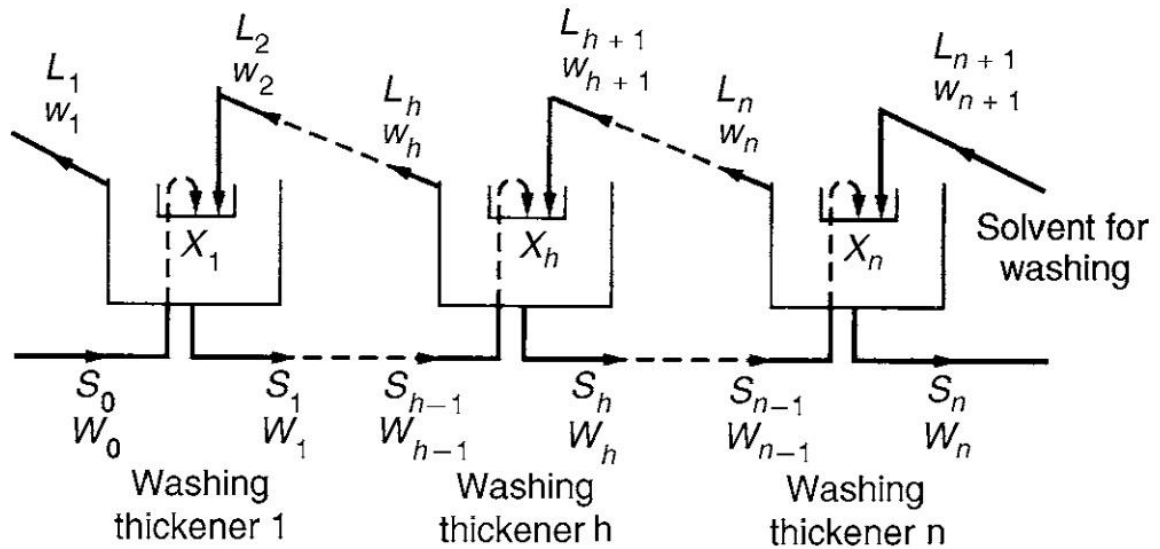
$$B = (500) (1 - 0.28) = 360 \text{ kg}$$

$$\begin{aligned} \text{Total solute} &= \text{unleached} + \text{remaining in the solid after washing} \\ &= 55.3 + 2.06 \times 10^{-2} = 55.32 \text{ kg} \end{aligned}$$

$$\text{Solute conc. } (X_A) = \frac{55.32}{55.32 + 360} = 0.133$$

Continuous Leaching

1. Counter current:



Where:

L : is the of flow solute in overflow (kg/s).

S : is the of flow solute in underflow (kg/s).

V : is the of flow solution in overflow (kg/s).

W : is the of flow solution in underflow (kg/s).

B : is the of flow insoluble solid in underflow (kg/s).

Case I: Constant under flow:

The amount of solvent removed with the insoluble solid in the underflow is constant, and independent of the concentration of the solution in the thickener, then the amount of solvent leaving each thickener in the underflow will then be the same, and therefore the amount of solvent in the overflow will also be the same. Hence the ratio of the solvent discharged in the overflow to that in the underflow is constant. This will be taken as R , where:

$$R = \frac{\text{Amount of solvent discharge in overflow}}{\text{Amount of solvent discharge in underflow}} = \frac{V - L}{W - S}$$

Also:

$$R = \frac{\text{Amount of solute discharge in overflow}}{\text{Amount of solute discharge in underflow}} = \frac{L}{S}$$

Also:

$$R = \frac{\text{Amount of solution discharge in overflow}}{\text{Amount of solution discharge in underflow}} = \frac{V}{W}$$

Solute material balance on stage (n)

$$S_{n-1} + L_{n+1} = L_n + S_n$$

$$S_{n-1} + R S_{n+1} = R S_n + S_n \quad \text{when } L = R S$$

$$\frac{1}{R} S_{n-1} + S_{n+1} = S_n + \frac{1}{R} S_n$$

$$S_{n+1} - \left[1 + \frac{1}{R}\right] S_n + \frac{1}{R} S_{n-1} = 0$$

Using E-operator

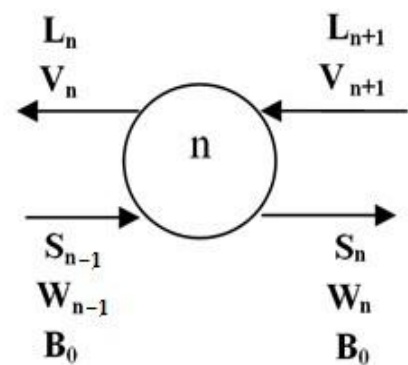
$$E S_n - \left[1 + \frac{1}{R}\right] S_n + \frac{1}{R} E^{-1} S_n = 0 \quad \dots \dots \dots (\times E)$$

$$E^2 S_n - \left[1 + \frac{1}{R}\right] E S_n + \frac{1}{R} S_n = 0$$

Change E to ρ :

$$\rho^2 - \left[1 + \frac{1}{R}\right] \rho + \frac{1}{R} = 0$$

$$\rho_1 = 1 \quad \text{and} \quad \rho_2 = \frac{1}{R}$$



2nd order difference equation with the solution:

$$S_n = A \rho_1^n + B \rho_2^n$$

$$S_n = A + B \left(\frac{1}{R}\right)^n \dots \dots \dots (1)$$

Using boundary conditions to find the constants (A and B):

B.C.1:

At $n = 0 \implies S_n = S_0$

Sub. in equation (1):

$$S_0 = A + B \left(\frac{1}{R}\right)^0 \implies S_0 = A + B \dots \dots \dots (2)$$

B.C.2:

At $n = 1 \implies S_n = S_1$

Sub. in equation (1):

$$S_1 = A + B \left(\frac{1}{R}\right)^1 \implies S_1 = A + B \left(\frac{1}{R}\right) \text{ when } L = R S$$

$$\frac{L_1}{R} = A + B \left(\frac{1}{R}\right) \implies L_1 = RA + B \dots \dots \dots (3)$$

Subtraction Eq.(3) from Eq.(2) to obtain:

$$L_1 - S_0 = A(R - 1)$$

$$A = \frac{L_1 - S_0}{R - 1}$$

In the case for using solute free solvent (pure solvent), $L_{n+1}=0$:

$$L_1 = S_0 - S_n$$

$$A = \frac{S_0 - S_n - S_0}{R - 1} = \frac{-S_n}{R - 1}$$

$$\implies \boxed{A = \frac{-S_n}{R - 1} \dots \dots \dots (4)}$$

Substitute Eq.(4) in Eq.(2):

$$S_0 = \frac{-S_n}{R - 1} + B \implies B = S_0 - \frac{L_1 - S_0}{R - 1} = \frac{S_0(R - 1) - L_1 + S_0}{R - 1}$$

$$B = \frac{S_0 R - S_0 - L_1 + S_0}{R-1} = \frac{S_0 R - L_1}{R-1} = \frac{S_0 R - S_0 + S_n}{R-1}$$

$$B = \frac{S_0(R-1) + S_n}{R-1} \dots\dots\dots 5$$

$$S_n = A + B \left(\frac{1}{R}\right)^n$$

$$S_n R^n = A R^n + B \implies S_n R^n = \frac{-S_n}{R-1} R^n + \frac{S_0(R-1) + S_n}{R-1}$$

$$S_n R^n (R-1) = -S_n R^n + S_0(R-1) + S_n$$

$$S_n R^n (R-1) = S_n (1 - R^n) + S_0(R-1)$$

$$S_n R^n (R-1) - S_n (1 - R^n) = S_0(R-1)$$

$$S_n R^n (R-1) - S_n + S_n R^n = S_0(R-1)$$

$$S_n (R^{n+1} - 1) = S_0(R-1)$$

$$\frac{S_n}{S_0} = \frac{(R-1)}{(R^{n+1}-1)}$$

$$f = \frac{S_N}{S_0} = \frac{(R-1)}{(R^{N+1}-1)}$$

Where: f = fractional of solute discharge.

$$\text{Recovery} = 1 - f$$

$$N = \frac{\log \left[1 + \left(\frac{R-1}{f} \right) \right]}{\log (R)} - 1$$

- We can know the type of continuous leaching (constant under flow) from the sentence:
(The residue from each stage contain 0.25 kg water/kg insoluble solid)

Example:

1.6 kg/s of sand-salt mixture containing 62.5% sand is leached with 0.5 kg/s of water in a counter-current. The residue from each stage containing 0.25 kg water per kg insoluble solid. Find the number of stages such that the sand from the final stage contains 10% salt when dried.

Solution:

$$B = (0.625) (1.6) = 1 \text{ kg/s}$$

$$S_0 = 1.6 - 1 = 0.6 \text{ kg/s}$$

$$V_{N+1} = 0.5 \text{ kg/s}$$

$$L_{N+1} = 0 \text{ (Pure solvent)}$$

$$\text{The amount of solvent in under flow} = W - S = (0.25) (B) = 0.25 \text{ kg/s}$$

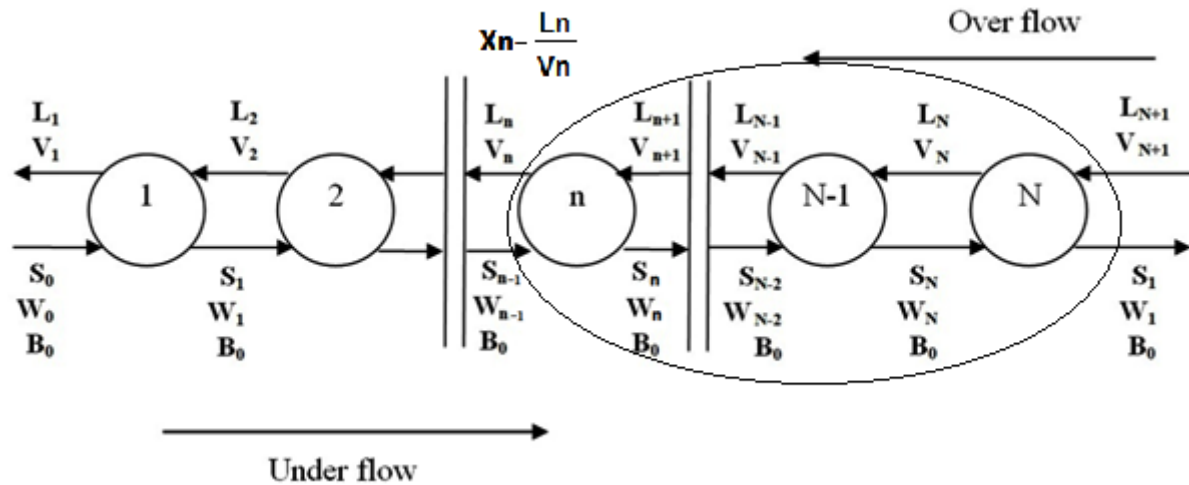
$$0.1 = \frac{S_N}{B + S_N} = \frac{S_N}{1 + S_N} \implies S_N = 0.11 \text{ kg/s}$$

$$R = \frac{V - L}{W - S} = \frac{0.5}{0.25} = 2$$

$$f = \frac{S_N}{S_0} = \frac{0.11}{0.6} = 0.183$$

$$N = \frac{\log \left[1 + \left(\frac{R-1}{f} \right) \right]}{\log (R)} - 1$$

$$N = \frac{\log \left[1 + \left(\frac{2-1}{0.183} \right) \right]}{\log (2)} - 1 = 1.69 \approx 2 \text{ stages}$$

Case II: Variable under flow:Variable under flow:

$W_1 \neq W_2 \neq W_3 \neq W_n \neq W_N \implies$ solution under flow is variable

Let: $X = \frac{L}{V} = \frac{S}{W} = \frac{\text{kg solute}}{\text{kg solution}} = \text{weight fraction}$

Solute material balance on the last section:

$$S_{n-1} + L_{N+1} = L_n + S_N \dots \dots \dots (1)$$

Solution material balance on the last section:

$$W_{n-1} + V_{N+1} = V_n + W_N \dots \dots \dots (2)$$

From Eq.(1) and Eq.(2):

$$L_n = L_{N+1} + S_{n-1} - S_N \dots \dots \dots (3)$$

$$V_n = V_{N+1} + W_{n-1} - W_N \dots \dots \dots (4)$$

Divided Eq.(3) by Eq.(4):

$$\frac{L_n}{V_n} = \frac{L_{N+1} + S_{n-1} - S_N}{V_{N+1} + W_{n-1} - W_N} \implies x_n = \frac{L_{N+1} + S_{n-1} - S_N}{V_{N+1} + W_{n-1} - W_N} \dots \dots \dots (S_1)$$

$$W = a + b X \dots \dots \dots (S_2)$$

$$S = X W \dots \dots \dots (S_3)$$

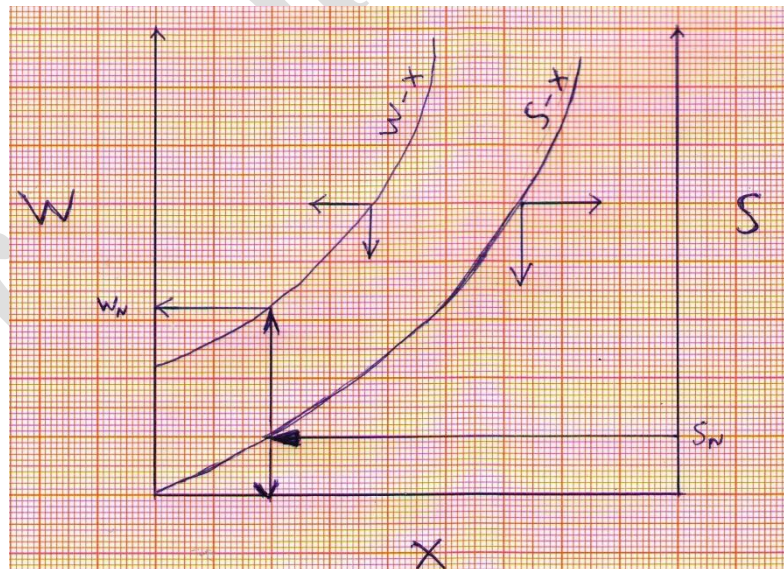
Where:

$$x = \frac{\text{kg solute}}{\text{kg solution}}$$

$$w = \frac{\text{kg solution}}{\text{kg insoluble solid}}$$

To find the number of stages:

1. We always take basis 1 kg of insoluble solid (B).
 2. We substitute S_0 and W_0 given in Eq. (S₁) to gain X_1 .
 3. We substitute X_1 in Eq. (S₂) to gain W_1 .
 4. We substitute X_1 and W_1 in Eq. (S₃) to gain S_1 .
 5. We repeat for S_1 and W_1 in Eq. (S₁) to gain X_2 then in Eq. (S₂) to gain W_2 then in Eq. (S₃) to gain S_2 and we continue until $S_n \leq S_N$.
- Another way to obtain W_n and X_n is to make a plot between (W vs. X) and (S vs. X).



We take S_N and intersect a line to the (S vs. X) curve and read downwards X_N , and from there we go upwards and intersect a line to the (W vs. X) curve and then go to the left and read W_N .

Example:

A vegetable seed material containing 0.4 kg oil/kg insoluble solid is washed with hydrocarbon solvent in order to recover 90% of the oil in a counter current unit. It is found that the under flow varies with the concentration of classifier as given below:

Amount of solution in under flow, $W = \frac{\text{kg solution}}{\text{kg insoluble solid}}$	Concentration of solute in solution, $X = \frac{\text{kg solute}}{\text{kg solution}}$
0.30	0
0.32	0.1
0.34	0.2
0.36	0.3
0.38	0.4
0.40	0.5

If the solvent input flow 0.5 kg/kg insoluble solid. Find the number of stages required?

Solution:

Basis: 1 kg/s of insoluble solid (B_0)

$$x_n = \frac{L_{N+1} + S_{n-1} - S_N}{V_{N+1} + W_{n-1} - W_N}$$

$L_{N+1} = 0$ (pure solvent)

$V_{N+1} = 0.5$ kg/s

$$\frac{S_N}{S_0} = f = (1 - \text{recovery})$$

$$S_N = S_0 (1 - \text{recovery}) = (0.4)(1 - 0.9) = 0.04 \text{ kg/s}$$

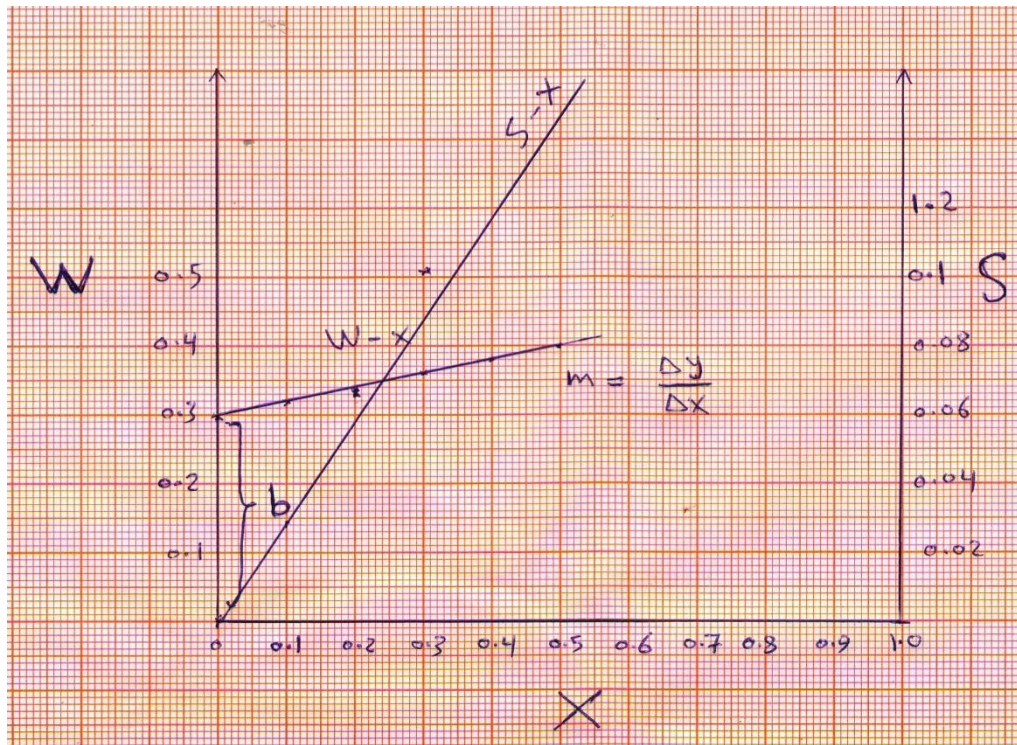
From the plot:

$$m = \frac{\Delta y}{\Delta x} = \frac{0.4 - 0.3}{0.5 - 0} = 0.2$$

Intercept = 0.3

Then: $W = a + b X = 0.3 + 0.2 X$

Where: $S = X W$



$$W_N = 0.3 + 0.2 X = 0.3 + 0.2 \left(\frac{S_N}{W_N} \right)$$

Put: $S_N = 0.04 \text{ kg/s}$

$$W_N^2 - 0.3 W_N + 0.2 (0.04) = 0$$

$$W_N^2 - 0.3 W_N + 0.008 = 0$$

$$W_N = \frac{+0.3 \mp \sqrt{(0.3)^2 - 4(-0.008)}}{2} \implies W_N = 0.3246 \text{ kg/s}$$

$$X_n = \frac{0 - 0.04 + S_{n-1}}{0.5 - 0.325 + W_{n-1}}$$

$$X_n = \frac{-0.04 + S_{n-1}}{0.175 + W_{n-1}} \dots \dots \dots (1)$$

$$W_n = 0.2 X_n + 0.3 \dots \dots \dots (2)$$

$$S_n = X_n * W_n \dots \dots \dots (3)$$

Stages calculations:

Stage (1) $\implies n = 1$

$$S_0 = 0.4, \quad W_0 = \text{solvent} + \text{solute} = 0 + 0.4 = 0.4 \text{ kg/s}$$

$$\text{From Eq.(1):} \quad x_1 = \frac{-0.04 + S_0}{0.175 + W_0} = \frac{-0.04 + 0.4}{0.175 + 0.4} = 0.626$$

$$\text{From Eq.(2):} \quad W_1 = 0.2 X_1 + 0.3 = 0.2 (0.625) + 0.3 = 0.425$$

$$\text{From Eq.(3):} \quad S_1 = X_1 * W_1 = (0.625)(0.425) = 0.266$$

$$S_1 > S_N \implies \text{continue}$$

Stage (2) $\implies n = 2$

$$\text{From Eq.(1):} \quad x_2 = \frac{-0.04 + S_1}{0.175 + W_1} = \frac{-0.04 + 0.266}{0.175 + 0.425} = 0.376$$

$$\text{From Eq.(2):} \quad W_2 = 0.2 X_2 + 0.3 = 0.2 (0.376) + 0.3 = 0.375$$

$$\text{From Eq.(3):} \quad S_2 = X_2 * W_2 = (0.376)(0.375) = 0.141$$

$$S_2 > S_N \implies \text{continue}$$

Stage (3) $\implies n = 3$

$$\text{From Eq.(1):} \quad x_3 = \frac{-0.04 + S_2}{0.175 + W_2} = \frac{-0.04 + 0.141}{0.175 + 0.375} = 0.183$$

$$\text{From Eq.(2):} \quad W_3 = 0.2 X_3 + 0.3 = 0.2 (0.183) + 0.3 = 0.3366$$

$$\text{From Eq.(3):} \quad S_3 = X_3 * W_3 = (0.183)(0.3366) = 0.01615$$

$$\text{Now:} \quad S_3 < S_N$$

Sheet No.(8)

Q1:- 2 Kg/s of a solid containing 30% by mass a water soluble component is to be leached in a counter – current unit . Given that the recovery is 98% and that the under flow from each stage contains 0.5 Kg water / Kg insoluble solid . Find :

(A) The number of stages required if the water flow rate is 1.6 Kg/s .

(B) The water flow rate if the number of stages is 3 .

Q2:- 2.3 Kg/s of solid containing 0.55 Kg/s soluble material is to be washed with 2.8 Kg/s pure solvent in a counter – current unit . The final residue is to contain not more than 0.1% solute when dried . Find the number of stages given that the amount of solution in under flow varies with solute concentration according to the relation : $W = 0.2 X + 0.3$, Also find the concentration and quantities in and out from each stage .

Q3:- Fresh halibut livers containing 25.7% (weight percent) oil are to be extracted with pure ethyl ether to remove 95% of the oil in a counter – current multistage leaching process . The feed rate is 1000 Kg of the fresh liver per hour . The final exit over flow solution is to be contain 70% by mass oil . The molar flow varies as follows .

Kg solution / kg inert solid	Kg oil / kg solution
0.205	0
0.286	0.2
0.405	0.4
0.599	0.6
0.719	0.81

Calculate the amount & composition of the exit streams and the total number of the theoretical stages.

Q4:- A treated ore containing inert solid gangue and copper sulfate is to be leached in a counter – current multistage extractor using pure water to leach CuSO_4 . The solid charge rate per hour consist of 10000 Kg of inert gangue , 1200 Kg of CuSO_4 and 400 Kg of water . The exit wash solution is to be contain 92% by wt. water and 8% CuSO_4 . A total of 95% of the CuSO_4 in the inlet ore is to be recovered . The under flow is constant at 2 Kg solvent per inert gangue solid . Calculate the number of stages required and the amount of all streams .

Q5:- 1000 Kg/hr of waxed paper is to be dewaxed by counter-current leaching with kerosene in stage type equipment . The paper contains 30% paraffin wax and 70% paper pulp . The pulp leaving the leaching process must not contain over 0.002 Kg wax per Kg pulp .The kerosene solvent entering the process is to contain 0.0005 Kg wax / Kg kerosene .Experiments show that the pulp retains 1 Kg of kerosene per Kg pulp as it is transferred from stage to stage . The kerosene leaving the process contains 0.06 Kg wax / Kg kerosene . Assuming overall efficiency of 90% , determine the actual number of stages required .

Co – Current (Cross Current)

Q6:- Cod liver oil is to be extracted from ground – up cod fish heads , containing 25% oil and 75% insoluble solids , the leaching operating is to be done in a cross – flow (co – current) apparatus consisting of the equivalent of three equilibrium stages , 1000 Kg/hr of fish heads is fed in to the system and the under flow contains 0.2 Kg solution / Kg insoluble solid , if 300 Kg/hr of hexane solvent is added to each stage , determine the percentage of oil remaining in fish heads leaving the system .

Q7:- 0.8 Kg/s of seeds containing 30% by wt. of oil are extracted on a counter-current unit and 95% of the oil recovered in a solution containing 50% by wt. oil . Calculate the number of stages of the final conc. from stage one (over flow) twice final conc. from stage N counter flow .

Q8:- 1.5 Kg/s of sand-salt mixture containing 70% sand leach with 0.84 Kg/s of water in a counter-current unit , the residue from each stage contain 0.4 Kg water per Kg insoluble solid . Calculate the number of stages such that the sand from final stage containing 9.5% salt when 80% water evaporated ?

Q9:- 2 Kg/s of sand-salt mixture is leach with 0.8 Kg/s water in 2 stages counter-current unit . The salt percent discharge from N is 2.68% before drying . Calculate the salt conc. discharge from one (over flow) of 90% recovery ?

Q10:- 1.8 Kg/s of sand-salt mixture containing 25% Kg salt per Kg insoluble solid is leach with one Kg/s water in a counter-current unit . The under flow from such stage containing 0.5 Kg solution / Kg insoluble solid . Calculate the number of stages of the efficiency of 80% and percentage of extraction is 98% ?

Q11:- 1.2 Kg/s of sand-salt mixture is leach with water . The amount of water reduced by two times . The discharge from such contain 0.5 Kg water per Kg insoluble solid . Calculate the number of stages to extract 0.4 Kg/s of salt , such that the sand from final stage containing salt when drying 6.25% , also calculate the final conc. discharge from such over flow ?

Q12:- 1.6 Kg/s of sand-salt mixture containing 30% by wt. of salt is leach with 0.25 Kg of water containing 5% salt on a counter-current plant . The percentage of water discharge from the stage N is 10% .Calculate the conc. of salt discharge from stage one (over flow) , the recovery is 90% ?

Chapter (11)

Distillation

The separation of liquid mixtures into their various components is one of the major operations in the process industries, and distillation, the most widely used method of achieving this end, is the key operation in any oil refinery. In processing, the demand for purer products, coupled with the need for greater efficiency, has promoted continued research into the techniques of distillation. In engineering terms, distillation columns have to be designed with a larger range in capacity than any other types of processing equipment, with single columns 0.3–10 m in diameter and 3–75 m in height.

Distillation: is the separation of liquid mixture by partial evaporation. The essential requirement is to have a vapor composition different from liquid.

Boiling point: is the temperature at which the $\sum P_i = P_T$ and for pure component $P_T = P^\circ$ (vapor pressure).

- If $P_A^\circ > P_B^\circ$

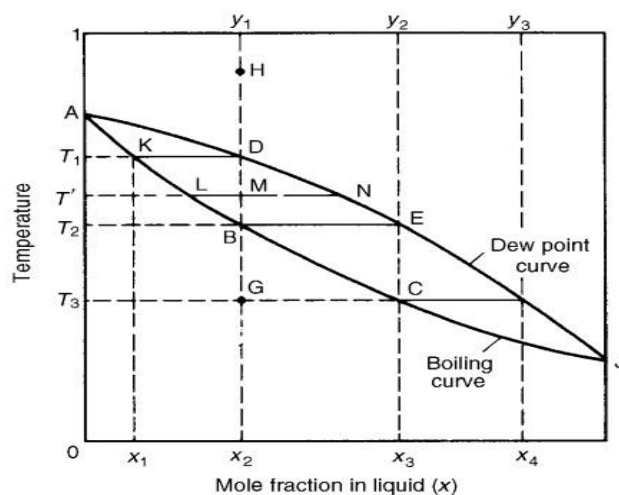
Volatility of (A) > Volatility of (B)

- If $P_B^\circ > P_A^\circ$

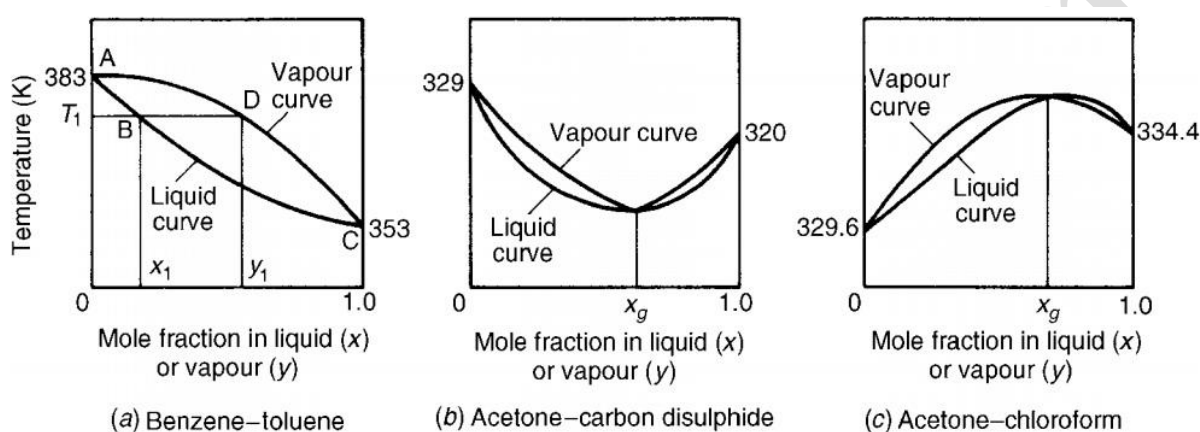
Volatility of (B) > Volatility of (A)

Vapour–Liquid Equilibrium

The composition of the vapour in equilibrium with a liquid of given composition is determined experimentally using an equilibrium still. The results are conveniently shown on a temperature–composition diagram as shown in Figure below.



In the normal case shown in Figure (a), the curve ABC shows the composition of the liquid which boils at any given temperature, and the curve ADC the corresponding composition of the vapour at that temperature. Thus, a liquid of composition x_1 will boil at temperature T_1 , and the vapour in equilibrium is indicated by point D of composition y_1 . It is seen that for any liquid composition x the vapour formed will be richer in the more volatile component, where x is the mole fraction of the more volatile component in the liquid, and y in the vapour. Examples of mixtures giving this type of curve are benzene–toluene and n-heptane–toluene.



In Figures (b) and (c), there is a critical composition x_g where the vapour has the same composition as the liquid, so that no change occurs on boiling. Such critical mixtures are called **azeotropes**. For compositions other than x_g , the vapour formed has a different composition from that of the liquid. It is important to note that these diagrams are for **constant pressure conditions**, and that the composition of the vapour in equilibrium with a given liquid will change with pressure.

For distillation purposes it is more convenient to **plot y against x** at a constant pressure, since the majority of industrial distillations take place at substantially constant pressure.

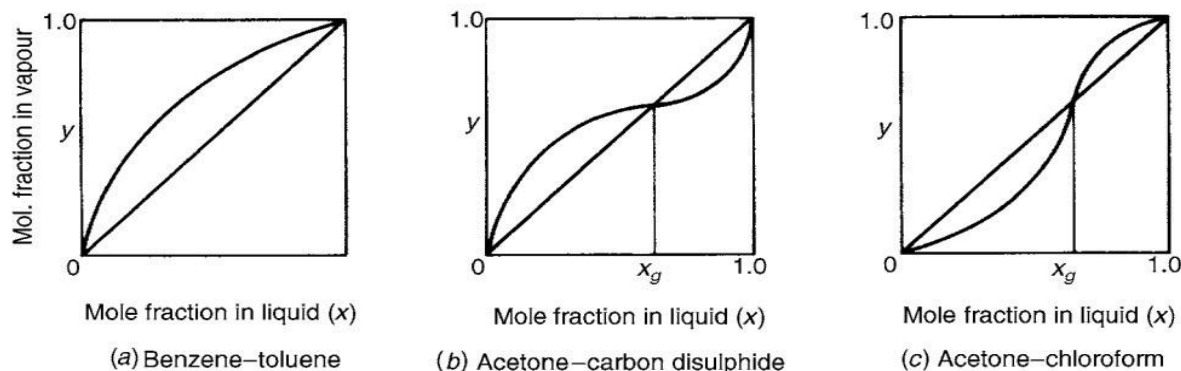


Figure 11.4. Vapour composition as a function of liquid composition at constant pressure

The vapour-liquid equilibrium data is calculated from:

1. Raoult's and Dalton's law for ideal system:

For an ideal mixture, the partial pressure is related to the concentration in the liquid phase by **Raoult's law** which may be written as:

$$P_A = P_A^0 x_A \quad \text{and} \quad P_B = P_B^0 x_B \quad \text{or} \quad P_B = P_B^0 (1 - x_A)$$

Where:

P_A : is the partial pressure of component A.

P_A^0 : is the vapour pressure of component A.

P_B : is the partial pressure of component B.

P_B^0 : is the vapour pressure of component B.

x_A : is the mole fraction of component A in liquid phase.

This relation (**Raoult's law**) is usually found to be true only for high values of x_A , or correspondingly low values of x_B , although mixtures of organic isomers and some hydrocarbons follow the law closely.

By **Dalton's law** of partial pressures:

$$P_T = \sum P_i$$

Where: $P_A = y_A P_T$

$$P_T = P_A + P_B$$

$$P_T = P_A^0 x_A + P_B^0 (1 - x_A)$$

$$P_T = P_A^0 x_A + P_B^0 - P_B^0 x_A$$

$$P_T - P_B^0 = x_A (P_A^0 - P_B^0)$$

$$x_A = \frac{P_T - P_B^0}{P_A^0 - P_B^0} \dots \dots \dots (1)$$

$$y_A = \frac{P_A^0}{P_T} x_A \dots \dots \dots (2)$$

Temp.	P_A^0	P_B^0	$x_A = \frac{P_T - P_B^0}{P_A^0 - P_B^0}$	$y_A = \frac{P_A^0}{P_T} x_A$
-	-	-	Calculated from eq.1	Calculated from eq.2
-	-	-	Calculated from eq.1	Calculated from eq.2

2. Relative volatility (α):

The relationship between the composition of the vapour y_A and of the liquid x_A in equilibrium may also be expressed in a way, which is particularly useful in distillation calculations:

$$\alpha_A = \frac{P_A^0}{P_T} \quad \text{and} \quad \alpha_B = \frac{P_B^0}{P_T} \quad \text{difference for distillation}$$

As the difference increase, the distillation would be easier.

Where:

α_A : is the volatility of component A.

α_B : is the volatility of component B.

The relative volatility (α_{AB}):

$$\alpha_{AB} = \frac{\alpha_A}{\alpha_B} = \frac{P_A^0/P_T}{P_B^0/P_T} > 1$$

For separation to be achieved, α_{AB} must not equal 1 and, considering the more volatile component, as α_{AB} increases above unity, y increases and the separation becomes much easier.

$$\alpha_{AB} = \frac{\alpha_A}{\alpha_B} = \frac{P_A^0}{P_B^0} = \frac{P_A/x_A}{P_B/x_B} = \frac{P_A/P_T x_A}{P_B/P_T x_B} = \frac{y_A/x_A}{y_B/x_B} = \frac{y_A/x_A}{(1-y_A)/(1-x_A)}$$

$$y_A = \frac{\alpha_{AB} * x_A}{1 + (\alpha_{AB} - 1) x_A} \dots \dots \dots (*)$$

Equilibrium relation in distillation

$$x_A = \frac{y_A}{\alpha_{AB} - (\alpha_{AB} - 1)y_A}$$

To plot x_A against y_A , we use Eq.(*) for given value of x_A between (0 – 1.0) which are arbitrary:

x_A	y_A
0	-
0.1	-
0.2	-
0.3	-
0.4	-
0.5	-
0.6	-
0.7	-
0.8	-
0.9	-
1.0	=

3. Henry's law for non-ideal systems:

For low values of x_A , a linear relation between P_A and x_A again exists, although the proportionality factor is Henry's constant H , and not the vapour pressure P_A^0 of the pure material. For a liquid solute A in a solvent liquid B, Henry's law takes the form:

$$y_A = H x_A$$

Where: H = is Henry's constant

or we use the equilibrium constant or we call the "distribution coefficient, k "

$$y_A = k_A x_A$$

Where: k_A = is the distribution coefficient or equilibrium constant
 $k = f(T)$

For non-ideal binary mixture the partial pressure may be expressed in the form:

$$P_A = \frac{\gamma_A P_A^0 x_A}{P_T} \quad \text{and} \quad P_B = \frac{\gamma_B P_B^0 x_B}{P_T}$$

Where: γ_A = is the activity coefficient for component A.

γ_B = is the activity coefficient for component B.

$$\alpha_{AB} = \frac{k_A}{k_B} = \frac{y_A/x_A}{y_B/x_B} = \frac{\gamma_A P_A^0}{\gamma_B P_B^0}$$

The liquid phase activity coefficients γ_A and γ_B depend upon temperature, pressure and concentration. Typical values taken from Perry's Chemical Engineers' Handbook are shown in Figure 11.8 for the systems n-propanol–water and acetone–chloroform.

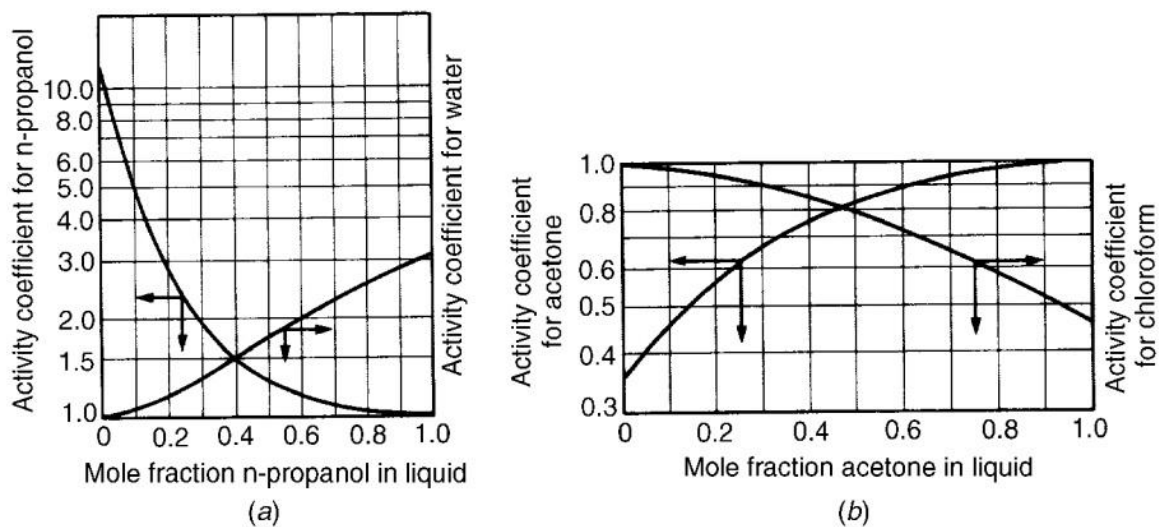


Figure 11.8. Activity coefficient data

Example: The following vapour pressure were obtained for phenol and ortho-cresol:

Temp. (K)	Vapour pressure of ortho-cresol (kN/m ²)	Vapour pressure of phenol (kN/m ²)
387	7.7	10
387.9	7.94	10.4
388.7	8.21	10.8
389.6	8.5	11.2
390	8.76	11.6
391.1	9.06	12.0
391	9.4	12.4
392.7	9.73	12.9
393.3	10.0	13.3

Assuming Raoult's and Dalton's laws apply. Find the following data for a total pressure of 10.0 kN/m².

- A temperature-composition diagram.
- A vapour-liquid equilibrium data.
- Relative volatility against mole fraction of phenol in liquid.

Solution:

P_A^o	P_B^o	$\alpha_{AB} = \frac{P_A^o}{P_B^o}$	x_A	$y_A = \frac{\alpha_{AB}(\text{avg.}) * x_A}{1 + (\alpha_{AB}(\text{avg.}) - 1) x_A}$
10	7.7	1.3	0	
10.4	7.94	1.31	0.1	
10.8	8.21	1.315	0.2	
11.2	8.5	1.318	0.3	
11.6	8.76	1.324	0.4	
12.0	9.06	1.325	0.5	
12.4	9.4	1.319	0.6	
12.9	9.73	1.326	0.7	
13.3	10.0	1.330	0.8	
			0.9	
			1.0	
		$\alpha_{AB}(\text{average}) = 1.318$		

$$\alpha_{AB}(\text{avg.}) = \sqrt[9]{(1.3)(1.31)(1.315)(1.318)(1.324)(1.325)(1.319)(1.326)(1.330)} = 1.318$$

Methods of Distillation -Two Component Mixtures

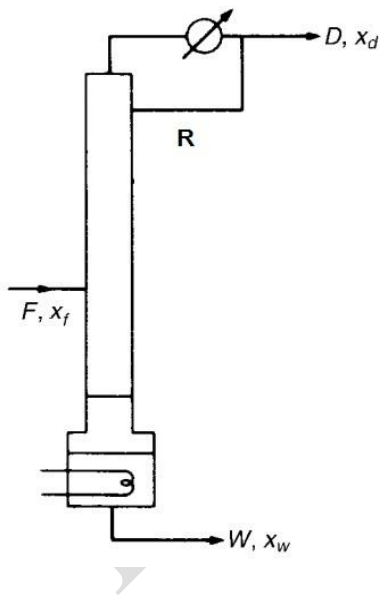
For a binary mixture with a normal $x_A - y_A$ curve, the vapour is always richer in the more volatile component than the liquid from which it is formed. There are two main methods used in distillation practice which all rely on this basic fact. These are:

1. Continuous Distillation.

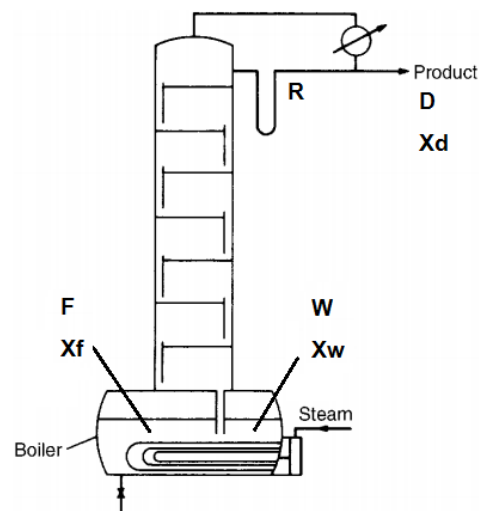
- a. Rectifying (fractionation) distillation.
- b. Flash (equilibrium) distillation.

2. Non- Continuous Distillation.

- a. Differential distillation.
- b. Batch distillation.
 - i. Operation at constant reflux ratio.
 - ii. Operation at constant product composition (variable reflux).



Rectifying distillation



Batch Distillation

1. Differential distillation:

The simplest example of batch distillation is a single stage (differential distillation) starting with a still pot, initially full, heated at a constant rate. In this process:

1. The vapour formed on boiling the liquid is removed at once from the system.
2. Vapour is richer in the more volatile component than the liquid, and the liquid remaining becomes steadily weaker in this component, so this result that the composition of the product progressively alters.
3. The vapour formed over a short period is in equilibrium with the liquid.
4. The total vapour formed is not in equilibrium with the residual liquid. At the end of the process the liquid which has not been vaporized is removed as the bottom product.

If S_o = Numbers of moles of feed in the still initially.

S = Numbers of moles of liquid mixture in the still after concentrated.

D = Numbers of moles of product (distillate).

x_o = Mole fraction of A (more volatile component) in the feed.

x = Mole fraction of A in the waste (residue).

x_d = Mole fraction of A in the distillate (product).

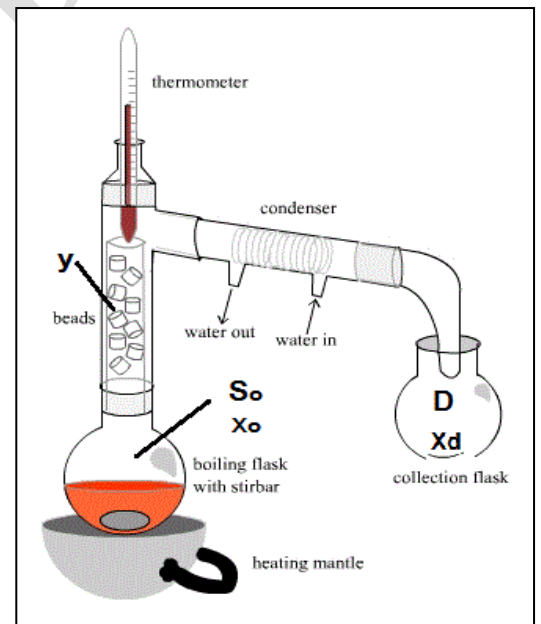
Overall material balance gives:

$$S_o = D + S$$

Material balance on more volatile component gives:

$$(S_o)(x_o) = (D)(x_d) + (S)(x)$$

$$x_d = \frac{(S_o)(x_o) - (S)(x)}{D}$$



The boundary conditions of the differential distillation:

At time = 0, $\implies S = S_o$ and $x = x_o$ (liquid condition)

At time = t, $\implies S = S$ and $x = x$ (variable)

In time (dt):

ds: is the amount of liquid vaporized from the still.

dx: is the concentration difference in the still.

So we take a material balance :

$$y dS = d(Sx) = S dx + x dS$$

$$(y - x) dS = S dx$$

$$\int_{S_0}^S \frac{dS}{S} = \int_{x_0}^x \frac{dx}{y-x}$$

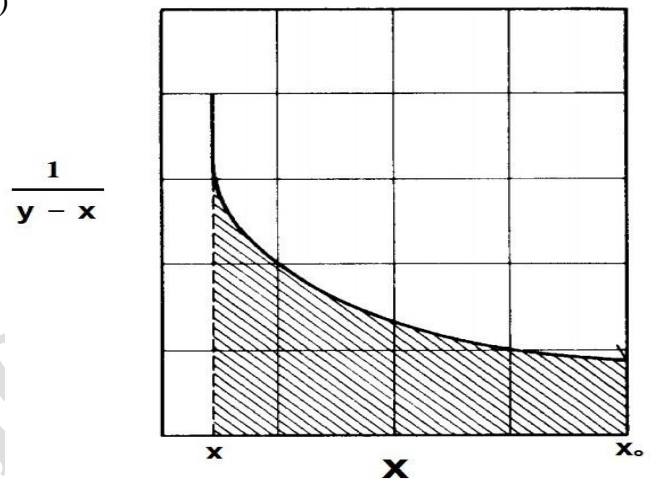
$$\ln \frac{S}{S_0} = \int_{x_0}^x \frac{dx}{y-x} \dots\dots\dots (*)$$

The integral on the right-hand side of this equation may be solved by three ways:

1. Graphical solution:

Taking values of x and y from the equilibrium relationship.
(If the equilibrium data given as points)

x	y	$\frac{1}{y-x}$
-	-	-
-	-	-
-	-	-



$$\ln \frac{S}{S_0} = \text{Area under the curve} = \int_{x_0}^x \frac{dx}{y-x}$$

2. If the equilibrium data is a straight line of the form $y = m x + c$

$$\ln \frac{S}{S_0} = \left(\frac{1}{m-1} \right) \ln \left[\frac{(m-1)x + c}{(m-1)x_0 + c} \right]$$

3. If the equilibrium data is given by **volatility (α)**:

$$y = \frac{\alpha * x}{1 + (\alpha - 1)x}$$

$$\ln \frac{S}{S_0} = \int_{x_0}^x \frac{dx}{y - x} = \int_{x_0}^x \frac{dx}{\left[\frac{\alpha * x}{1 + (\alpha - 1)x} \right] - x}$$

$$\ln \frac{S}{S_0} = \left(\frac{1}{\alpha - 1} \right) \ln \left[\frac{x(1 - x_0)}{x_0(1 - x)} \right] + \ln \left[\frac{1 - x_0}{1 - x} \right]$$

Example: 100 kmol of a mixture (A and B) is fed to a simple still. The feed contains 50 mol% of A and a remain in the still is 5 mol% of A. Calculate the quantity and the average composition of the product obtained? The equilibrium data are:

x	1.0	0.9	0.6	0.5	0.4	0.3	0.2	0.1	0.05
y	1.0	0.932	0.745	0.67	0.57	0.46	0.34	0.2	0.1

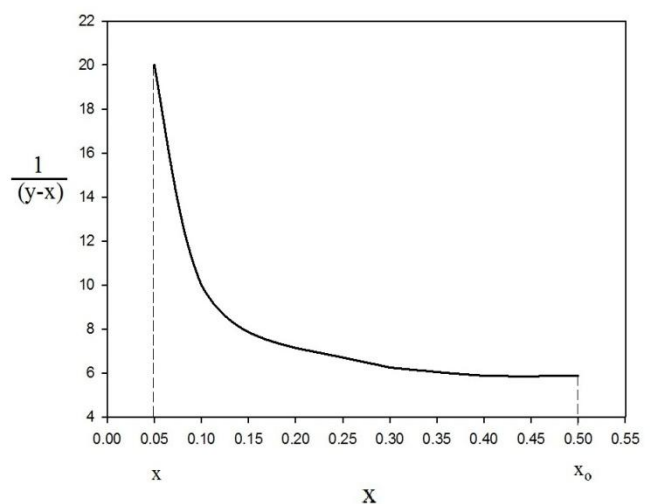
Solution:

$S_0 = 100$ kmol, $x_0 = 0.5$ and $x = 0.05$

Since the type is differential distillation, then:

$$\ln \frac{S}{S_0} = \int_{x_0}^x \frac{dx}{y - x}$$

	x	y	$\frac{1}{y - x}$
$x_0 \rightarrow$	0.5	0.67	5.88
	0.4	0.57	5.88
	0.3	0.46	6.25
	0.2	0.34	7.14
	0.1	0.2	10.0
$X \rightarrow$	0.05	0.1	20.0



The No. of squares = 16

The area of one square = $(\Delta x)(\Delta y) = (0.1)(2) = 0.2$

The area under the curve = (No. of squares) (area of one square) = $(16)(0.2) = 3.22$

From plot:

The area under the curve = 3.2 $\implies \ln \frac{S}{S_0} = 3.2$

Then, $S = 4.076 \text{ kmol}$

Overall material balance:

$$D = S_0 - S = 100 - 4.076 = 95.92 \text{ kmol}$$

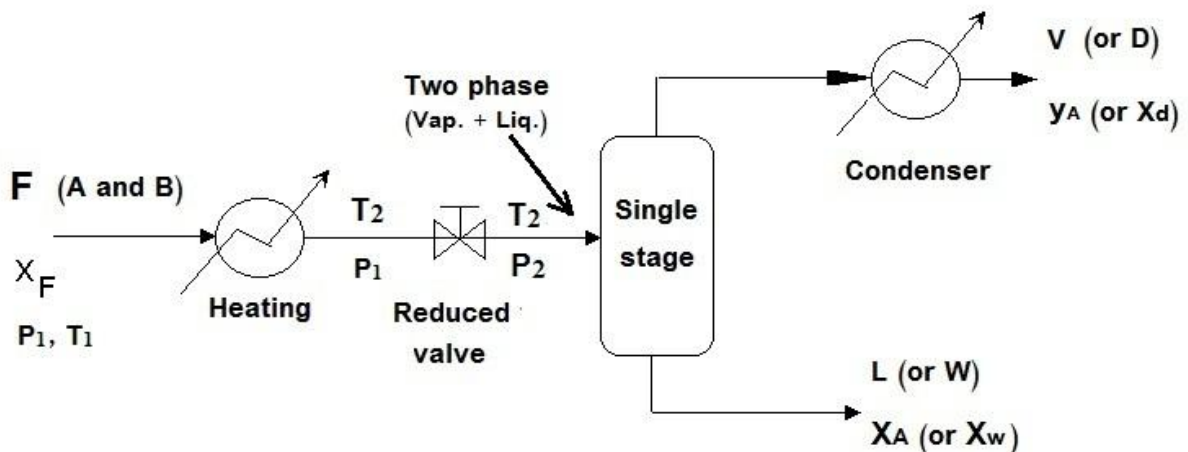
Material balance on more volatile component gives:

$$(S_0)(x_0) = (D)(x_d) + (S)(x)$$

$$x_d = \frac{(S_0)(x_0) - (S)(x)}{D} = \frac{(100)(0.5) - (4.076)(0.05)}{95.92} = 0.519$$

2. Flash or equilibrium distillation:

Flash or equilibrium distillation, frequently carried out as a continuous process, consists of vaporizing a definite fraction of the liquid feed in such a way that the vapour evolved is in equilibrium with the residual liquid. The feed is usually pumped through a fired heater and enters the still through a valve where the pressure is reduced. The still is essentially a separator in which the liquid and vapour produced by the reduction in pressure have sufficient time to reach equilibrium. The vapour is removed from the top of the separator and is then usually condensed, while the liquid leaves from the bottom.



1. Overall mass balance gives:

$$F = V + L \quad \dots\dots\dots(1)$$

2. Material balance on more volatile component gives:

$$(F) (x_F) = (V) (y_A) + (L) (x_A) \quad \dots\dots\dots(2)$$

The values of x_A and y_A required must satisfy, not only the equation, but also the appropriate equilibrium data. Thus these values may be determined depends on the equilibrium relationship:

a. Graphically using an x – y diagram.

x	-	-	-	-	-
y	-	-	-	-	-

First, plot the equilibrium data (x, y), then assume the value of x_A and find the value of y_A from the equilibrium plot. Substitute the assumed value of x_A and the calculated value of y_A in Eq.(2). If the right side of Eq.(2) equal to the left side then the assumed x_A and the calculated y_A represents the mole fraction of more volatile component in liquid and vapour phase, respectively. If not, repeat the assumption.

b. Analytically if the equilibrium relationship is linear:

$$y_A = m x_A \quad \dots\dots\dots(3)$$

Substitute Eq.(3) into Eq.(2) to find x_A then the calculated value of x_A substitute into the equilibrium relation Eq.(3) to find y_A .

c. Analytically if the equilibrium relationship is given by the relative volatility (α_{AB}):

$$y_A = \frac{\alpha_{AB} * x_A}{1 + (\alpha_{AB} - 1) x_A} \quad \dots\dots\dots(4)$$

Substitute Eq.(4) into Eq.(2) to find x_A then the calculated value of x_A substitute into the equilibrium relation Eq.(4) to find y_A .

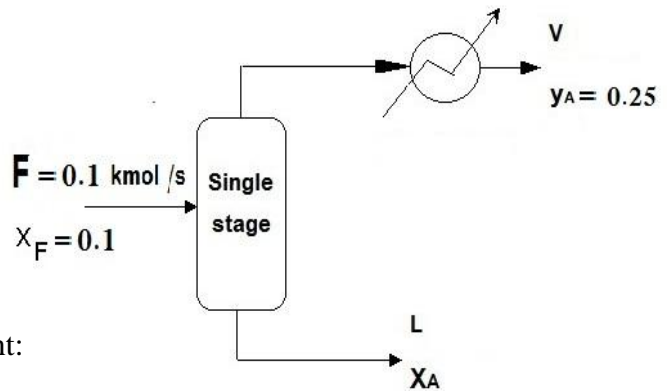
Example (1): An aqueous solution at its boiling point containing 10 mol% of ammonia is fed to the flash distillation to produce a distillate containing 25 mol% of ammonia. At equilibrium, the mole fraction of ammonia in the vapour phase is 6.3 times that in the liquid phase and the feed flow rate is 0.1 kmol/s. Calculate the number of moles distillate obtainable from the flash distillation.

Solution

Overall material balance:

$$F = V + L = 0.1$$

$$V = 0.1 - L \quad \dots\dots\dots(1)$$



Material balance on more volatile component:

$$(F) (x_f) = (V) (y_A) + (L) (x_A)$$

$$(0.1) (0.1) = (0.1 - L) (y_A) + (L) (x_A) \quad \dots\dots\dots(2)$$

Equilibrium relationship:

$$y_A = 6.3 x_A \quad \dots\dots\dots(3)$$

$$\text{at } y_A = 0.25 \quad \Longrightarrow \quad x_A = \frac{0.25}{6.3} = 0.0346$$

Substitute x_A into Eq.(2) to gain L:

$$(0.1) \quad (0.1) = (0.1 - L) (0.25) + (L) (0.0346)$$

$$L = 0.0712 \text{ kmol/s}$$

$$V = 0.1 - 0.0721 = 0.0287 \text{ kmol /s}$$

Example (2): A liquid mixture containing 40 mol% of n-heptane and 60 mol% of n-octane is to be continuously flash vaporized at 1 atm. The product vapour is 70% of the feed. What will be the composition of the vapour and liquid. Given $\alpha_{AB} = 2.16$.

Solution

$$F = 100 \text{ kmol/s} \quad \text{and} \quad V = 70 \text{ kmol/s}$$

Overall material balance:

$$F = V + L$$

$$100 = 70 + L \quad \implies L = 30 \text{ kmol/s}$$

Material balance on more volatile component:

$$(F) (x_f) = (V) (y_A) + (L) (x_A)$$

$$(100) (0.4) = (70) (y_A) + (30) (x_A) \quad \dots\dots\dots(1)$$

Equilibrium relationship:

$$y_A = \frac{\alpha_{AB} * x_A}{1 + (\alpha_{AB} - 1) x_A} = \frac{2.16 x_A}{1 + (2.16 - 1) x_A}$$

$$y_A = \frac{2.16 x_A}{1 + 1.16 x_A} \quad \dots\dots\dots(2)$$

Substitute Eq.(2) into Eq.(1) to get:

$$34.8 x_A^2 + 134.8 x_A - 40 = 0$$

$$x_A = \frac{-134.8 \mp \sqrt{(134.8)^2 - 4(34.8)(-40)}}{2 (34.8)}$$

$$x_A = 0.276$$

Substitute x_A value into Eq.(2) to get y_A :

$$y_A = 0.452$$

3. Continuous (Rectification) distillation:

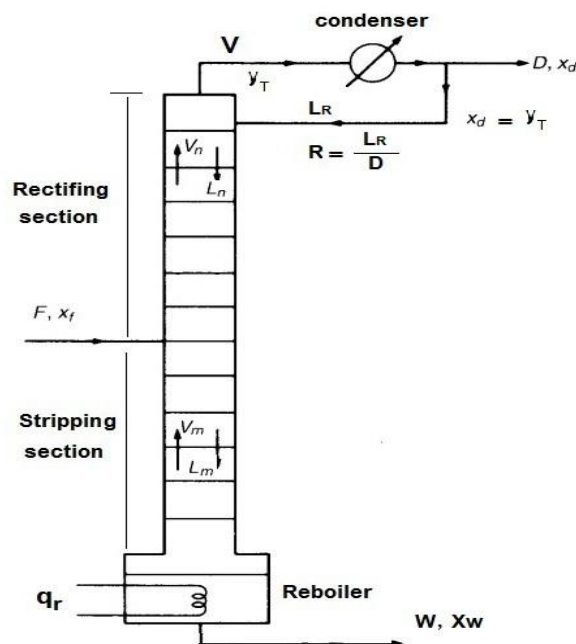
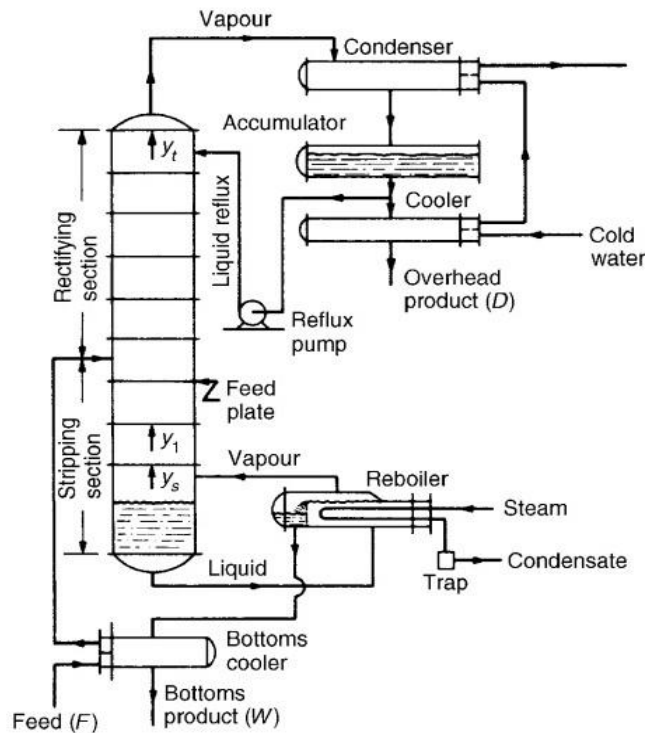


Fig: Continuous fractionating column with rectifying and stripping sections

We must know some main points in this tower (fractionating tower):

1. The temperature varies along the tower ($T_w > T_f > T_t$).
2. The feed differs from each process where it could be:
 - a. Cold liquid (subcooled).
 - b. Liquid at boiling point (saturated liquid).
 - c. Vapour at boiling point (saturated vapour).
 - d. Partially vaporized.
 - e. Superheated vapour.
3. Whenever we increase L_R the tower height will decrease.
4. We have a reflux ratio of: $R = \frac{L_R}{D}$
5. Whenever we find a process with a reboiler means that the tower used is a distillation tower.
6. The feed is pumped from anywhere:
 - a. From top or middle or bottom of the tower.
 - b. From reboiler.

and the position of feed is determined by x_f :

If the x_f is small, the feed is pumped from the bottom tower.

If the x_f is large, the feed is pumped from the top tower.

- When the feed pumped from the reboiler, all the tower is rectifying tower.
 - When the feed pumped from the top, all the tower is stripping tower.
7. Reboiler is a single mass transfer stage with 100% efficiency.

$$\text{No. of stages} = \text{No. of plates} + 1$$

Continuous distillation can be divided depending on the number of components in the feed stream into:

1. Binary mixture.
2. Multi-component mixtures.

The most common things that need to be calculated in the distillation column tower are:

1. Actual and minimum number of plates.
2. Reflux ratio and minimum reflux ratio.
3. The heat added in the boiler (Q_R).
4. The heat removed in the condenser (Q_C).

The McCabe-Thiele Method

The simplifying assumption for the McCabe-Thiele method is that :

latent heat of vaporization of component A \neq latent heat of vaporization of component B

$$\lambda_A \neq \lambda_B$$

i. Rectifying section operating line equation:

Overall material balance between plate (n) and the top product indicated by the loop I:

$$V_n = L_{n+1} + D$$

Since the molar liquid and vapour overflow is constant:

$$L_n = L_{n+1} = L_{n-1} = L_R$$

$$V_n = V_{n+1} = V_{n-1} = V$$

Then the overall material balance equation becomes:

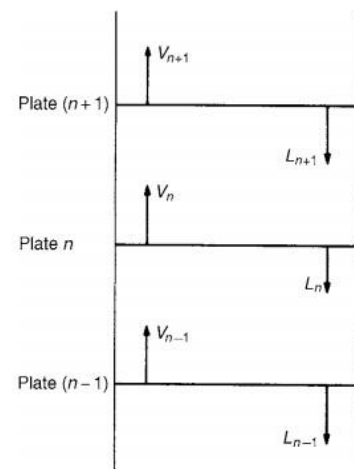
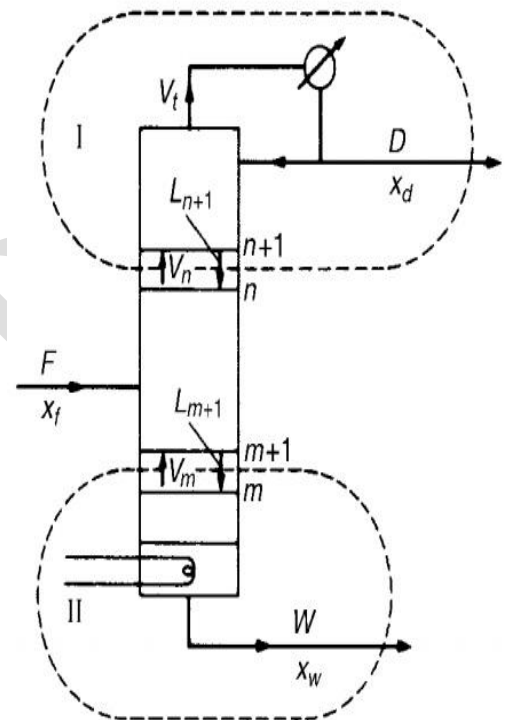
$$V_n = L_n + D$$

Material balance on (M.V.C) between plate (n) and the top product indicated by the loop I:

$$V_n y_n = L_{n+1} x_{n+1} + D x_d$$

$$y_n = \frac{L_{n+1}}{V_n} x_{n+1} + \frac{D x_d}{V_n}$$

$$y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D x_d}{V_n} \dots\dots\dots(1)$$



We can write Eq.(1) in the form of reflux ratio as follows:

$$R = \frac{L_n}{D}$$

$$y_n = \frac{RD}{RD+D} x_{n+1} + \frac{D x_d}{RD+D}$$

$$y_n = \frac{R}{R+1} x_{n+1} + \frac{x_d}{R+1} \dots\dots\dots(2)$$

We can plot the **rectifying (top) operating line** from the slope and intercept in the Eq.(1) and Eq.(2):

- **slope** = $\frac{L_n}{V_n} = \frac{R}{R+1}$
- **intercept** = $(0, \frac{D x_d}{V_n})$ and $(0, \frac{x_d}{R+1})$

Or we can plot the top operating line from two points:

- $(0, \frac{D x_d}{V_n})$ and (x_d, x_d)
- $(0, \frac{x_d}{R+1})$ and (x_d, x_d)

ii. Stripping section operating line equation:

Overall material balance between plate (m) and the bottom product indicated by the loop II:

$$L_{m+1} = V_m + W$$

Since the molar liquid and vapour is constant:

$$L_m = L_{m+1} = L_{m-1}$$

$$V_m = V_{m+1} = V_{m-1}$$

Then the overall material balance equation becomes:

$$L_m = V_m + W$$

Material balance on (M.V.C) between plate (m)
and the bottom product indicated by the loop II:

$$V_m y_m = L_{m+1} x_{m+1} - W x_w$$

$$\boxed{y_m = \frac{L_m}{V_m} x_{m+1} - \frac{W x_w}{V_m}} \dots \dots \dots (1)$$

We can plot the **stripping (bottom) operating line** from the slope and intercept in the Eq.(1):

- **slope** = $\frac{L_m}{V_m}$
- **intercept** = $(0, \frac{-W x_w}{V_m})$

iii. The q- line equation:

If the two operating lines intersect at a point with coordinates (x_q, y_q) , then from equations (top and bottom operating lines):

$$V_n y_q = L_n x_q + D x_d \dots \dots \dots (1)$$

$$V_m y_q = L_m x_q - W x_w \dots \dots \dots (2)$$

Subtraction Eq.(2) from Eq.(1):

$$y_q (V_m - V_n) = (L_m - L_n) x_q - (D x_d + W x_w) \dots \dots \dots (*)$$

A material balance over the feed plate gives:

$$F + L_n + V_m = L_m + V_n$$

$$V_m - V_n = L_m - L_n - F \dots \dots \dots (**)$$

To obtain a relation between L_n and L_m , it is necessary to make an enthalpy balance over the feed plate, and to consider what happens when the feed enters the column. If the feed is all in the form of liquid at its boiling point, then:

$$L_m = L_n + F$$

If the feed is a liquid at a temperature T_f , that is less than the boiling point, *some vapour rising from the plate below will condense to provide sufficient heat to bring the feed liquor to the boiling point.*

If: H_f = is the enthalpy per mole of feed.

H_{fs} is the enthalpy of one mole of feed at its boiling point.

The heat to be supplied to bring feed to the boiling point = $F(H_{fs} - H_f)$.

The number of moles of vapour to be condensed to provide this heat = $\frac{F(H_{fs} - H_f)}{\lambda}$

Where: λ is the molar latent heat of the vapour.

The reflux liquor is then:

$$L_m = L_n + F + \frac{F(H_{fs} - H_f)}{\lambda}$$

$$L_m = L_n + F \left(\frac{\lambda + H_{fs} - H_f}{\lambda} \right)$$

$$L_m = L_n + q F$$

Where:

$$q = \frac{\text{heat to vaporize 1 mole of feed}}{\text{molar latent heat of feed}}$$

Thus, from Eq.(**):

$$V_m - V_n = q F - F$$

A material balance of the more volatile component over the whole column gives:

$$F x_f = D x_d + W x_w$$

Thus, from Eq.(*):

$$F(q - 1) y_q = q F x_q - F x_f$$

$$y_q = \left(\frac{q}{q-1} \right) x_q - \left(\frac{x_f}{q-1} \right) \dots \dots \dots (q\text{-line equation})$$

Thus, the point of intersection of the two operating lines lies on the straight line of slope $\left(\frac{q}{q-1}\right)$ passing through the point (x_f, x_f) .

* We can plot the q -line from the point (x_f, x_f) with slope of $\left(\frac{q}{q-1}\right)$.

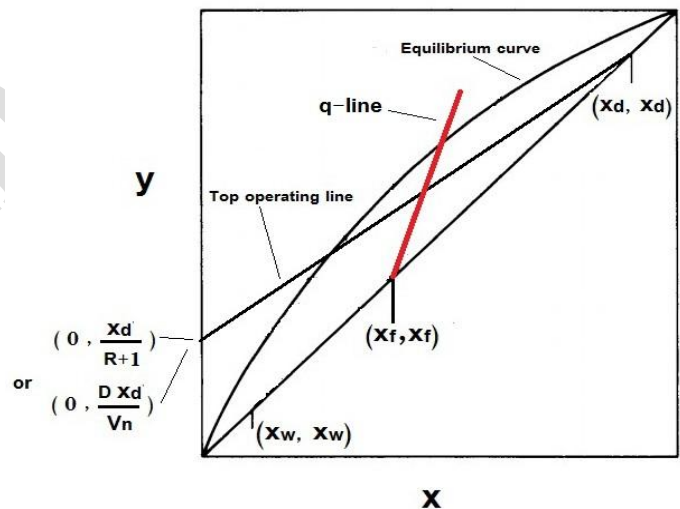
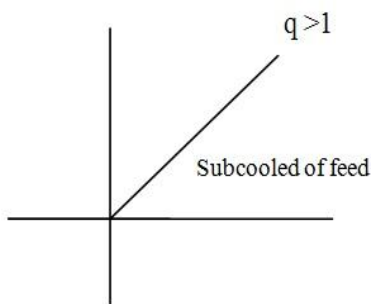
From the definition of q , it follows that the slope of the q -line is governed by the nature of the feed as follows:

1. Cold feed as liquor (the feed is subcooled)

$$T_f < T_{BP}$$

$$q = \frac{C_P (T_{BP} - T_f) + \lambda}{\lambda} > 1$$

$$\text{Slope} = \frac{q}{q-1} = \frac{+ve}{+ve}$$

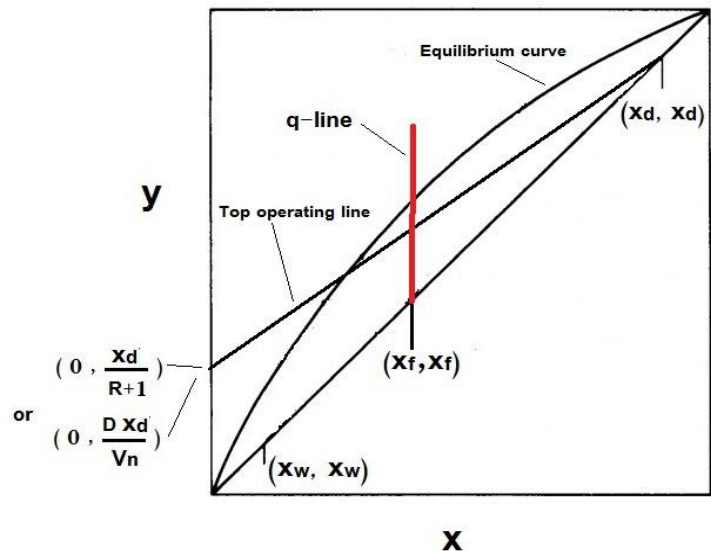
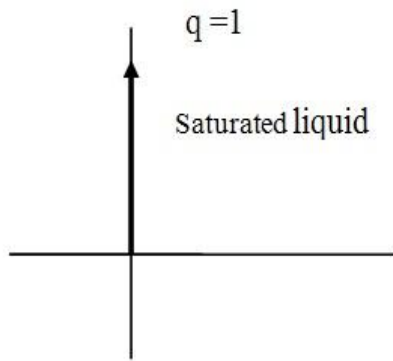


2. Feed at boiling point (saturated liquid)

$$T_f = T_{BP}$$

$$q = \frac{0 + \lambda}{\lambda} = 1$$

$$\text{Slope} = \frac{q}{q-1} = \frac{1}{0} = \infty$$



3. Feed partly vapour (partially vaporized feed)

$$T_f = T_{BP}$$

Two phase vapour and liquid feed quality = 20% vapour, this means that:

saturated vapour = 20%

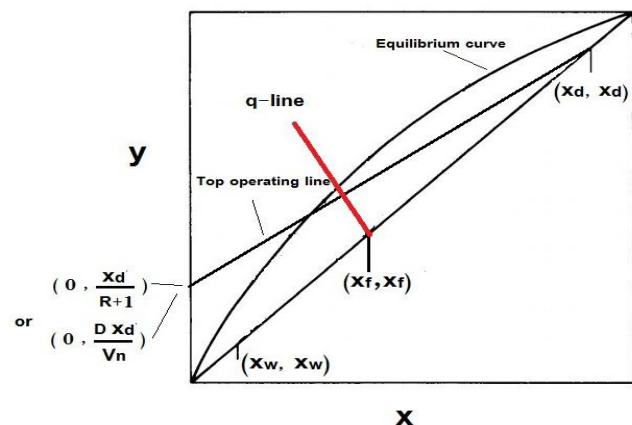
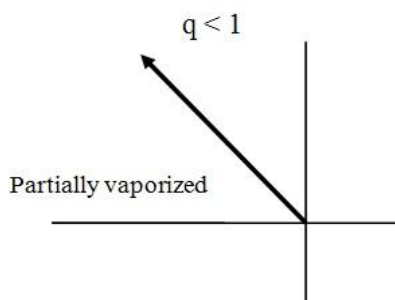
saturated liquid = 80%

$$q = \frac{(\text{Fraction of liquid}) \lambda}{\lambda} = 0.8 < 1$$

q = fraction of liquid

$$\text{Slope} = \frac{q}{q-1} = \frac{0.8}{0.8-1} = \frac{0.8}{-0.2} = -4$$

$$\text{Slope} = \frac{\text{fraction of liquid}}{- \text{fraction of vapour}}$$

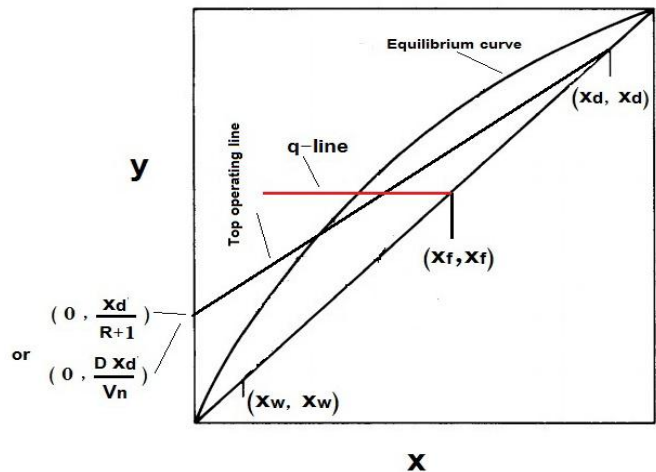
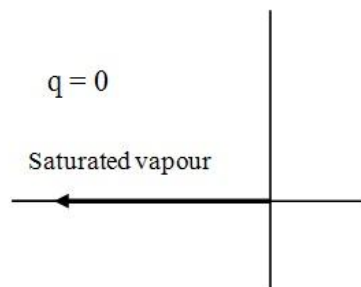


4. Feed saturated vapour (single vapour phase):

$$T_f = T_{BP}$$

$$q = \frac{0}{\lambda} = 0$$

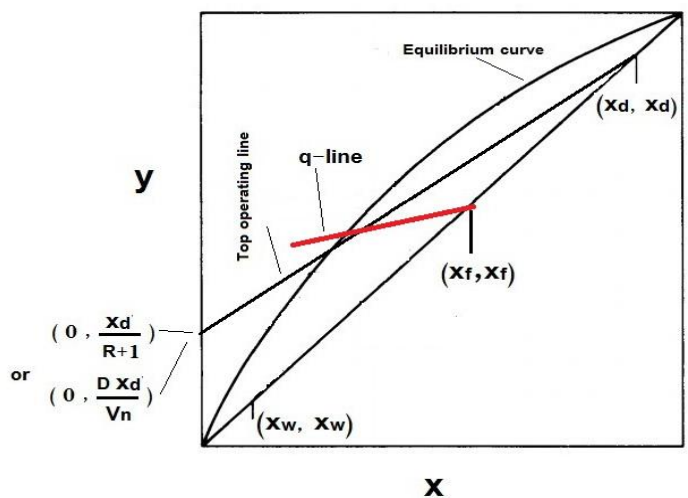
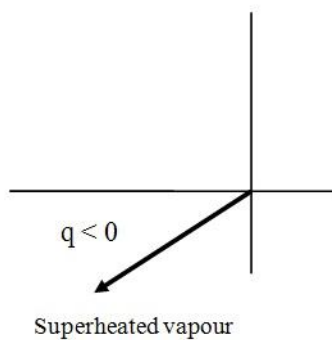
$$\text{Slope} = \frac{q}{q-1} = \frac{0}{-1} = 0$$

**5. Feed superheated vapour:**

$$T_f > T_{BP}$$

$$q = \frac{[C_p (T_{BP} - T_f) - \lambda] + \lambda}{\lambda} < 0$$

$$\text{Slope} = \frac{q}{q-1} = \frac{-ve}{-ve}$$



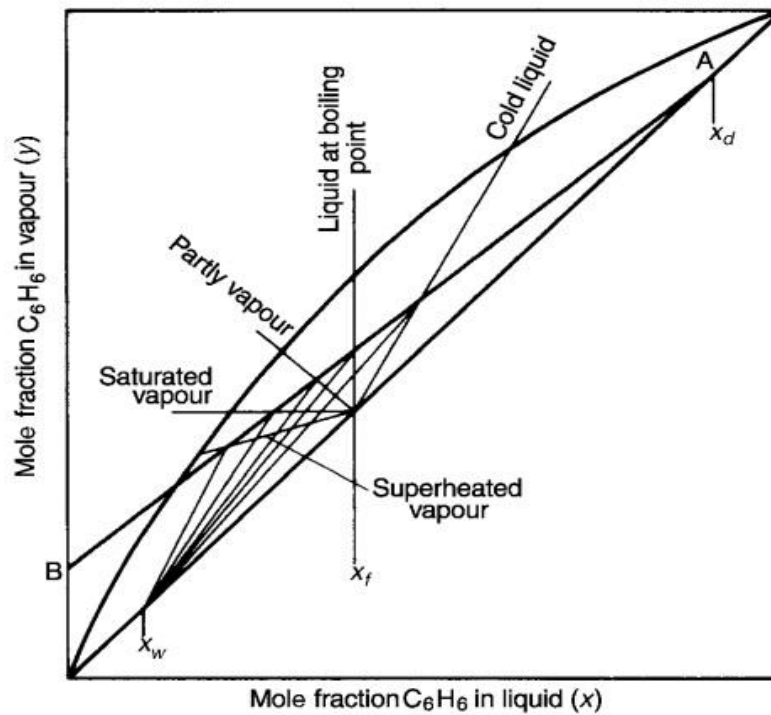


Fig. 11.16. Effect of the condition of the feed on the intersection of the operating lines for a fixed reflux ratio.

Calculation of the theoretical number of plates of the continuous distillation column by McCabe-Thiele Method (graphically):

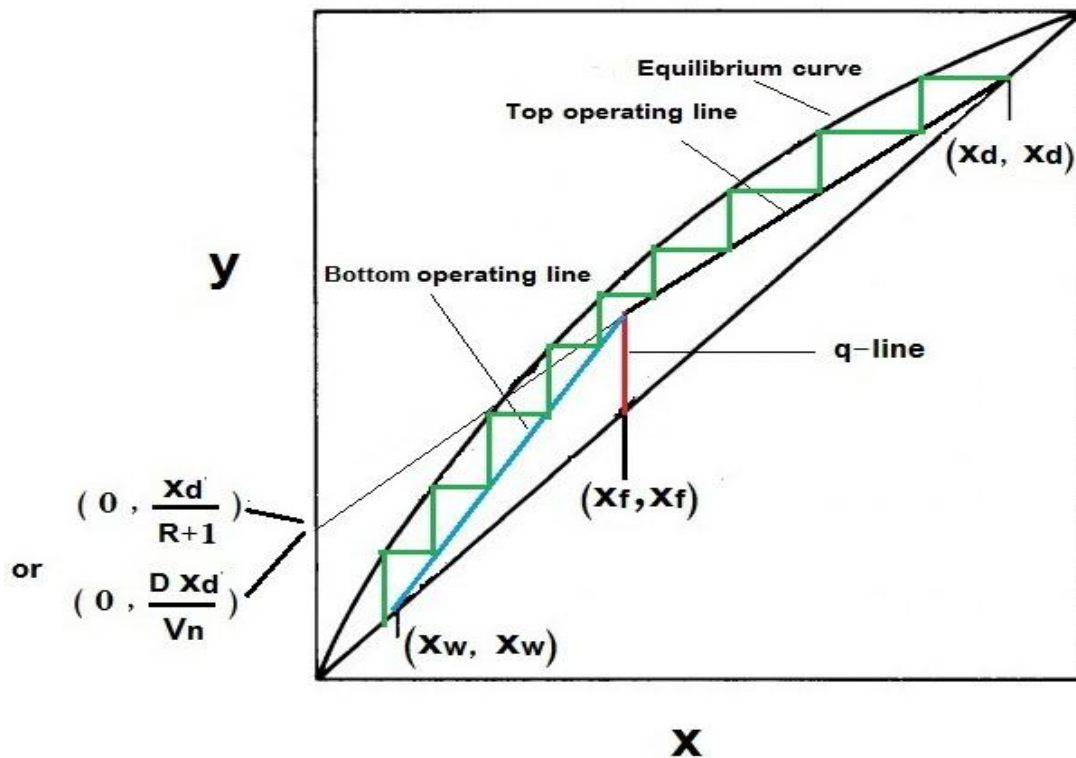
1. We draw the top operating line from two points:

$$\left[(x_d, x_d) \text{ and } \left(0, \frac{D x_d}{V_n} \right) \right] \quad \text{or} \quad \left[(x_d, x_d) \text{ and } \left(0, \frac{x_d}{R+1} \right) \right]$$

or from one point (x_d, x_d) and slope $= \frac{L_n}{V_n} = \frac{R}{R+1}$

2. We draw the q-line from (x_f, x_f) until its intersect with the top operating at (x_q, x_q) .
3. We draw the bottom operating line by joining point (x_q, x_q) to point (x_w, x_w) .
4. We draw the vertical and horizontal lines from (x_d, x_d) to (x_w, x_w) which represents **the number of stages**.

No. of plates = No. of stages – 1



Example: A continuous rectifying column handles a mixture consisting of 40 per cent of benzene by mass and 60 per cent of toluene at the rate of 4 kg/s, and separates it into a product containing 97 per cent of benzene and a liquid containing 98 per cent toluene. The feed is liquid at its boiling-point.

- Calculate the mass flows of distillate and waste liquor.
- If a reflux ratio of 3.5 is employed, how many plates are required in the rectifying part of the column?

Mole fraction of benzene in liquid	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mole fraction of benzene in vapour	0.22	0.38	0.51	0.63	0.7	0.78	0.85	0.91	0.96

Solution:

$$\text{Mole fraction of benzene in feed, } x_f = \frac{(40/78)}{(40/78) + (60/92)} = 0.440$$

$$\text{Similarly: } x_d = 0.974 \quad \text{and} \quad x_w = 0.024$$

As the feed is a liquid at its boiling-point, the q -line is vertical and may be drawn at $x_f = 0.44$.

(a) A mass balance over the column and on the more volatile component in terms of the mass flow rates gives:

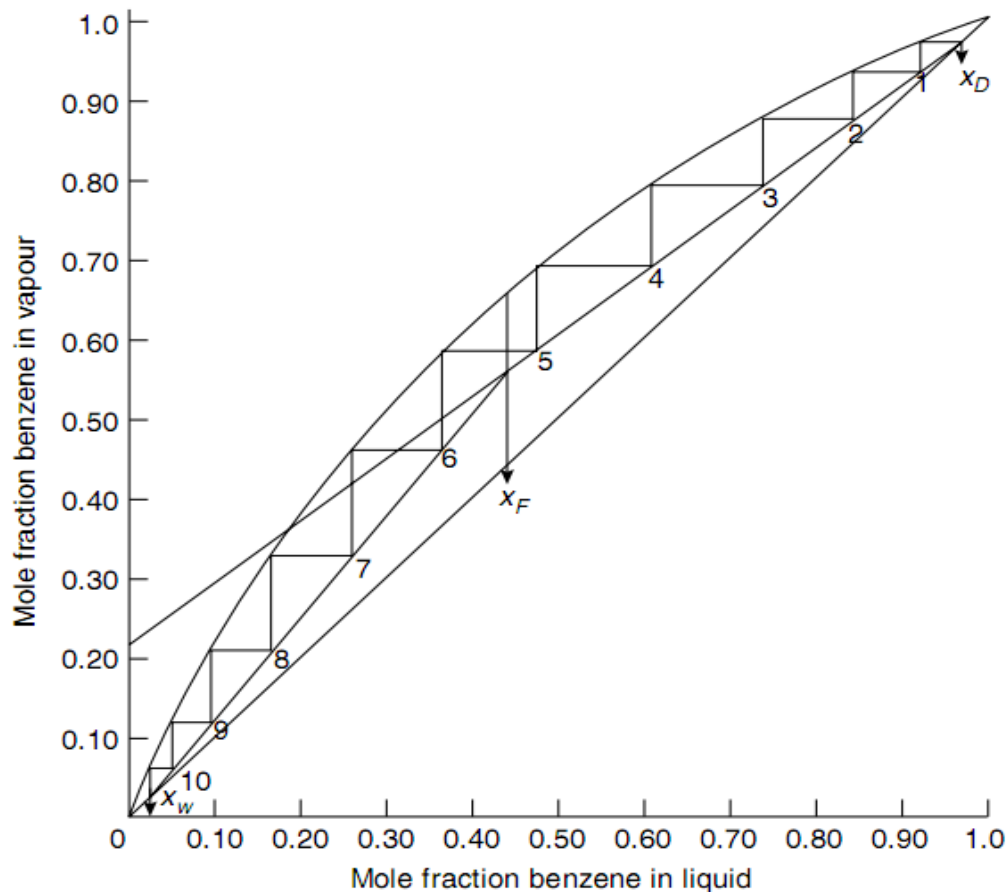
$$4.0 = W' + D'$$

$$(4 \times 0.4) = 0.02W' + 0.97D'$$

from which: bottoms flowrate, $W' = \underline{\underline{2.4 \text{ kg/s}}}$

and: top product rate, $D' = \underline{\underline{1.6 \text{ kg/s}}}$

(b) If $R = 3.5$, the intercept of the top operating line on the y-axis is given by $x_d/(R + 1) = (0.974/4.5) = 0.216$, and thus the operating lines may be drawn as shown in Figure 11h. The plates are stepped off as shown and 10 theoretical plates are required.



Minimum Reflux ratio (R_{\min})

If the reflux ratio is reduced, the slope of the operating line is reduced and more stages are required to pass from x_f to x_d , as shown by the line AK in Figure 11.17. Further reduction in R will eventually bring the operating line to AE, where an infinite number of stages is needed to pass from x_d to x_f . This arises from the fact that under these conditions the steps become very close together at liquid compositions near to x_f , and no enrichment occurs from the feed plate to the plate above. These conditions are known as minimum reflux (R_{\min}).

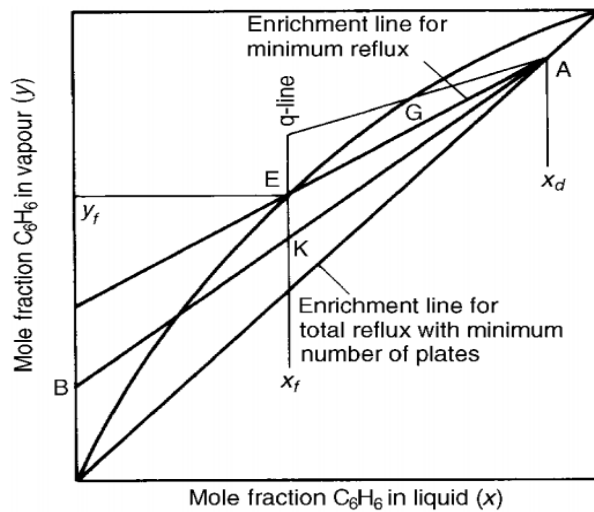


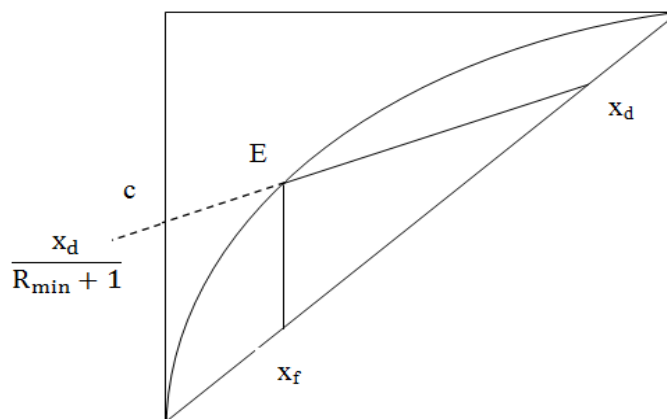
Figure 11.17. Influence of reflux ratio on the number of plates required for a given separation

Here: The number of plates (from this figure) will equal ($N = \infty$) because the triangles will reach point E, and will not come out of that point.

There are two methods to estimate (R_{\min}):

1. Calculation of (R_{\min}) by graphical method.

(R_{\min}) is obtained when the top operating line (Rectifying line) intersects the equilibrium curve at the feed point.



We read (R_{\min}) from the y-axis, let's say (C) so:

$$C = \frac{x_d}{R_{\min} + 1} \implies R_{\min} = \text{any number}$$

2. Calculation of (R_{\min}) by equation from volatility:

$$R_{\min} = \frac{1}{(\alpha - 1)} \left[\frac{x_d}{x_f} - \frac{\alpha (1 - x_d)}{(1 - x_f)} \right]$$

$$R_{\text{act.}} = (1.1 - 1.5) R_{\min}$$

* If we have different α_{AB} in the feed, waste and distillate product, so we take the average α_{AB} .

$$\alpha_{AB} = [(\alpha_{AB})_w \cdot (\alpha_{AB})_f \cdot (\alpha_{AB})_d]^{1/3}$$

Minimum number of stages (N_{\min})

1. Calculation of (N_{\min}) by graphical method.

If no product is withdrawn from the still, that is $D = 0$, then the column is said to **operate under conditions of total reflux** and the top operating line has its maximum slope of unity, and coincides with the line $x = y$. If the reflux ratio is reduced, the slope of the operating line is reduced and more stages are required to pass.

$$\text{When } D = 0 \implies R_{\max} = \frac{L_R}{D} = \frac{L_R}{0} = \infty$$

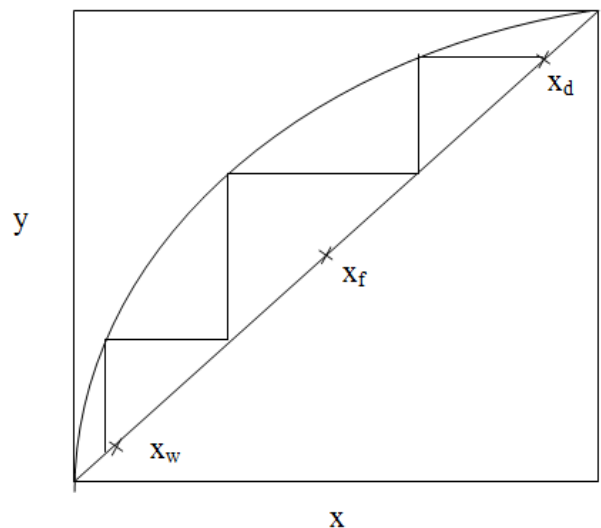
(at total reflux)

$$\text{Slope} = \frac{R}{R + 1} = \frac{\infty}{\infty + 1} = 1.0$$

$$\text{Intercept} = \frac{x_d}{R + 1} = \frac{x_d}{\infty + 1} = 0$$

$$N_{\min} = 3 \text{ stages}$$

$$= 2 \text{ plates}$$



2. Calculation of (N_{\min}) by Fenske equation.

$$N_{\min} + 1 = \frac{\log \left[\left(\frac{x_d}{1 - x_d} \right) \times \left(\frac{1 - x_w}{x_w} \right) \right]}{\log(\alpha_{av.})}$$

Efficiency of Column

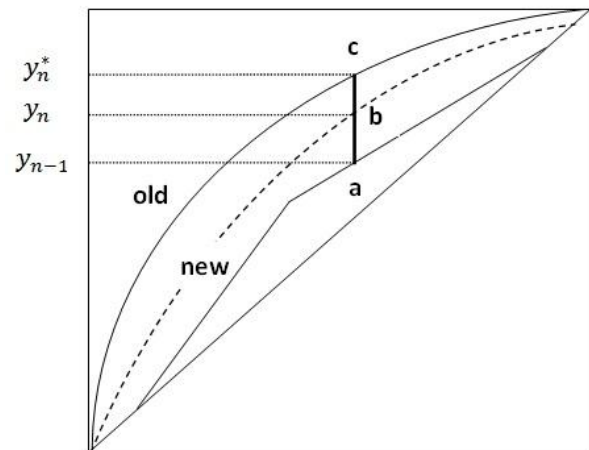
1. Overall efficiency (η_c):

$$\eta_c = \frac{\text{Theoritical No.}}{\text{Actual No.}}$$

2. Plate efficiency:

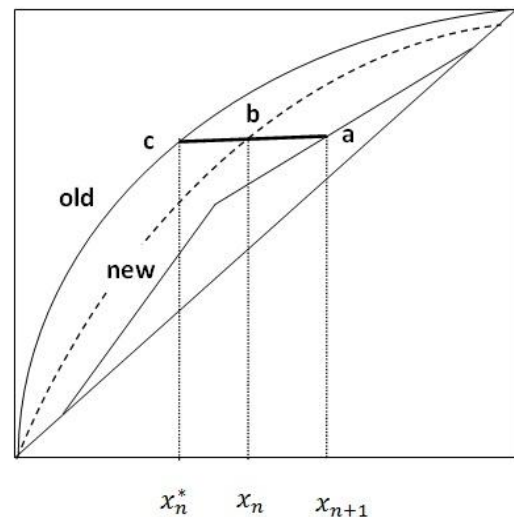
i. Efficiency based on vapour phase (Emv).

$$Emv = \frac{y_n - y_{n-1}}{y_n^* - y_{n-1}} = \frac{ab}{ac}$$



ii. Efficiency based on liquid phase (Eml).

$$Eml = \frac{x_{n+1} - x_n}{x_{n+1} - x_n^*} = \frac{ab}{ac}$$



Heat Balance on Distillation Column

Overall heat balance on distillation column:

$$F \cdot h_f + q_r = D \cdot h_d + W \cdot h_w + q_c \dots\dots\dots(1)$$

Where:

q_r : heat added to the reboiler (kw).

q_c : heat removed from condenser (kw).

h : enthalpy of the liquid (kJ/kmol).

H : enthalpy of the vapour (kJ/kmol).

Heat balance on condenser

$$V \cdot H_v = q_c + (D + L) h_d$$

$$V \cdot H_v = q_c + V \cdot h_d$$

$$q_c = V (H_v - h_d)$$

$$\text{But, } V = D + L = D + DR = D(R + 1)$$

$$q_c = D (R + 1) (H_v - h_d) \dots\dots\dots(2)$$

Then the amount of heat added to the reboiler is:

$$q_r = D \cdot h_d + W \cdot h_w + q_c - F \cdot h_f \dots\dots\dots(3)$$

Calculation the amount of water required in the condenser:

$$q_c = m_{H_2O} \cdot C_{P(H_2O)} (T_{in} - T_{out})$$

$$m_{H_2O} = \frac{q_c}{C_{P(H_2O)} (T_{in} - T_{out})}$$

Calculation the amount of steam required in the reboiler:

$$q_r = (m^* \cdot \lambda)_{\text{steam}}$$

$$m^*_{\text{steam}} = \frac{q_r}{\lambda_{\text{steam}}}$$

Where: λ = heat of vaporization of steam (kJ/kg).

Multiple feeds and side streams

In general, a side stream is defined as any product stream other than the overhead product and the residue such as the streams S' , S'' and S''' in Figure 11.21. In a similar way, F_1 and F_2 are separate feed streams to the column. Side streams are most often removed with multi-component systems, although they may be used with binary mixtures.

- The number of operating lines in this type of distillation are equal to the number of feed and side streams plus one.
- The arrangement of these streams in the distillation column depends on the mole fraction of more volatility component in these streams.

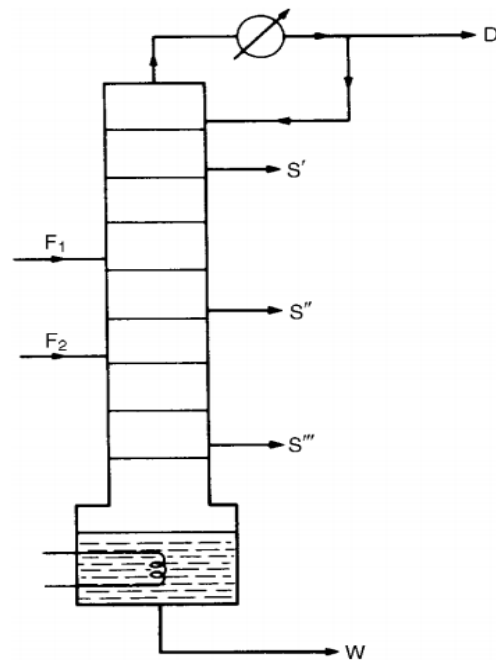


Figure 11.21. Column with multiple feeds and sidestreams

Example: It is desired to distill a feed (F_1) of 200 kmol/hr partially vaporized so that 80% vapour and 20% liquid containing 20% heptane and 80% ethylbenzene and (F_2) of 150 kmol/hr saturated liquid containing 40% heptane to give a distillate containing 95% heptane and a bottom containing 3mol% heptane. It is proposed to withdraw 20% of heptane in the entering streams as a side stream with a mole fraction of 0.7 heptane:

- Determine the number of theoretical plates required and the position of the feeds and side stream plates, if the used reflux ratio is (3).
- If the stream at 138 kPa gauge pressure is used for heating, how much steam required per hr.
- If cooling water enters the condenser at 27°C and leaves at 65°C, how much cooling water is required per hr.

The vapour-liquid equilibrium:

X	0	0.08	0.25	0.485	0.79	1.0
Y	0	0.23	0.514	0.73	0.904	1.0

Where the heptane boiling point and latent heat are 98.4°C and 95.11 kcal/kmol, respectively. Whereas the ethylbenzene boiling point and latent heat are 136.2°C and 95.4 kcal/kmol, respectively.

Solution:

$$S = 0.2 \left[\frac{(0.2)(200) + (0.4)(150)}{0.7} \right] = 28.5 \text{ kmol/hr}$$

Overall material balance:

$$F_1 + F_2 = D + S + W$$

$$200 + 150 = D + 28.5 + W \dots\dots\dots(1)$$

Material balance on more volatile component:

$$(F_1) (x_{F1}) + (F_2) (x_{F2}) = (D) (x_d) + (S) (x_s) + (W) (x_w)$$

$$(200) (0.2) + (150) (0.4) = (D) (0.95) + (28.5) (0.7) + (W) (0.03) \dots\dots\dots(2)$$

$$D = 76.53 \text{ kmol/hr}$$

$$W = 244.9 \text{ kmol/hr}$$

$$L_n = R D = (3) (76.53) = 229.39 \text{ kmol/hr}$$

$$V_n = (R+1) D = 306.1 \text{ kmol/hr}$$

Top operating line equation:

$$y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D x_d}{V_n}$$

$$y_n = \frac{229.39}{306.1} x_{n+1} + \frac{(76.53)(0.95)}{306.1}$$

$$y_n = 0.75 x_{n+1} + 0.237 \dots\dots\dots(1)$$

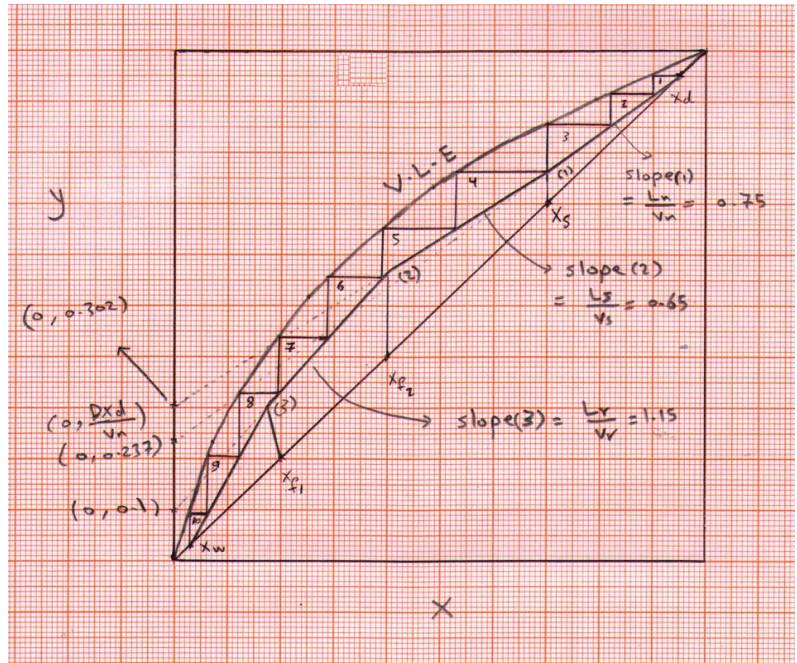
We can plot the top operating line from two points:

$$(x_d, x_d) \text{ and } (0, \frac{D x_d}{V_n})$$

$$(0.95, 0.95) \text{ and } (0, 0.237)$$

q- line of the side stream (S):

Since the side stream is saturated liquid, then the q-line is vertical line from (x_s, x_s) . The top operating line intersect with q-line at point (1).



Drive the equation for the operating line between (S) and (F₂):

Material balance on more volatile component:

$$V_s y_s = L_{s+1} x_{s+1} + D x_d + S x_s$$

$$y_s = \frac{L_{s+1}}{V_s} x_{s+1} + \frac{D x_d + S x_s}{V_s}$$

$$y_s = \frac{L_s}{V_s} x_{s+1} + \frac{D x_d + S x_s}{V_s} \dots\dots\dots(2)$$

Where:

$$L_s = L_n - S = 229.39 - 28.2 = 201 \text{ kmol/hr}$$

$$V_s = V_n = 306.1 \text{ kmol/hr}$$

$$y_s = \frac{201}{306.1} x_{s+1} + \frac{(0.95)(76.53) + (0.7)(28.5)}{306.1}$$

$$y_s = 0.65 x_{s+1} + 0.302$$

We draw this operating line from two points
(point (1)) and (0, 0.302)

q- line of the feed (F₂):

Since the feed (F₂) is saturated liquid, then the q-line is vertical line from (x_{f2}, x_{f2}). The second operating line intersect with q-line of (F₂) at point (2).

Drive the equation for the operating line between (F₂) and (F₁):

Material balance on more volatile component:

$$V_r y_r + F_2 x_{f2} = L_{r+1} x_{r+1} + D x_d + S x_s$$

$$y_r = \frac{L_{r+1}}{V_r} x_{r+1} + \frac{D x_d + S x_s - F_2 x_{f2}}{V_r}$$

$$y_r = \frac{L_r}{V_r} x_{r+1} + \frac{D x_d + S x_s - F_2 x_{f2}}{V_r} \dots\dots\dots(3)$$

Where:

$$V_r = V_s = 306.1 \text{ kmol/hr}$$

$$L_{r+1} = L_{s+1} + q F_2 = L_{s+1} + (1) F_2 = 201 + 150 = 351 \text{ kmol/hr}$$

$$y_r = 1.15 x_{r+1} + 0.1$$

We draw this operating line from two points
(point (2)) and (0, 0.1)

q- line of the feed (F₁):

Since the feed (F₁) is partially vaporized:

q = fraction of liquid = 0.2

$$\text{slope} = \frac{q}{q+1} = \frac{-\text{fraction of liquid}}{\text{fraction of vapour}} = \frac{-0.2}{0.8} = \frac{-1}{4}$$

Then the q-line of (F₁) is draw form (x_{f1}, x_{f1}) with slope = $\frac{-1}{4}$

The third operating line intersect with q-line of (F₁) at point (3).

Bottom operating line equation:

$$y_m = \frac{L_m}{V_m} x_{n+1} - \frac{W x_w}{V_m}$$

Where:

$$L_m = L_r + q F_1 = 351 + (0.2)(200) = 391 \text{ kmol/hr}$$

$$V_m = L_m - W = 391 - 244.9 = 146 \text{ kmol/hr}$$

$$y_m = 2.67 x_{n+1} - 0.05 \dots\dots\dots(4)$$

We draw the bottom operating line from two points

(point (3)) and (0, -0.05)

Or point (3) and (x_w , x_w)

From plot:

No. of stages = 10

No. of plates = 10 - 1 = 9

The side stream position = 3 stage from top = 2 plate

The feed stream (F_2) position = 5 stage from top = 4 plate

The side stream (F_1) position = 8 stage from top = 7 plate

b. $Q_r = V_m \lambda$

where: $V_m = 146 \text{ kmol/hr}$

$$\lambda = 95.4 \text{ kcal/kmol}$$

$$Q_r = (146) (95.4) = 13928 \text{ kcal/hr}$$

For steam: $Q_r = m_{\text{steam}} \cdot \lambda_{\text{steam}}$

From steam table: λ_{steam} at 239.3 kPa = 526 kcal/kg

$$Q_r = m_{\text{steam}} \cdot \lambda_{\text{steam}} \implies 13928 = m_{\text{steam}} \cdot (526)$$

$$m_{\text{steam}} = 26.4 \text{ kg/hr}$$

c. $Q_c = V_n \lambda$

where: $V_n = 306.1 \text{ kmol/hr}$

$$\lambda = 95.11 \text{ kcal/kmol}$$

$$Q_c = V_n \lambda = (306.1) (95.11) = 29.11 \cdot 10^3 \text{ kcal/hr}$$

For water: $Q_c = m_{\text{water}} \cdot c_p \Delta T$

$$29.11 \cdot 10^3 = m_{\text{water}} \cdot (1) (65-27)$$

$$\implies m_{\text{water}} = 760 \text{ kg/hr}$$

The Lewis–Sorel method

This method is used to calculate the mole fraction of components on the plates and the number of plates by stage to stage calculations for binary mixture with one feed only. There are two types of calculations:

1. Calculations from top to bottom section:

a. $x_d = y_t$

b. x_t is to be found from the equilibrium data (Graphically or by equation):

$$x_t = \frac{y_t}{\alpha_{AB} - (\alpha_{AB} - 1)y_t}$$

c. y_{t-1} is to be calculated from rectifying operating line equation:

$$y_{t-1} = \frac{L_n}{V_n} x_t + \frac{D x_d}{V_n} \quad \text{or} \quad y_{t-1} = \frac{R}{R+1} x_t + \frac{x_d}{R+1}$$

d. x_{t-1} is to be found from the equilibrium data (Graphically or by equation):

$$x_{t-1} = \frac{y_{t-1}}{\alpha_{AB} - (\alpha_{AB} - 1)y_{t-1}}$$

e. y_{t-2} is to be calculated from rectifying operating line equation:

$$y_{t-2} = \frac{L_n}{V_n} x_{t-1} + \frac{D x_d}{V_n} \quad \text{or} \quad y_{t-2} = \frac{R}{R+1} x_{t-1} + \frac{x_d}{R+1}$$

a. Continue until reach x_f , then use stripping operating line equation to calculate the mole fraction in the vapour (y) and stop the calculation when reach x_w .

2. Calculations from bottom to top section:

b. $x_w = x_1$

c. y_1 is to be found from the equilibrium data (Graphically or by equation):

$$y_1 = \frac{\alpha_{AB} * x_1}{1 + (\alpha_{AB} - 1) x_1}$$

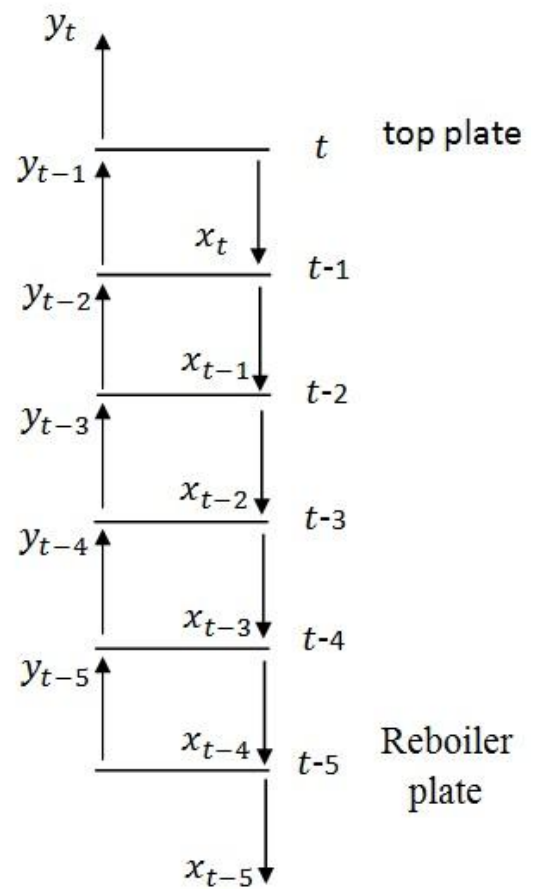
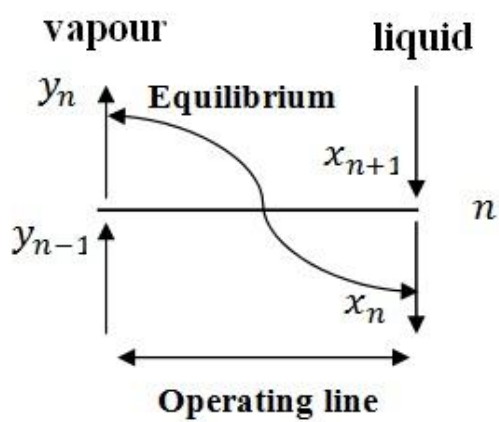
d. x_2 is to be calculated from stripping operating line equation:

$$y_1 = \frac{L_m}{V_m} x_2 - \frac{W x_w}{V_m}$$

- e. y_2 is to be found from the equilibrium data (Graphically or by equation):

$$y_2 = \frac{\alpha_{AB} * x_2}{1 + (\alpha_{AB} - 1) x_2}$$

- f. Continue until reach x_f , then use rectifying operating line equation to calculate the mole fraction in the liquid (x) and stop the calculation when reach x_d .



Example: A mixture of benzene and toluene containing 50 mole per cent benzene is to be separated to give a product containing 90 mole per cent benzene at the top, and a bottom product containing not more than 10 mole per cent benzene. The feed enters the column at its boiling point. It is proposed to operate the unit with an (L_n/D) ratio of 3.5:1 kmol/kmol product. It is required to find the composition of the liquid on the third theoretical plate from top and on the third theoretical plate from bottom. Take the relative volatility as 2.16.

Solution:

Basis: 100 kmol/hr of feed

$$F = D + W$$

$$100 = D + W \quad \dots\dots\dots(1)$$

$$F (x_f) = D (x_d) + W (x_w)$$

$$100 (0.5) = D (0.9) + W (0.1) \quad \dots\dots\dots(2)$$

From Eq.(1) & Eq.(2) :

$$D = 50 \text{ kmol/hr}, \quad W = 50 \text{ kmol/hr}$$

$$L_n = R D = 3.5 (50) = 175 \text{ kmol/hr}$$

$$V_n = (R+1) D = (3.5+1) (50) = 225 \text{ kmol/hr}$$

Top operating line equation:

$$y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D x_d}{V_n}$$

$$y_{t-1} = 0.778 x_t + 0.2 \quad \dots\dots\dots(3)$$

Equilibrium relation:

$$x_A = \frac{y_A}{\alpha_{AB} - (\alpha_{AB} - 1)y_A} = \frac{y_A}{2.16 - 1.16 y_A}$$

Calculations from top to bottom section:

$$y_t = x_d = 0.9$$

From equilibrium relation:

$$x_t = \frac{y_t}{2.16 - 1.16 y_t} = \frac{0.9}{2.16 - 1.16 (0.9)} = 0.806$$

From top operating line equation:

$$y_{t-1} = 0.778 x_t + 0.2 = 0.778 (0.806) + 0.2 = 0.827$$

From equilibrium relation:

$$x_{t-1} = \frac{y_{t-1}}{2.16 - 1.16 y_{t-1}} = \frac{0.827}{2.16 - 1.16 (0.827)} = 0.688$$

From top operating line equation:

$$y_{t-2} = 0.778 x_{t-1} + 0.2 = 0.778 (0.688) + 0.2 = 0.735$$

From equilibrium relation:

$$x_{t-2} = \frac{y_{t-2}}{2.16 - 1.16 y_{t-2}} = \frac{0.735}{2.16 - 1.16 (0.735)} = 0.562$$

Calculations from bottom to top section:

$$L_m = L_n + q F \quad (\text{for saturated liquid } q=1)$$

$$L_m = 175 + (1) 100 = 275 \text{ kmol/hr}$$

$$V_m = V_n + (q-1) F$$

$$V_m = V_n = 225 \text{ kmol/hr}$$

Bottom operating line equation:

$$y_m = \frac{L_m}{V_m} x_{m+1} - \frac{W x_w}{V_m}$$

$$y_1 = 1.22 x_2 - 0.022$$

Equilibrium relation:

$$y_A = \frac{\alpha_{AB} * x_A}{1 + (\alpha_{AB} - 1) x_A}$$

$$y_1 = \frac{2.16 x_1}{1 + 1.16 x_1}$$

$$x_1 = x_w = 0.1$$

From equilibrium relation:

$$y_1 = \frac{2.16 x_1}{1 + 1.16 x_1} = \frac{2.16 (0.1)}{1 + 1.16 (0.1)} = 0.193$$

From bottom operating line equation:

$$\begin{aligned} y_1 &= 1.22 x_2 - 0.022 \\ 0.193 &= 1.22 x_2 - 0.022 \\ x_2 &= 0.173 \end{aligned}$$

From equilibrium relation:

$$y_2 = \frac{2.16 x_2}{1 + 1.16 x_2} = \frac{2.16 (0.173)}{1 + 1.16 (0.173)} = 0.315$$

From bottom operating line equation:

$$\begin{aligned} y_2 &= 1.22 x_3 - 0.022 \\ 0.315 &= 1.22 x_3 - 0.022 \\ x_3 &= 0.276 \end{aligned}$$

From equilibrium relation:

$$y_3 = \frac{2.16 x_3}{1 + 1.16 x_3} = \frac{2.16 (0.276)}{1 + 1.16 (0.276)} = 0.451$$

From bottom operating line equation:

$$\begin{aligned} y_3 &= 1.22 x_4 - 0.022 \\ 0.451 &= 1.22 x_4 - 0.022 \\ x_4 &= 0.55 \end{aligned}$$

(Non-ideal mixture)
The Ponchon - Savarit method
The Enthalpy - Composition diagram
The $H - x$ diagram

For a non-ideal system, where the molar latent heat is no longer constant and where there is a substantial heat of mixing, the calculations become much more tedious. For binary mixtures of this kind a graphical model has been developed by Ponchon and Savarit, based on the use of an enthalpy–composition chart. A typical enthalpy–composition or $H - x$ chart is shown in Figure 11.24, where the upper curve **V** is the dew-point curve, and the lower curve **L** the boiling-point curve. The use of this diagram is based on the geometrical properties, as illustrated in Figure 11.25. A quantity of mixture in any physical state is known as a “phase” and is denoted by mass, composition and enthalpy. The phase is shown upon the diagram by a point which shows enthalpy and composition, however it does not show the mass.

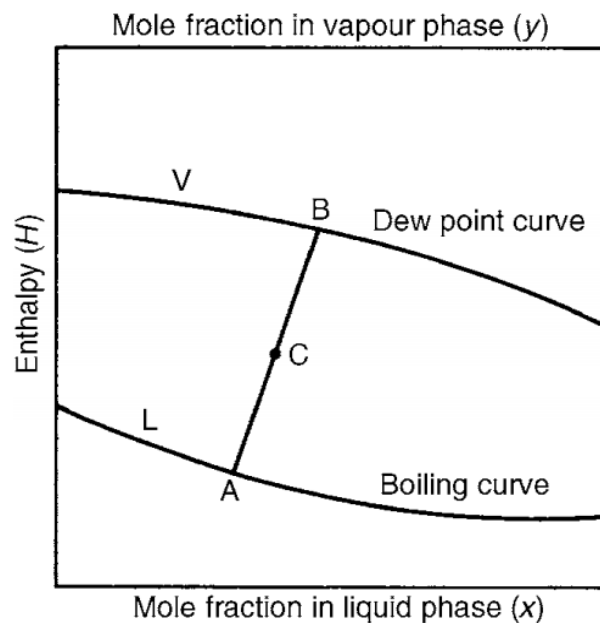


Figure 11.24. Enthalpy–composition diagram, showing the enthalpies of liquid and vapour

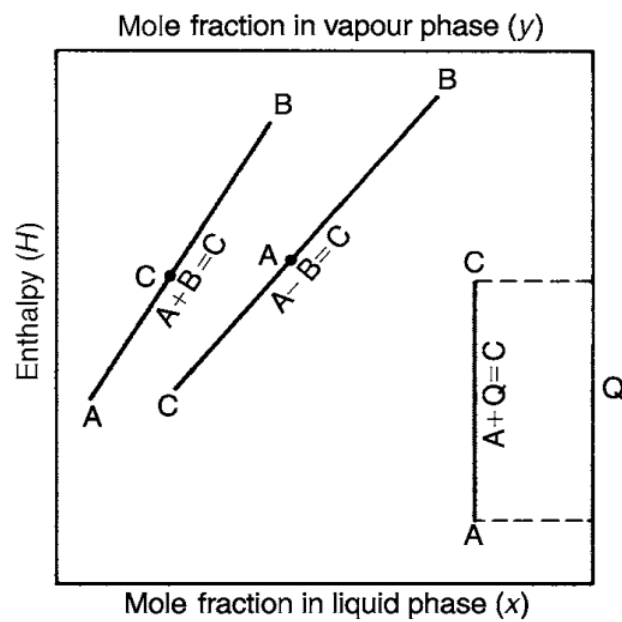


Figure 11.25. Combination and separation of a mixture on an enthalpy–composition diagram

If m is the mass, x the composition and H the enthalpy per unit mass, then the addition of two phases **A** and **B** to give phase **C** is governed by:

$$m_A + m_B = m_C$$

$$m_A x_A + m_B x_B = m_C x_C$$

$$m_A H_A + m_B H_B = m_C H_C$$

Rectifying section:

For non-ideal mixture:

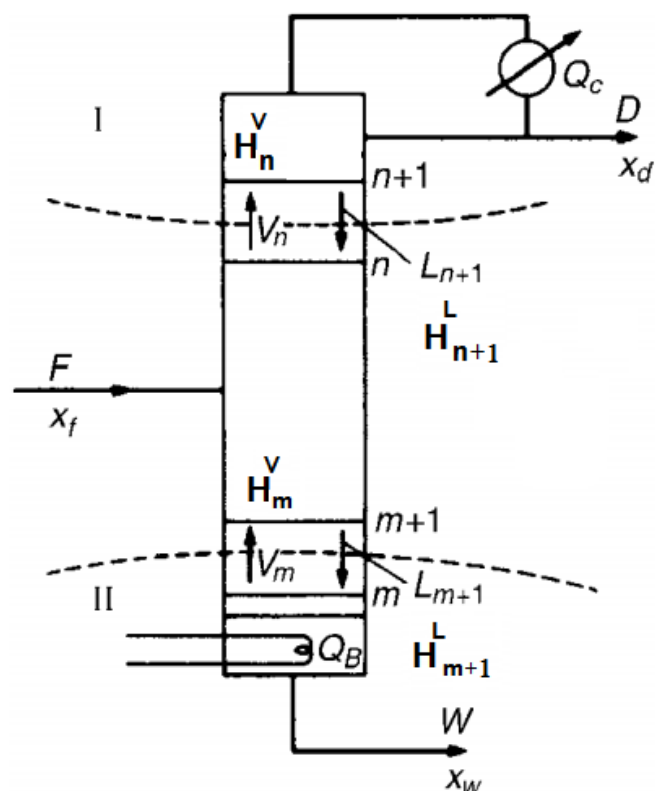
$$\lambda_A \neq \lambda_B$$

$$L_{n+1} \neq L_n \neq L_{n-1}$$

$$V_{n+1} \neq V_n \neq V_{n-1}$$

Overall material balance on condenser:

$$V_n = L_{n+1} + D$$



More volatile component material balance on condenser:

$$V_n y_n = L_{n+1} x_{n+1} + D x_d$$

$$y_n = \frac{L_{n+1}}{V_n} x_{n+1} + \frac{D x_d}{V_n}$$

$$(L_{n+1} + D) y_n = L_{n+1} x_{n+1} + D x_d$$

$$L_{n+1} y_n + D y_n = L_{n+1} x_{n+1} + D x_d$$

$$L_{n+1} y_n - L_{n+1} x_{n+1} = D x_d - D y_n$$

$$L_{n+1} (y_n - x_{n+1}) = D (x_d - y_n)$$

$$\frac{L_{n+1}}{D} = \frac{x_d - y_n}{y_n - x_{n+1}}$$

Enthalpy balance:

$$V_n H_n^V = L_{n+1} H_{n+1}^L + D H_D^L + Q_C$$

$$(L_{n+1} + D) H_n^V = L_{n+1} H_{n+1}^L + D (H_D^L + \frac{Q_C}{D})$$

$$\text{Let: } (H_D^L + \frac{Q_C}{D}) = H_D'$$

$$(L_{n+1} + D) H_n^V = L_{n+1} H_{n+1}^L + D H_D'$$

$$\frac{L_{n+1}}{D} = \frac{H_D' - H_n^V}{H_n^V - H_{n+1}^L}$$



$$\frac{L_{n+1}}{D} = \frac{H_D' - H_n^V}{H_n^V - H_{n+1}^L} = \frac{x_d - y_n}{y_n - x_{n+1}}$$

$$y_n = \left[\frac{H'_D - H_n^V}{H'_D - H_{n+1}^L} \right] x_{n+1} + \left[\frac{H_n^V - H_{n+1}^L}{H'_D - H_{n+1}^L} \right] x_d$$

Stripping section:

Overall material balance on loop II:

$$V_m = L_{m+1} - W$$

More volatile component material balance on loop II:

$$V_m y_m = L_{m+1} x_{m+1} - W x_w$$

$$y_m = \frac{L_{m+1}}{V_m} x_{m+1} - \frac{W x_w}{V_m}$$

$$(L_{m+1} - W) y_m = L_{m+1} x_{m+1} - W x_w$$

$$L_{m+1} y_m - W y_m = L_{m+1} x_{m+1} - W x_w$$

$$L_{m+1} y_m - L_{m+1} x_{m+1} = W y_m - W x_w$$

$$L_{m+1} (y_m - x_{m+1}) = W (y_m - x_w)$$

$$\frac{L_{m+1}}{W} = \frac{y_m - x_w}{y_m - x_{m+1}}$$

Enthalpy balance:

$$L_{m+1} H_{m+1}^L + Q_r = V_m H_m^v + W H_w^L$$

$$(L_{m+1} - W) H_m^v = L_{m+1} H_{m+1}^L + W \left(\frac{Q_r}{W} - H_w^L \right)$$

$$\text{Let: } H_w^L - \frac{Q_r}{W} = H'_w$$

$$(L_{m+1} - W) H_m^v = L_{m+1} H_{m+1}^L + W H'_w$$

$$\frac{L_{m+1}}{W} = \frac{H_m^v - H_w'}{H_m^v - H_{m+1}^L} = \frac{y_m - x_w}{y_m - x_{m+1}}$$

$$y_m = \left[\frac{H_m^v - H_w'}{H_{m+1}^L - H_w'} \right] x_{m+1} - \left[\frac{H_m^v - H_{m+1}^L}{H_{m+1}^L - H_w'} \right] x_w$$

The equilibrium relation is given as below:

Temp.	X	y	H^V	H^L
-	/ - tie line-	/	-	-
-	/ - tie line-	/	-	-
-	/ - tie line-	/	-	-
-	/ - tie line-	/	-	-
-	/ - tie line-	/	-	-

Determination of the number of plates on the $H - x$ diagram

1. We draw a plot (H^L vs. x) and (H^V vs. y) on the same graph (which represents the equilibrium curves), where the upper equilibrium curve is for the vapour and the lower equilibrium curve is for the liquid.

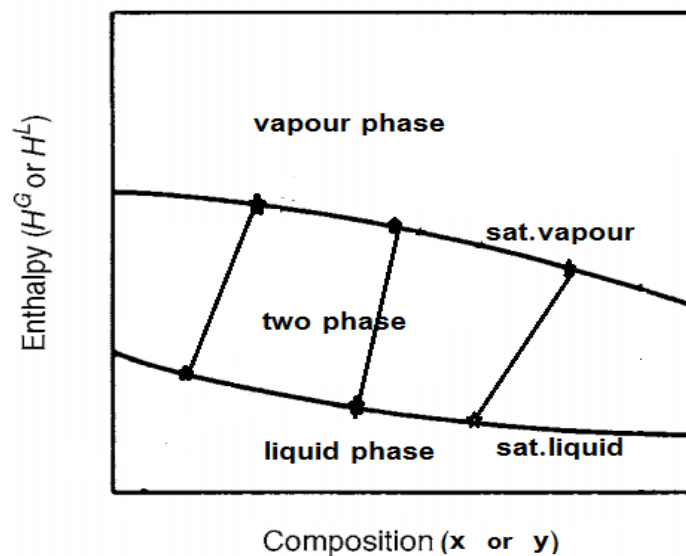


Figure 11.27. Enthalpy–composition diagram

2. We draw the tie line between the mole fraction of liquid (x) on liquid (lower) curve and the mole fraction of vapour (y) on vapour (upper) curve.
3. We locate:- x_w , x_f and x_d on the plot.
4. We find the pole **N** by the point (x_d, H'_D). Where: $H'_D = (H_D^L + \frac{Q_C}{D})$.
5. Pole **M** is located on the extension of **N-F** cutting the ordinate at x_w in **M**.
6. The condition of the vapour leaving the top plate is shown at **V₇** on the saturated vapour curve with abscissa x_d .
7. The condition of the liquid on the top plate is then found by drawing the tie line **T₇** from **V₇** to **L₇** on the saturated liquid curve.

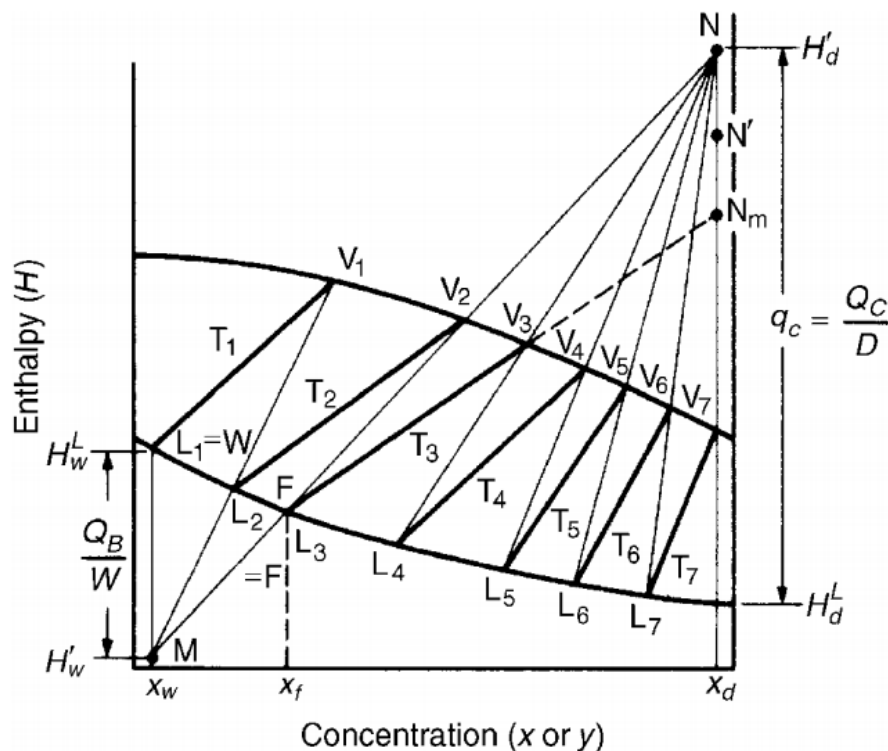


Figure 11.28. Determination of the number of plates using the enthalpy–composition diagram

8. The condition **V₆** of the vapour on the second plate is found by drawing **L₇-N** to cut the saturated vapour curve on **V₆**.
9. **L₆** is then found on the tie line **T₆**.

10. The conditions of vapour and liquid V_5, V_4, V_3 and L_5, L_4 are then found by similar construction. Tie line T_3 gives L_3 , which has the same composition as the feed.
11. V_2 is then found using the line $M-F-V_2$ as this represents the vapour on the top plate of the stripping section.
12. L_2, L_1 and V_1 are then found by a similar construction. L_1 has the required composition of the bottoms, x_w .
13. No. of plates = No. of stages - 1

Determination of the minimum number of plates (N_{\min})

The maximum reflux ratio (R_{\max}) at $D = 0$ (Total reflux)

$$y_n = \frac{L_{n+1}}{V_n} x_{n+1} + \frac{D x_d}{V_n}$$

$$y_n = \frac{L_{n+1}}{L_{n+1} + D} x_{n+1} + \frac{D x_d}{L_{n+1} + D}$$

$$y_n = \frac{L_{n+1}}{L_{n+1}} x_{n+1} + 0$$

$$y_n = x_{n+1}$$

$$N_{\min} = 8 \text{ stages} = 7 \text{ plates}$$

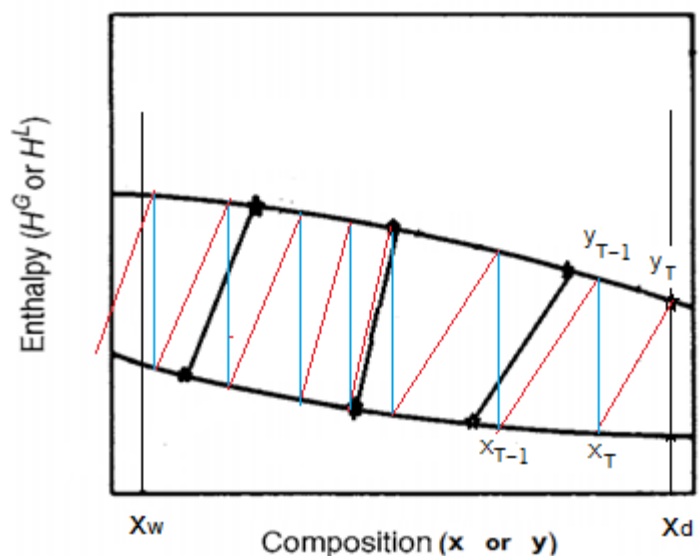


Figure 11.27. Enthalpy–composition diagram

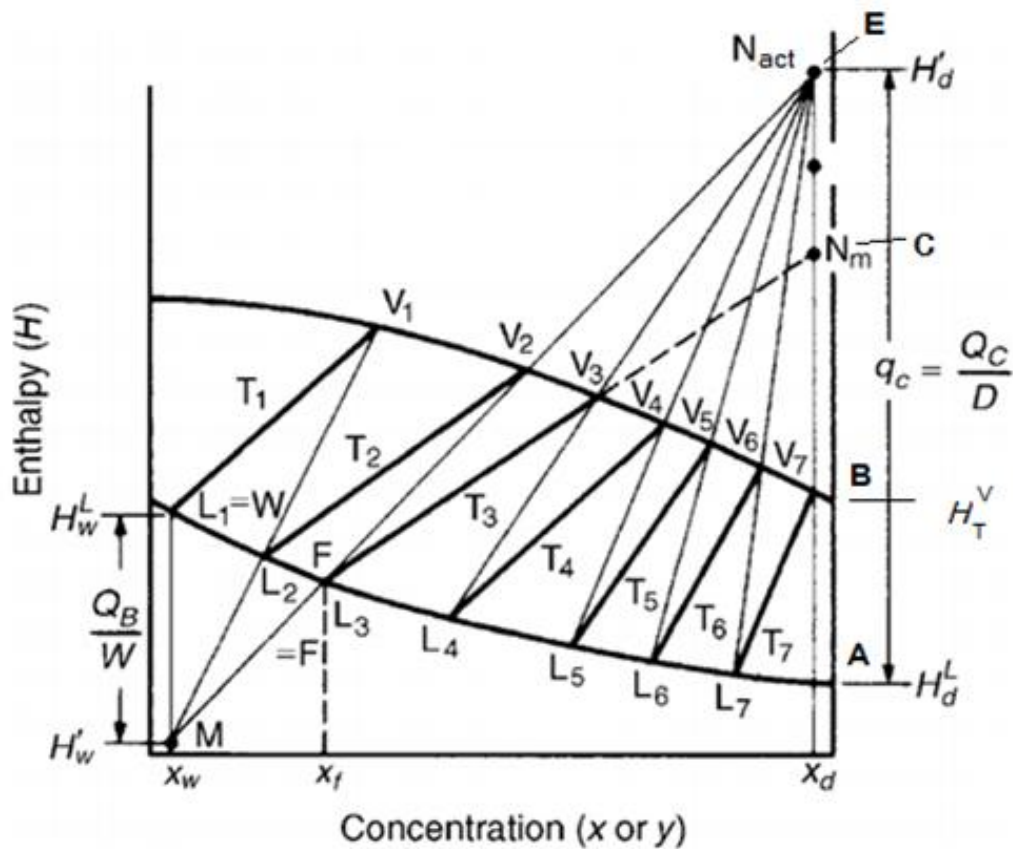
Determination of the minimum reflux ratio (R_{\min})

The minimum reflux ratio (R_{\min}) is determined at maximum number of plates (N_{\max}):

$$R_{\min} = \frac{\text{Lenght CB}}{\text{Lenght BA}}$$

$$R_{\text{act}} = (1.1 - 1.5)R_{\min}$$

$$R_{\text{act}} = \frac{\text{Lenght EB}}{\text{Lenght BA}} = \frac{H'_D - H_T^V}{H_T^V - H_D^L}$$



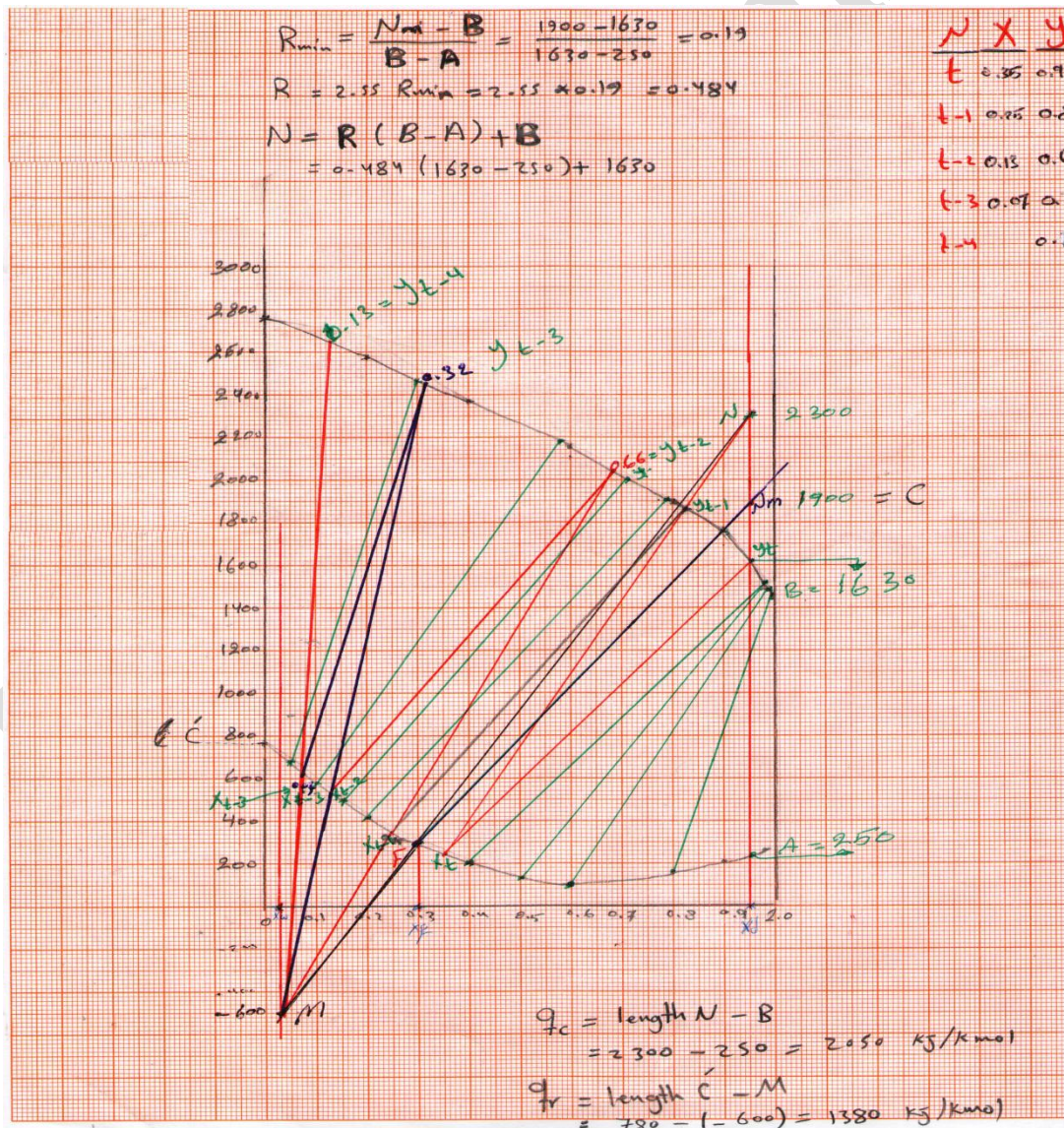
Example : A 30 mol% solution of A and B at its boiling point is to be continuous distilled to give a top product of 95 mol% of A and a bottom product containing not more than 3 mol% of A, the reflux ratio is to be 2.55 R_{\min} value and an estimate of the number of theoretical plates required to effect the separation is to be obtained by Ponchon-Savarit method. Also determine the condenser duty and the heat required supplied in the reboiler. Given the following data:

x_A	0	0.05	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.8
y_A	0	0.30	0.58	0.71	0.79	0.9	0.98	0.98	0.99	0.995

Enthalpy data kJ/kmol

x_A	0	0.2	0.4	0.6	0.8	0.9	1.0
H^L	767	418	198	116	163	209	279
H^V	2765	2580	2370	2160	1905	1765	1490

Solution:



Batch Distillation

Similar to differential distillation with rectifying section only and reflux ratio. In batch distillation the whole of a batch is run into the boiler of the still and, on heating, the vapour is passed into a fractionation column, as shown in Figure 11.33. As with continuous distillation, the composition of the top product depends on the still composition, the number of plates in the column and on the reflux ratio used. There are two possible modes of operation in batch distillation column:

1. Operation at constant product composition (x_d), the reflux ratio may be increased continuously [mean, constant (x_d) and variable (R)].
2. Operation at constant reflux ratio (R), the composition of the top product will decrease with time [mean, constant (R) and variable (x_d)].

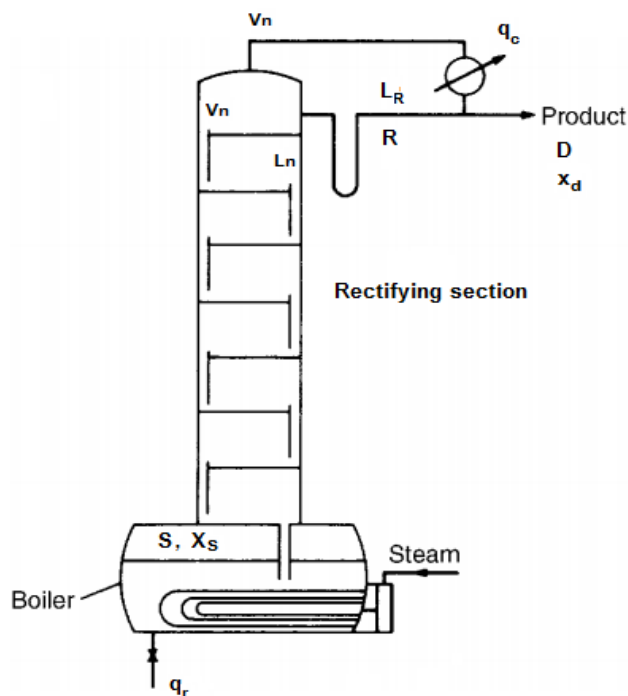


Figure 11.33. Column for batch distillation

1. Operation at constant product composition (x_d)

The batch distillation column operates with a rectifying section only. Therefore, there is one operating line equation represents this section which is the rectifying operating line equation. There are some symbols are used in this type of distillation:

S_1 : is the number of moles of feed in the still initially.

X_{S1} : is the mole fraction of more volatile component in the feed

S_2 : is the number of moles of liquid mixture in the still after concentrated (distillation).

X_{S2} : is the mole fraction of more volatile component in the still after concentrated (distillation).

Over all material balance

$$S_1 - S_2 = D$$

Over all material balances on more volatile component:

$$S_1 (x_{s1}) - S_2 (x_{s2}) = D (x_d)$$

$$S_1 (x_{s1}) - (S_1 - D) (x_{s2}) = D (x_d)$$

$$S_1 (x_{s1}) - S_1 (x_{s2}) = D (x_d) - D (x_{s2})$$

$$D = S_1 \left[\frac{x_{s1} - x_{s2}}{x_d - x_{s2}} \right] \dots \dots \dots (1)$$

The heat to be supplied in the boiler to provide this reflux during the total distillation Q_R is given by:

$$Q_R = \int_0^{L_n} \lambda \, dL_n = \lambda \int_{R=R_1}^{R=R_2} R \, dD = \lambda \cdot \text{Area} \dots \dots \dots (2)$$

Where: λ is the latent heat per mole.

Q_R is the amount of heat required in the reboiler to vaporize (L_n).

$$Q_T = Q_R + \lambda \dots \dots \dots (3)$$

Equation (2) may be integrated graphically if the relation between R and D is known. For any desired value of R , x_s may be obtained by drawing the operating line, and marking off the steps corresponding to the given number of stages. The amount of product D is then obtained from equation (1) and, if the corresponding values of R and D are plotted, graphical integration will give the value of $R \, dD$.

Example (1): A mixture of ethyl alcohol and water with 0.55 mole fraction of alcohol is distilled to give a top product of 0.75 mole fraction of alcohol. The column has four ideal plates and the distillation is stopped when the reflux ratio has to be increased beyond 4.0. What is the amount of distillate obtained, and the heat required per kmol of product?

The equilibrium data are given as:

Mole fraction of ethyl alcohol in liquid	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Mole fraction of ethyl alcohol in vapour	0	0.42	0.52	0.58	0.61	0.65	0.7	0.75	0.81	0.9

Solution:

The distillation with constant (x_d) and variable (R).

$$S_1 = 100 \text{ kmol}$$

$$N = 4 \text{ plates} = 5 \text{ stages}$$

$$x_{s1} = 0.55, \quad x_{s2} = ?, \quad D = ?, \quad x_d = 0.75$$

$$R_2 = 4, \quad R_1 = ?$$

Values of x_s are found as shown in Figure 11.35 for the two values of R of 0.85 and 4. The amount of product is then found from equation (1). Thus, for $R = 4$:

$$D = S_1 \left[\frac{x_{s1} - x_{s2}}{x_d - x_{s2}} \right] = 100 \left[\frac{0.55 - 0.05}{0.75 - 0.05} \right] = 41.4 \text{ kmol}$$

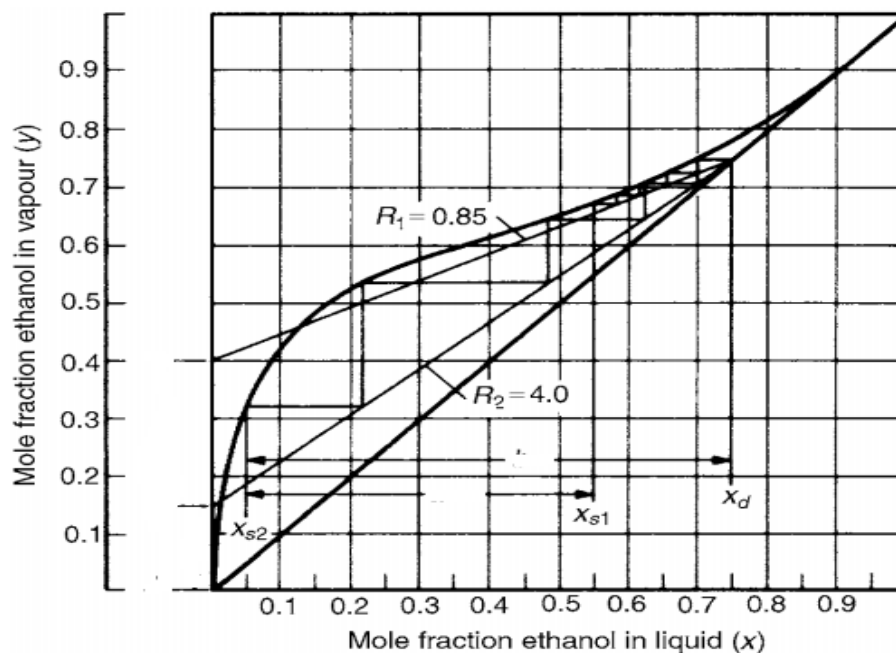
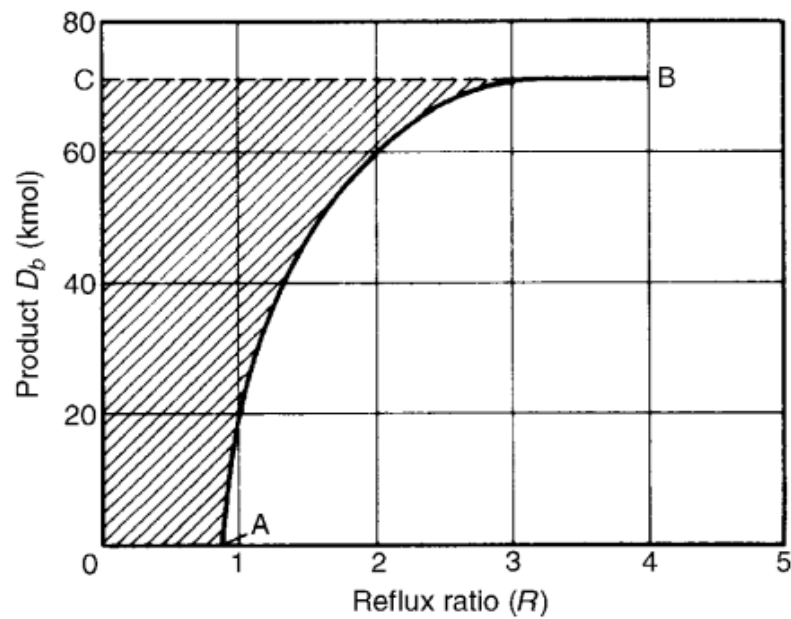


Figure 11.35. Batch distillation—constant product composition

values of D found in this way are:

R	x_s	D
0.85	0.55	0
1.0	0.50	20.0
1.5	0.37	47.4
2.0	0.20	63.8
3.0	0.075	70.5
4.0	0.05	71.4



The area under the curve = 96 kmol

$$Q_R = \lambda \int_{R=R_1}^{R=R_2} R dD = \lambda \cdot \text{Area} = (4000)(96) = 380000 \text{ kJ} = 380 \text{ MJ}$$

The heat to be supplied to provide the reflux per kmol of product is then $(380/71.4) = 5.32 \text{ MJ}$ and the total heat is $(5.32 + 4.0) = \underline{\underline{9.32 \text{ MJ/kmol product}}}$.

2. Operation at constant reflux ratio (R)

If the same column is operated at a constant reflux ratio (**R**), the concentration of the more volatile component in the top product will continuously fall. Over a small interval of time dt , the top-product composition with respect to the more volatile component will change from x_d to $x_d + dx_d$, where dx_d is negative for the more volatile component. If in this time the amount of product obtained is dD , then a material balance on the more volatile component gives:

Over all material balance

$$D = S_1 - S_2 \dots\dots\dots(1)$$

Over all material balances on more volatile component:

$$S_1 (x_{s1}) - S_2 (x_{s2}) = D (x_d)$$

$$(x_d) = \frac{S_1 (x_{s1}) - S_2 (x_{s2})}{D} \dots\dots\dots(2)$$

$$\text{More volatile component removed in product} = dD \left[x_d + \frac{dx_d}{2} \right]$$

which, neglecting second – order terms, gives: $= x_d dD$

$$\text{and:} \quad x_d dD = -d(Sx_s)$$

$$\text{but:} \quad dD = -dS$$

$$\text{and hence:} \quad -x_d dS = -S dx_s - x_s dS$$

$$\text{and:} \quad S dx_s = dS (x_d - x_s)$$

$$\text{Thus:} \quad \int_{S_1}^{S_2} \frac{dS}{S} = \int_{x_{s1}}^{x_{s2}} \frac{dx_s}{(x_d - x_s)}$$

$$\ln \frac{S_1}{S_2} = \int_{x_{s2}}^{x_{s1}} \frac{dx_s}{(x_d - x_s)} \dots\dots\dots(3)$$

The heat to be supplied to provide the reflux Q_R is given by:

$$Q_R = \lambda R D \dots\dots\dots(4)$$

$$Q_T = Q_R + \lambda \dots\dots\dots(5)$$

Example (2): If the same batch as in Example (1) is distilled with a constant reflux ratio of $R = 2.1$, what will be the heat required and the average composition of the distillate if the distillation is stopped when the composition in the still has fallen to 0.105 mole fraction of ethanol?

Solution:

The initial composition of the top product will be 0.78, as shown in Figure 11.37, and the final composition will be 0.74. Values of x_d , x_s , $x_d - x_s$ and of $1/(x_d - x_s)$ for various values of x_s and a constant reflux ratio are:

x_s	x_d	$x_d - x_s$	$1/(x_d - x_s)$
0.550	0.780	0.230	4.35
0.500	0.775	0.275	3.65
0.425	0.770	0.345	2.90
0.310	0.760	0.450	2.22
0.225	0.750	0.525	1.91
0.105	0.740	0.635	1.58

Values of x_s and $1/(x_d - x_s)$ are plotted in Figure 11.38 from which $\int_{0.105}^{0.55} (dx_s/(x_d - x_s)) = 1.1$.

From equation 11.103: $\ln(S_1/S_2) = 1.1$ and $(S_1/S_2) = 3.0$.

Product obtained, $D_b = S_1 - S_2 = (100 - 100/3) = 66.7$ kmol.

Amount of ethanol in product = $x_1 S_1 - x_2 S_2$

$$= (0.55 \times 100) - (0.105 \times 33.3) = 51.5 \text{ kmol}$$

Thus: average composition of product = $(51.5/66.7) = \underline{\underline{0.77 \text{ mole fraction ethanol}}}$.

The heat required to provide the reflux = $(4000 \times 2.1 \times 66.7) = 560,380$ kJ.

Heat required to provide reflux per kmol of product = $(560,380/66.7) = \underline{\underline{8400 \text{ kJ}}}$.

Thus in Example 2 the total heat required per kmol of product is $(5320 + 4000) = 9320$ kJ and at constant reflux ratio (Example 1) it is $(8400 + 4000) = 12,400$ kJ, although the average quality of product is 0.77 for the second case and only 0.75 for the first.

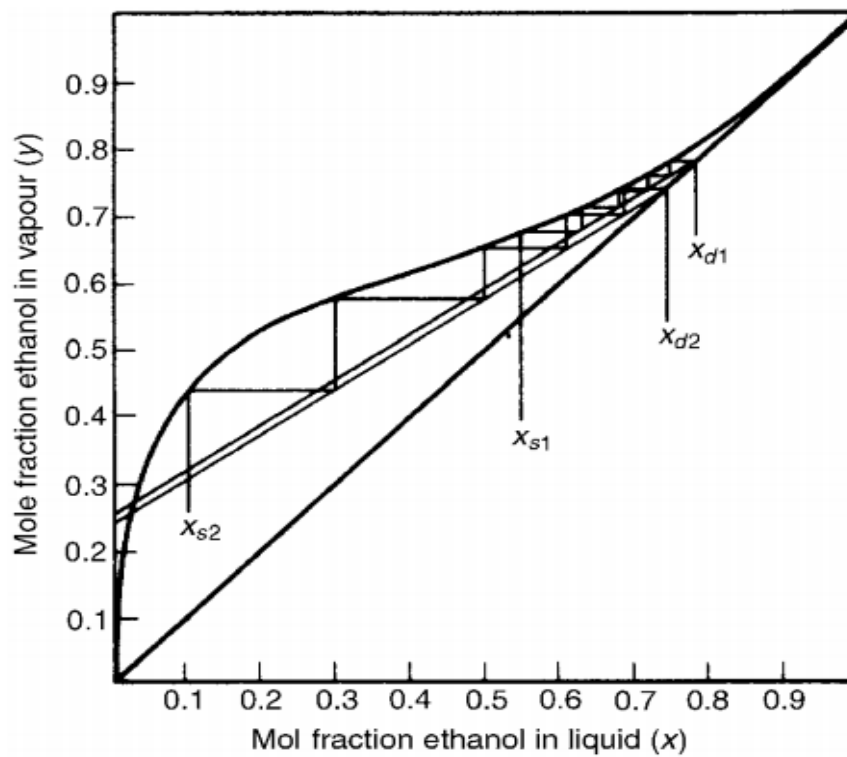


Figure 11.37. Batch distillation–constant reflux ratio (Example 11.13)

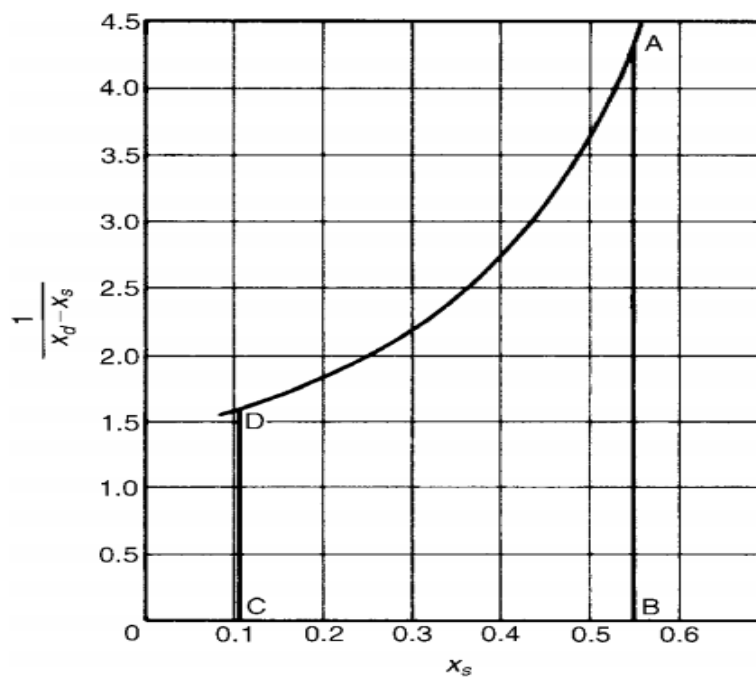


Figure 11.38. Graphical integration for Example 11.13

Multicomponent Distillation

Multicomponent distillation refers to the separation of mixture containing more than two components. In the fractionation of multicomponent mixtures, the essential requirement is often the separation of two components. Such components are called the key components. *The concept of the key components is introduced where the light and heavy key components are those whose specifications are fixed in the top and bottom streams.*

There are two types of key components:

1. **Light key component:** the component which appears in the top and bottom streams and has high concentration in the top (largest volatility, α).
2. **Heavy key component:** the component which appears in the top and bottom streams and has high concentration in the bottom (lowest volatility, α).

Example (1):

Comp.	Distillate	Waste	
A	0.03	0.0	
B	0.07	0.0	
C	0.147	0.003	Light key
D	0.1255	0.2045	
E	0.003	0.297	Heavy key
F	0.0	0.12	

Example (2):

Comp.	Feed (%)	Distillate (%)	Waste (%)	
A	1	1.1	0.0	
B	6	15.1	0.0	
C	33	83.8	2.3	Light key
D	47	0.0	76.9	Heavy key
E	13	0.0	20.8	

* إذا ظهر مركب واحد فقط مشترك فنلاحظ تركيزه فإذا كان تركيزه في الأعلى (Top) اكبر من تركيزه في الأسفل (Bottom) فيعتبر هو المركب الخفيف (Light key) والعكس صحيح.

Calculation of the theoretical number of plates

One of the most successful methods for calculating the number of plates necessary for a given separation is due to **Lewis and Matheson**. This is based on the **Lewis–Sorel method**, described previously for binary mixtures. If the composition of the liquid on any plate is known, then the composition of the vapour in equilibrium is calculated from a knowledge of the vapour pressures or relative volatilities of the individual components. The composition of the liquid on the plate above is then found by using an operating equation, as for binary mixtures, although in this case there will be a separate equation for each component.

The vapour-liquid equilibrium data for multicomponent mixture is calculated from:

1. Volatility:

If a mixture of components **A, B, C, D**, and so on has mole fractions x_A, x_B, x_C, x_D , and so on in the liquid and y_A, y_B, y_C, y_D , and so on in the vapour, then:

$$y_A + y_B + y_C + y_D + \dots = 1$$

$$\text{and:} \quad \frac{y_A}{y_B} + \frac{y_B}{y_B} + \frac{y_C}{y_B} + \frac{y_D}{y_B} + \dots = \frac{1}{y_B}$$

$$\alpha_{AB} \frac{x_A}{x_B} + \alpha_{BB} \frac{x_B}{x_B} + \alpha_{CB} \frac{x_C}{x_B} + \alpha_{DB} \frac{x_D}{x_B} + \dots = \frac{1}{y_B}$$

$$\Sigma(\alpha_{AB} x_A) = \frac{x_B}{y_B}$$

$$y_B = \frac{x_B}{\Sigma(\alpha_{AB} x_A)}$$

$$\text{and, similarly:} \quad y_A = \frac{x_A \alpha_{AB}}{\Sigma(\alpha_{AB} x_A)}; \quad y_C = \frac{x_C \alpha_{CB}}{\Sigma(\alpha_{AB} x_A)}; \quad y_D = \frac{x_D \alpha_{DB}}{\Sigma(\alpha_{AB} x_A)}$$

In general:

$$y_i = \frac{\alpha_i x_i}{\Sigma \alpha_i x_i}$$

..... If the calculations from bottom to top.

$$x_i = \frac{y_i / \alpha_i}{\Sigma y_i / \alpha_i}$$

..... If the calculations from top to bottom.

2. Vapour pressure data for ideal system:

$$y_i = \frac{P_i^0 x_i}{\sum P_i^0 x_i} \dots \dots \dots \text{If the calculations from bottom to top.}$$

$$x_i = \frac{y_i/P_i^0}{\sum y_i/P_i^0} \dots \dots \dots \text{If the calculations from top to bottom.}$$

3. K value for hydrocarbons:

$$y_i = \frac{k_i x_i}{\sum k_i x_i} \dots \dots \dots \text{If the calculations from bottom to top.}$$

$$x_i = \frac{y_i/k_i}{\sum y_i/k_i} \dots \dots \dots \text{If the calculations from top to bottom.}$$

The rectifying operating line equation:

The equations for the operating lines depends on the number of components and may be written as:

$$(y_i)_n = \frac{L_n}{V_n} (x_i)_{n+1} + \frac{D}{V_n} (x_i)_d$$

First we have to find D , L_n , V_n and x_d for each component. Then there is an operating line equation for each component which can describe its composition.

The stripping operating line equation:

The equations for the stripping operating lines depends on the number of components and may be written as:

$$(y_i)_m = \frac{L_m}{V_m} (x_i)_{m+1} - \frac{W}{V_m} (x_i)_w$$

First we have to find W , L_m , V_m and x_w for each component. Then there is an stripping operating line equation for each component which can describe its composition.

Calculation minimum reflux ratio (R_{\min}) using Underwood's method

For conditions where the relative volatilities remain constant, UNDERWOOD developed the following two equations from which R_m may be calculated:

$$\frac{\alpha_A x_{fA}}{\alpha_A - \theta} + \frac{\alpha_B x_{fB}}{\alpha_B - \theta} + \frac{\alpha_C x_{fC}}{\alpha_C - \theta} + \dots = 1 - q$$

and:

$$\frac{\alpha_A x_{dA}}{\alpha_A - \theta} + \frac{\alpha_B x_{dB}}{\alpha_B - \theta} + \frac{\alpha_C x_{dC}}{\alpha_C - \theta} + \dots = R_m + 1$$

where: x_{fA} , x_{fB} , x_{fC} , x_{dA} , x_{dB} , x_{dC} , etc., are the mole fractions of components **A**, **B**, **C**, etc., in the feed and distillate, **A** being the light and **B** the heavy key,

q is the ratio of the heat required to vaporise 1 mole of the feed to the molar latent heat of the feed, as in equation ,

α_A , α_B , α_C , etc., are the volatilities with respect to the least volatile component, and

θ is the root of equation , which lies between the values of α_A and α_B .

Example: A mixture of hexane, heptane, and octane is to be separated to give the following products. What will be the value of the minimum reflux ratio, if the feed is liquid at its boiling point?

Component	Feed		Product		Bottoms		Relative volatility
	F (kmol)	x_f	D (kmol)	x_d	W (kmol)	x_w	
Hexane	40	0.40	40	0.534	0	0	2.70
Heptane	35	0.35	34	0.453	1	0.04	2.22
Octane	25	0.25	1	0.013	24	0.96	1.0

Solution:

the light key (**A**) is heptane and the heavy key (**B**) is octane. With $q = 1$ then:

$$\left(\frac{2.70 \times 0.40}{2.70 - \theta} \right) + \left(\frac{2.22 \times 0.35}{2.22 - \theta} \right) + \left(\frac{1 \times 0.25}{1 - \theta} \right) = 0$$

The required value of θ must satisfy the relation $\alpha_B < \theta < \alpha_A$, that is $1.0 < \theta < 2.22$. Assuming $\theta = 1.15$, then:

$$\Sigma \frac{\alpha x_f}{\alpha - \theta} = -0.243$$

Assuming $\theta = 1.17$, then:

$$\Sigma \frac{\alpha x_f}{\alpha - \theta} = -0.024$$

This is near enough and from equation :

$$\left(\frac{2.70 \times 0.534}{2.70 - 1.17} \right) + \left(\frac{2.22 \times 0.453}{2.22 - 1.17} \right) + \left(\frac{1.00 \times 0.013}{1.00 - 1.17} \right) = 1.827$$

Thus:

$$R_m = \underline{\underline{0.827}}$$

Calculation minimum number of plates (N_{\min}) using Fenske's equation

The number of plates required for a desired separation under conditions of total reflux can be found by applying Fenske's equation, equation 11.59, to the two key components.

Thus:

$$n + 1 = \frac{\log \left[\left(\frac{x_A}{x_B} \right)_d \left(\frac{x_B}{x_A} \right)_s \right]}{\log(\alpha_{AB})_{av}} \quad (11.116)$$

Where: A is the light component.

B is the heavy component

Example: A liquor consisting of phenol and cresols with some xylenols is fractionated to give a top product of 95.3 mole percent phenol. The compositions of the top product and of the phenol in the bottoms are:

	Compositions (mole percent)			
	Feed	Top	Bottom	volatility
phenol	35	95.3	5.24	1.25
o-cresol	15	4.55	—	1
m-cresol	30	0.15	—	0.63
xylenols	20	—	—	0.37

If a reflux ratio of 10 is used,

- Complete the material balance over the still for a feed of 100 kmol.
 - Calculate the composition on the second plate from the top.
 - Calculate the composition on the second plate from the bottom.
 - Calculate the minimum reflux ratio by Underwood's equation.
- The heavy key is m-cresol and the light key is phenol.

Solution:

(a) An overall mass balance and a phenol balance gives, on a basis of 100 kmol:

$$100 = D + W$$

and: $(100 \times 0.35) = 0.953D + 0.0524W$

from which: $D = 33.0 \text{ kmol}$ and $W = 67.0 \text{ kmol}$.

Balances on the remaining components give the required bottom product composition as:

$$o\text{-cresol: } (100 \times 0.15) = (0.0455 \times 33) + 67x_{wo} \text{ and } x_{wo} = \underline{\underline{0.2017}}$$

$$m\text{-cresol: } (100 \times 0.30) = (0.0015 \times 33) + 67x_{wm} \text{ and } x_{wm} = \underline{\underline{0.4472}}$$

$$x\text{ylenols: } (100 \times 0.20) = 0 + 67x_{wx}, \text{ and } x_{wx} = \underline{\underline{0.2987}}$$

(b) $L_n/D = 10$ and $L_n = 330 \text{ kmol}$

$$V_n = L_n + D \text{ and } V_n = 363 \text{ kmol}$$

The equation of the top operating line is:

$$y_n = \frac{L_n}{V_n}x_{n+1} + \frac{D}{V_n}x_d = (330/363)x_{n+1} + (33/363)x_d = 0.91x_{n+1} + 0.091x_d$$

The operating lines for each component then become:

$$\text{phenol: } y_{np} = 0.91x_{n+1} + 0.0867$$

$$o\text{-cresol: } y_{no} = 0.91x_{n+1} + 0.0414$$

$$m\text{-cresol: } y_{nm} = 0.91x_{n+1} + 0.00014$$

$$x\text{ylenols: } y_{nx} = 0.91x_{n+1}$$

Mean α -values are taken from the data given in Volume 2, Table 11.2 as:

$$\alpha_{pO} = 1.25, \quad \alpha_{OO} = 1.0, \quad \alpha_{MO} = 0.63, \quad \alpha_{XO} = 0.37$$

The solution may be set out as a table as follows, using the operating line equations and the equation:

$$x = \frac{y/\alpha}{\sum(y/\alpha)}$$

	$y = x_d$	y_t/α	x_t	y_{t-1}	y_{t-1}/α	x_{t-1}
phenol	0.953	0.762	0.941	0.943	0.754	0.928
<i>o</i> -cresol	0.0455	0.0455	0.056	0.054	0.054	0.066
<i>m</i> -cresol	0.0015	0.0024	0.003	0.003	0.005	0.006
xyleneols	—	—	—	—	—	—
	$\Sigma(y_t/\alpha) = 0.8099$		1.000	1.000	0.813	<u><u>1.000</u></u>

(c) In the bottom of the column:

$$L_m = L_n + F = 430 \text{ kmol}$$

$$V_m = L_m - W = 363 \text{ kmol}$$

and:
$$y_m = \frac{L_m}{V_m}x_{n+1} - \frac{W}{V_m}x_w = 1.185x_{m+1} - 0.185x_w$$

Hence for each component:

phenol: $y_{mp} = 1.185x_{m+1} - 0.0097$

o-cresol: $y_{mo} = 1.185x_{m+1} - 0.0373$

m-cresol: $y_{mm} = 1.185x_{m+1} - 0.0827$

xilenols: $y_{mx} = 1.185x_{m+1} - 0.0553$

Using these operating lines to calculate x and also $y = \alpha x / \sum \alpha x$ gives the following data:

	x_s	αx_s	y_s	x_1	αx_1	y_1	x_2
phenol	0.0524	0.066	0.100	0.093	0.116	0.156	0.140
<i>o</i> -cresol	0.2017	0.202	0.305	0.289	0.289	0.387	0.358
<i>m</i> -cresol	0.4472	0.282	0.427	0.430	0.271	0.363	0.376
xilenols	0.2987	0.111	0.168	0.188	0.070	0.094	0.126
	1.000	$\Sigma = 0.661$	1.000	1.000	$\Sigma = 0.746$	1.000	<u>1.000</u>

(d) Underwood's equations defined in, Problem 11.22, are used with $\alpha_p = 3.4$, $\alpha_o = 2.7$, $\alpha_m = 1.7$, $\alpha_x = 1.0$ to give:

$$\left(\frac{3.4 \times 0.35}{3.4 - \theta} \right) + \left(\frac{2.7 \times 0.15}{2.7 - \theta} \right) + \left(\frac{1.7 \times 0.30}{1.7 - \theta} \right) + \left(\frac{1.0 \times 0.20}{1 - \theta} \right) = (1 - q) = 0$$

$3.4 > \theta > 1.7$ and θ is found by trial and error to be 2.06.

Then:
$$\left(\frac{3.4 \times 0.953}{3.4 - 2.06} \right) + \left(\frac{2.7 \times 0.0455}{2.7 - 2.06} \right) + \left(\frac{1.7 \times 0.0015}{1.7 - 2.06} \right) = R_{m+1}$$

and:
$$\underline{\underline{R_{m+1} = 1.60}}$$

Chapter (13)

Humidification and Water Cooling

In the processing of materials it is often necessary either to increase the amount of vapour present in a gas stream, an operation known as **humidification**; or to reduce the vapour present, a process referred to as **dehumidification**. In **humidification**, the vapour content may be increased by passing the gas over a liquid which then evaporates into the gas stream. This transfer into the main stream takes place by diffusion, and at the interface simultaneous heat and mass transfer take place. In the reverse operation, that is **dehumidification**, partial condensation must be effected and the condensed vapour removed. The most common application of humidification and dehumidification involves the air-water system for example air conditioning. Air conditioning and gas drying also involve humidification and dehumidification operations. For example, moisture must be removed from wet chlorine so that the gas can be handled in steel equipment which otherwise would be severely corroded. Similarly, the gases used in the manufacture of sulphuric acid must be dried or dehumidified before entering the converters. Although the drying of wet solids is an example of a humidification operation, the reduction of the moisture content of the solids is the main objective, and the humidification of the air stream is a secondary effect.

In order that hot condenser water may be re-used in a plant, it is normally cooled by contact with an air stream. The equipment usually takes the form of a tower in which the hot water is run in at the top and allowed to flow downwards over a packing against a countercurrent flow of air which enters at the bottom of the cooling tower. The design of such towers forms an important part of the present chapter, though at the outset it is necessary to consider basic definitions of the various quantities involved in humidification.

Definitions:

1. **Humidity (\mathcal{H}):** mass of vapour associated with unit mass of dry gas.

$$\mathcal{H} = \frac{\text{mass (kg) of vapour}}{\text{mass (kg) of dry air}} = \frac{m_{\text{H}_2\text{O}}}{m_{\text{air}}}$$

But:
$$\frac{n_w}{n_{\text{air}}} = \frac{P_w}{P_{\text{air}}}$$

Also:
$$\frac{n_w}{n_T} = \frac{P_w}{P_T}$$

$$\mathcal{H} = \frac{m_w}{m_{\text{air}}} = \frac{n_w}{n_{\text{air}}} \times \frac{18}{29} = \frac{P_w}{P_{\text{air}}} \times \frac{18}{29}$$

$$\mathcal{H} = \frac{18 P_w}{29 (P_T - P_w)}$$

2. **Humidity of saturated gas (\mathcal{H}_0):** humidity of the gas when it is saturated with vapour at a given temperature.

$$\mathcal{H}_0 = \frac{\text{mass (kg) of vapour at saturations}}{\text{mass (kg) of dry air}}$$

$$\mathcal{H}_0 = \frac{18 P_w^0}{29 (P_T - P_w^0)}$$

Where: P_w is the partial pressure of water vapour in air.

P_w^0 is the partial pressure of water vapour in saturated air (vapour pressure).

3. **Percentage humidity ($\% \mathcal{H}$):**

$$\% \mathcal{H} = \frac{\mathcal{H}}{\mathcal{H}_0} \times 100 = \frac{P_w / (P_T - P_w)}{P_w^0 / (P_T - P_w^0)} \times 100$$

4. **Percentage relative humidity ($R\mathcal{H}\%$):**

$$R\mathcal{H} \% = \frac{P_w}{P_w^0} \times 100$$

5. **Humid heat (S) or (C_H):** heat required to raise unit mass of dry gas and its associated vapour through unit temperature difference at constant pressure.

$$S = C_p (\text{air}) + C_p (\text{water vapour}) \mathcal{H}$$

$$S = C_a + C_w \mathcal{H}$$

$$S = 1 + 1.9 \mathcal{H} \quad \text{kJ/ K. kg dry air}$$

6. **Humid volume (V_H):** volume occupied by unit mass of dry gas and its associated Vapour.

For ideal gas law (water-air) system:

$$V_H = \frac{RT}{29 P} + \frac{RT}{18 P} \mathcal{H} \quad \frac{m^3}{\text{kg dry air}}$$

$$V_H = \frac{RT}{P} \left[\frac{1}{29} + \frac{\mathcal{H}}{18} \right]$$

Form humid chart:

$$V_H = V_{\text{dry air}} + [V_{\text{sat.}} - V_{\text{dry}}] \mathcal{H}$$

7. **Enthalpy of humid gas (H):**

Total enthalpy of air- water vapour mixture (humid gas) per unit mass of dry gas is:

$$\text{then for an unsaturated gas: } H = H_a + H_w \mathcal{H} \quad (1)$$

$$\text{where: } H_a = C_a(\theta - \theta_0) \quad (2)$$

$$\text{and: } H_w = C_w(\theta - \theta_0) + \lambda \quad (3)$$

$$\text{Thus, in equation 1 : } H = (C_a + \mathcal{H} C_w)(\theta - \theta_0) + \mathcal{H} \lambda \quad (4)$$

$$= (\theta - \theta_0) s + \mathcal{H} \lambda$$

Where: θ : the temperature of humid gas.

θ_0 : the reference temperature.

C_a : specific heat of the gas at constant pressure, [For air, $C_a=1.003$ kJ/kg. K].

C_w : specific heat of the vapour at constant pressure, [For water, $C_w= 2.006$ kJ/kg. K].

λ : latent heat of vaporization of the liquid at θ_0 .

Wet-bulb temperature:

When a stream of unsaturated gas is passed over the surface of a liquid, the humidity of the gas is increased due to evaporation of the liquid. The temperature of the liquid falls below that of the gas and heat is transferred from the gas to the liquid. At equilibrium the rate of heat transfer from the gas just balances that required to vaporize the liquid and the liquid is said to be at the wet-bulb temperature. The rate at which this temperature is reached depends on the initial temperatures and the rate of flow of gas past the liquid surface. With a small area of contact between the gas and the liquid and a high gas flow rate, the temperature and the humidity of the gas stream remain virtually unchanged.

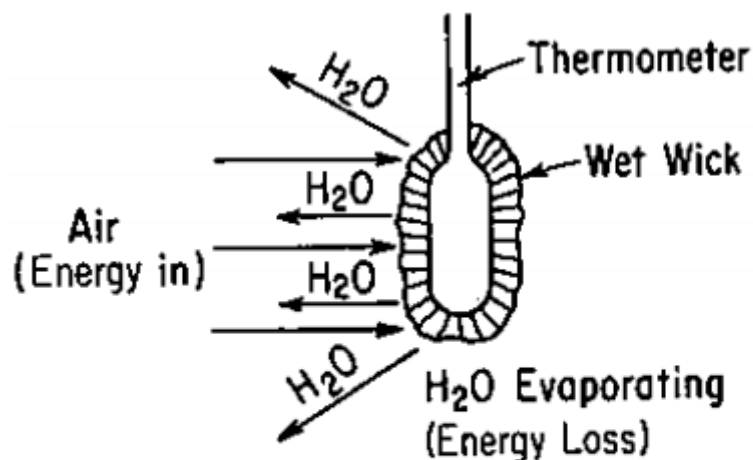


Figure (1): Wet-bulb temperature obtained with a sling psychrometer.

The rate of transfer of heat from the gas to the liquid can be written as:

$$Q = h A (\theta - \theta_w)$$

Where:

Q is the heat flow.

h is the heat transfer coefficient.

A is transfer area.

θ and θ_w are the temperatures of the gas and liquid phases.

The liquid evaporating into the gas is transferred by diffusion from the interface to the gas stream as a result of a concentration difference ($C_o - C$), where C_o is the concentration of the vapour at the surface (mass per unit volume) and C is the concentration in the gas stream.

The rate of evaporation is then given by:

$$W = h_D A (C_o - C) = h_D A \frac{M_w}{R T} (P_w^o - P_w)$$

Where:

h_D is the mass transfer coefficient.

P_w and P_w^o are the partial pressure of water vapour at θ and θ_w

M_w is the molecular weight of water.

If the partial pressures of the vapour P_w and P_w^o expressed in terms of the corresponding humidities \mathcal{H} and \mathcal{H}_w .

If P_w and P_w^o are small compare with P , $(P - P_w)$ and $(P - P_w^o)$ may be replaced by a mean partial pressure of the gas P_A and:

$$\begin{aligned} W &= h_{DA} A \frac{(\mathcal{H}_w - \mathcal{H}) M_w}{R T} \cdot \left(P_A \frac{M_A}{M_w} \right) \\ &= h_D A \rho_A (\mathcal{H}_w - \mathcal{H}) \end{aligned}$$

Heat balance on wick:

$$Q = h A (\theta - \theta_w) = W \lambda$$

$$(\mathcal{H} - \mathcal{H}_w) = -\frac{h}{h_D \rho_A \lambda} (\theta - \theta_w)$$

Note: For the air - water system, the ratio $(h/h_D \rho_A)$ is about 1.0 kJ/kg K and varies from 1.5 to 2.0 kJ/kg K for organic liquids.

Example (1): In a process in which it is used as a solvent, benzene is evaporated into dry nitrogen. At 297 K and 101.3 kN/m^2 , the resulting mixture has a percentage relative humidity of 60. It is required to recover 80 percent of the benzene present by cooling to 283 K and compressing to a suitable pressure. What should this pressure be? The vapour pressure of benzene is 12.2 kN/m^2 at 297 K and 6.0 kN/m^2 at 283 K.

Solution

From the definition of percentage relative humidity (RH):

$$P_w = P_{w0} \left(\frac{RH}{100} \right)$$

At 297 K:
$$P_w = (12.2 \times 1000) \times \left(\frac{60}{100} \right) = 7320 \text{ N/m}^2$$

In the benzene–nitrogen mixture:

$$\text{mass of benzene} = \frac{P_w M_w}{RT} = \frac{(7320 \times 78)}{(8314 \times 297)} = 0.231 \text{ kg}$$

$$\text{mass of nitrogen} = \frac{(P - P_w) M_A}{RT} = \frac{[(101.3 - 732) \times 1000 \times 28]}{(8314 \times 297)} = 1.066 \text{ kg}$$

Hence the humidity is:
$$\mathcal{H} = \left(\frac{0.231}{1.066} \right) = 0.217 \text{ kg/kg}$$

In order to recover 80 per cent of the benzene, the humidity must be reduced to 20 per cent of the initial value. As the vapour will be in contact with liquid benzene, the nitrogen will be saturated with benzene vapour

and hence at 283 K:

$$\mathcal{H}_0 = \frac{(0.217 \times 20)}{100} = 0.0433 \text{ kg/kg}$$

Thus in equation 13.2:

$$0.0433 = \left(\frac{6000}{P - 6000} \right) \left(\frac{78}{28} \right)$$

from which:

$$\underline{\underline{P = 3.92 \times 10^5 \text{ N/m}^2 = 392 \text{ kN/m}^2}}$$

Example (2): In a vessel at 101.3 kN/m^2 and 300 K , the percentage relative humidity of the water vapour in the air is 25. If the partial pressure of water vapour when air is saturated with vapour at 300 K is 3.6 kN/m^2 , calculate:

- the partial pressure of the water vapour in the vessel;
- the specific volumes of the air and water vapour;
- the humidity of the air and humid volume; and
- the percentage humidity.

Solution

(a) From the definition of percentage relative humidity:

$$P_w = P_{w0} \frac{RH}{100} = 3600 \times \left(\frac{25}{100} \right) = 900 \text{ N/m}^2 = \underline{\underline{0.9 \text{ kN/m}^2}}$$

(b) In 1 m^3 of air:

$$\text{mass of water vapour} = \frac{(900 \times 18)}{(8314 \times 300)} = 0.0065 \text{ kg}$$

$$\text{mass of air} = \frac{[(101.3 - 0.9) \times 1000 \times 29]}{(8314 \times 300)} = 1.167 \text{ kg}$$

$$\text{Hence: specific volume of water vapour at } 0.9 \text{ kN/m}^2 = \left(\frac{1}{0.0065} \right) = \underline{\underline{154 \text{ m}^3/\text{kg}}}$$

$$\text{specific volume of air at } 100.4 \text{ kN/m}^2 = \left(\frac{1}{1.167} \right) = \underline{\underline{0.857 \text{ m}^3/\text{kg}}}$$

$$\text{(c) Humidity: } \mathcal{H} = \left(\frac{0.0065}{1.1673} \right) = \underline{\underline{0.0056 \text{ kg/kg}}}$$

(Using the approximate relationship:

$$\mathcal{H} = \frac{(18 \times 900)}{(29 \times 101.3 \times 1000)} = 0.0055 \text{ kg/kg.})$$

\therefore Humid volume = volume of 1 kg air + associated vapour = specific volume of air at 100.4 kN/m^2

$$= \underline{\underline{0.857 \text{ m}^3/\text{kg}}}$$

$$\text{d. The percentage humidity} = \frac{P_w/(P_T - P_w)}{P_w^0/(P_T - P_w^0)} \times 100$$

$$= \frac{0.9/(101.3 - 0.9)}{3.6/(101.3 - 3.6)} \times 100 = 24.3 \%$$

Example (3): Moist air at 310 K has a wet-bulb temperature of 300 K. If the latent heat of vaporization of water at 300 K is 2440 kJ/kg, estimate the humidity of the air and the percentage relative humidity. The total pressure is 105 kN/m² and the vapour pressure of water vapour at 300 K is 3.60 kN/m² and 6.33 kN/m² at 310 K.

Solution

The humidity of air saturated at the wet-bulb temperature is given by:

$$\begin{aligned} \mathcal{H}_w &= \frac{P_{w0}}{P - P_{w0}} \frac{M_w}{M_A} & (\text{equation 13.2}) \\ &= \left(\frac{3.6}{105.0 - 3.6} \right) \left(\frac{18}{29} \right) = 0.0220 \text{ kg/kg} \end{aligned}$$

Therefore, taking $(h/h_D \rho_A)$ as 1.0 kJ/kg K, in equation 13.8:

$$(0.0220 - \mathcal{H}) = \left(\frac{1.0}{2440} \right) (310 - 300)$$

or:

$$\mathcal{H} = \underline{\underline{0.018 \text{ kg/kg}}}$$

At 310 K,

$$P_{w0} = 6.33 \text{ kN/m}^2$$

In equation 13.2:

$$0.0780 = \frac{18P_w}{(105.0 - P_w)29}$$

$$P_w = 2.959 \text{ kN/m}^2$$

$$\text{and the percentage relative humidity} = \frac{(100 \times 2.959)}{6.33} = \underline{\underline{46.7 \text{ per cent}}}$$

Water cooling ((Cooling towers))

Cooling of water can be carried out on a small scale either by allowing it to stand in an open pond or by the spray pond technique in which it is dispersed in spray form and then collected in a large, open pond. Cooling takes place both by the transference of sensible heat and by evaporative cooling as a result of which sensible heat in the water provides the latent heat, of vaporization.

On the large scale, air and water are brought into countercurrent contact in a cooling tower. The water flows down over a packing which give a large interfacial area and promote turbulence in the liquid. The air is humidified and heated as it rises, while the water is cooled mainly by evaporation.

There are two methods to calculate the height of the cooling tower:

1. Merkel and Mickley method ($z = HTU * NTU$).
2. Carey and Williamson method.

1. Equilibrium relation

The equilibrium is a relation between the air enthalpy (H_f) and water temperature (θ_f) at the interface with the liquid. The enthalpy-temperature curve for saturated air is plotted either using calculated data or can be calculated from the temperature and humidity.

1. Calculated data:

θ_f	-	-	-	-	-
H_f	-	-	-	-	-

2. Temperature and humidity data:

Use temperature and humidity data to calculate the enthalpy of air at the interface (H_f) using the equation below.

$$H = (C_a + \mathcal{H}C_w)(\theta - \theta_0) + \mathcal{H}\lambda$$

$$= (\theta - \theta_0) s + \mathcal{H}\lambda$$

$$S = 1 + 1.9\mathcal{H}$$

* we select value of Θ_f , then find the vapour pressure at this temperature (P_w^0) from steam table. After that we find the humidity at saturation (\mathcal{H}_o) from the equation below:

$$\mathcal{H}_o = \frac{18 P_w^0}{29 (P_T - P_w^0)}$$

Apply the enthalpy equation to calculate (H_f):

$$\begin{aligned} H &= (C_a + \mathcal{H} C_w)(\theta - \theta_0) + \mathcal{H} \lambda \\ &= (\theta - \theta_0) s + \mathcal{H} \lambda \end{aligned}$$

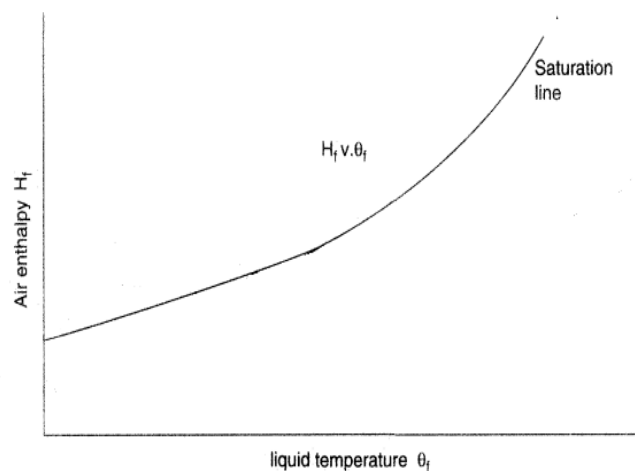
Repeat the above procedure for five or six temperature values to plot the equilibrium curve.

* We find the humidity at saturation from the humidity chart as follows:

1. For each temperature (θ_G) we draw vertical line (AB).
2. Point (B) on the RH% curve.
3. We draw a horizontal line (BC) and we read humidity at saturation (\mathcal{H}_o).
4. Then we calculate each (\mathcal{H}_o) for each (θ_G).
5. The enthalpy at saturation (H_f) is calculated from the equation below:

$$\begin{aligned} H &= (C_a + \mathcal{H} C_w)(\theta - \theta_0) + \mathcal{H} \lambda \\ 5 \quad &= (\theta - \theta_0) s + \mathcal{H} \lambda \end{aligned}$$

6. We draw the equilibrium between (θ_f , H_f)



2. Operating line:

The five basic equations for an incremental height: of column, dz , are:

1. Water balance: $dL' = G' d\mathcal{H}$
2. Enthalpy balance: $G' dH_G = L' dH_L$

since only a small proportion of the liquid is evaporated.

$$H_G = s(\theta_G - \theta_0) + \lambda \mathcal{H}$$

$$H_L = C_L(\theta_L - \theta_0)$$

$$G' dH_G = L' C_L d\theta_L$$

Thus:

and:
$$dH_G = s d\theta_G + \lambda d\mathcal{H}$$

Integration of this expression over the whole height of the column, on the assumption that the physical properties of the materials do not change appreciably, gives:

$$G'(H_{G2} - H_{G1}) = L'C_L(\theta_{L2} - \theta_{L1})$$

$$\boxed{H_{G2} = \frac{L'C_L}{G'}(\theta_{L2} - \theta_{L1}) + H_{G1}} \quad \dots \dots \dots \text{operating line equation}$$

Where:

L' is the liquid mass flow rate kg/hr or kg/hr.m².

G' is the gas mass flow rate kg/hr or kg/hr.m².

C_L is the heat capacity of liquid = 4.18 kJ/kg.K for water.

θ_{L1} is the temperature of the exit liquid.

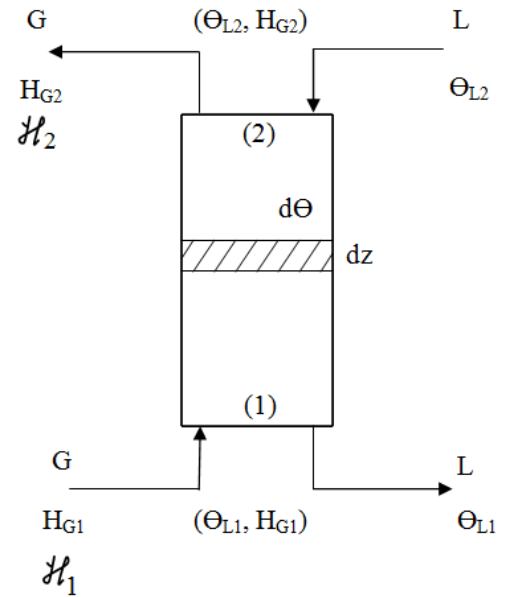
θ_{L2} is the temperature of the enter liquid.

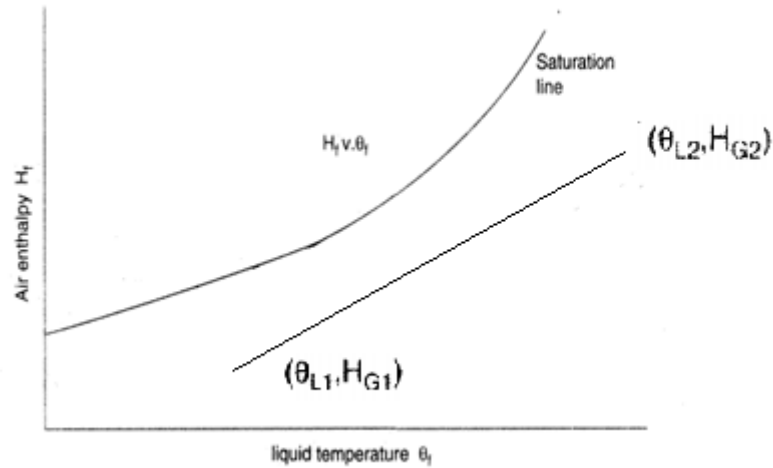
H_{G1} is the enthalpy of the enter gas.

H_{G2} is the enthalpy of the exit gas.

Where H_{G1} is calculated from:

$$\begin{aligned} H &= (C_a + \mathcal{H}C_w)(\theta - \theta_0) + \mathcal{H}\lambda \\ &= (\theta - \theta_0)s + \mathcal{H}\lambda \end{aligned}$$





3. Heat transfer from the body of the liquid to the interface:

$$h_L a \, dz (\theta_L - \theta_f) = L' C_L \, d\theta_L$$

$$\frac{d\theta_L}{(\theta_L - \theta_f)} = \frac{h_L a}{L' C_L} \, dz$$

4. Heat transfer from the interface to the bulk of the gas:

$$h_G a \, dz (\theta_f - \theta_G) = G' s \, d\theta_G$$

Where: h_G is the heat transfer coefficient in the gas phase.

Rearranging:
$$\frac{d\theta_G}{(\theta_f - \theta_G)} = \frac{h_G a}{G' s} \, dz$$

5. Mass transfer from the interface to the gas:

$$h_D \rho a \, dz (\mathcal{H}_f - \mathcal{H}) = G' \, d\mathcal{H}$$

Where: h_D is the mass transfer coefficient for the gas and ρ is the mean density of the air.

Rearranging:
$$\frac{d\mathcal{H}}{(\mathcal{H}_f - \mathcal{H})} = \frac{h_D a \rho}{G'} dz$$

Thus, writing $h_G = h_D \rho s$

$$G' s d\theta_G = h_D \rho a dz (s\theta_f - s\theta_G)$$

$$G' \lambda d\mathcal{H} = h_D \rho a dz (\lambda \mathcal{H}_f - \lambda \mathcal{H})$$

$$G' (s d\theta_G + \lambda d\mathcal{H}) = h_D \rho a dz [(s\theta_f + \lambda \mathcal{H}_f) - (s\theta_G + \lambda \mathcal{H})]$$

$$G' dH_G = h_D \rho a dz (H_f - H_G)$$

$$\frac{dH_G}{(H_f - H_G)} = \frac{h_D a \rho}{G'} dz$$

$$Z = \frac{G'}{h_D a \rho} \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H_f - H_G}$$

$$Z = \text{HTU} * \text{NTU}$$

Now how to join lines between the operating line and the equilibrium (saturation) curve:

$$G' dH_G = L' C_L d\theta_L \quad \dots\dots\dots(1)$$

$$h_L a dz (\theta_L - \theta_f) = L' C_L d\theta_L \quad \dots\dots\dots(2)$$

$$G' dH_G = h_D \rho a dz (H_f - H_G) \quad \dots\dots\dots(3)$$

Combining of above equations:

$$\frac{(H_G - H_f)}{(\theta_L - \theta_f)} = -\frac{h_L}{h_D \rho}$$

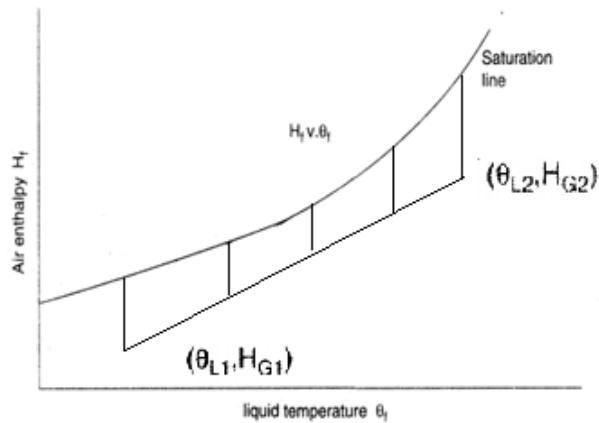
Where $(-\frac{h_L}{h_D \rho})$ is the slope of the equilibrium line which is draw between the operating line and the saturation curve.

Notes:

1. If the gas film control:

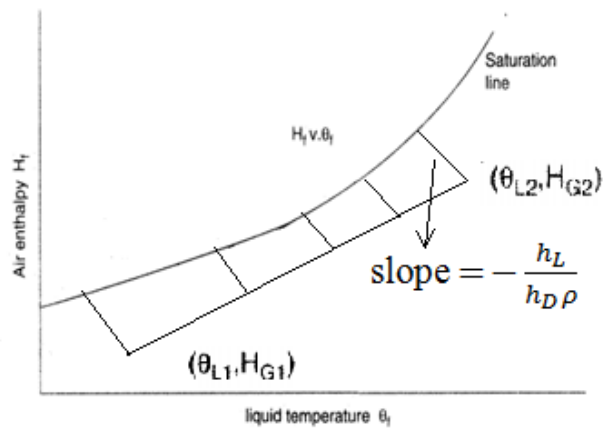
the equilibrium line slope, $-\frac{h_L}{h_D \rho} = \infty$

means that the equilibrium line is vertical.



2. If the gas and liquid films control the transfer:

the equilibrium line slope = $-\frac{h_L}{h_D \rho}$

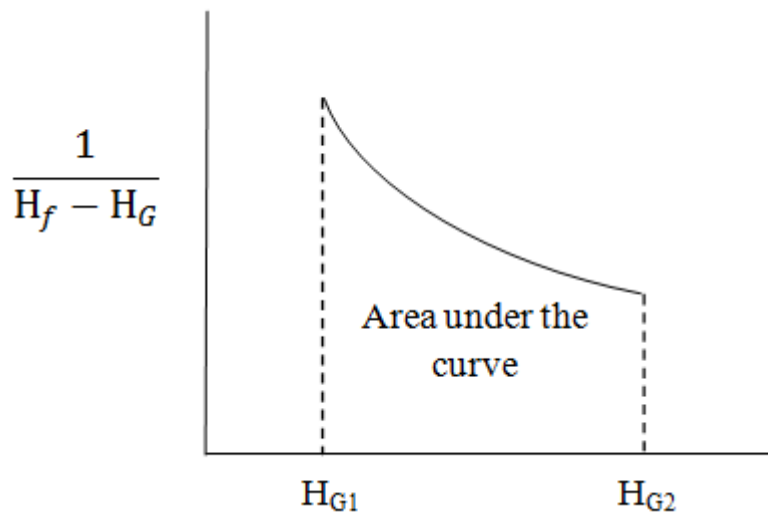


Calculation the height of the cooling tower:**1. Merkel and Mickley method:**

$$z = HTU * NTU$$

$$Z = \frac{G'}{h_D a \rho} \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H_f - H_G}$$

- We draw the equilibrium (saturation) curve.
- We draw the operating line.
- We draw the equilibrium line between the operating line and the saturation curve with slope $= -\frac{h_L}{h_D \rho}$
- We read H_f and H_G on the y-axis from horizontal lines drawn from the saturation curve and operating lines.
- We make a plot between H_G and $\frac{1}{H_f - H_G}$ and find the area under the curve which represent's (NTU).



2. Carey and Williamson method:

$$Z = \frac{G'}{h_D a \rho} \cdot \frac{H_{G2} - H_{G1}}{f \Delta H_m} \dots \dots \dots (*)$$

Where:

ΔH_m is the enthalpy difference at the mean water temperature in the column.

and, the mean temperature is, $\theta_m = \frac{\theta_{L1} + \theta_{L2}}{2}$

f is a factor for converting the driving force at the mean water temperature to the effective value.

How to gain ΔH_m :

$\Delta H_1 = H_{f1} - H_{G1}$ at the bottom of the column

$\Delta H_2 = H_{f2} - H_{G2}$ at the top of the column

$$\theta_m = \frac{\theta_{L1} + \theta_{L2}}{2}$$

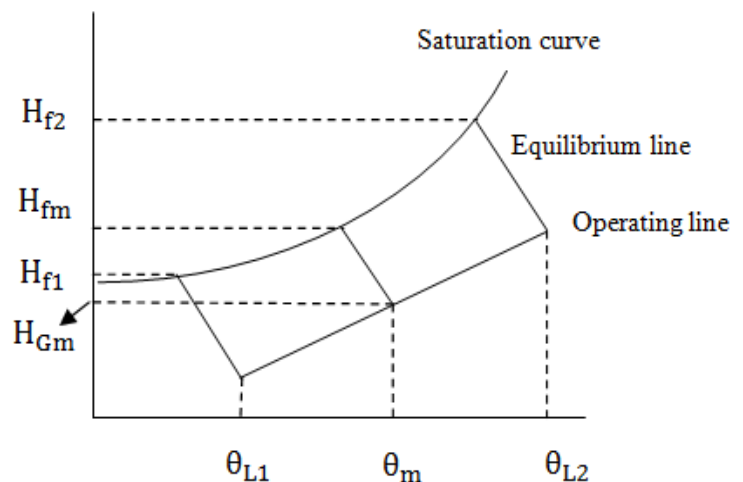
At θ_m we find H_{fm} and H_{Gm} :

$$\theta_m = \frac{\theta_{L1} + \theta_{L2}}{2}$$

$$\Delta H_m = H_{fm} - H_{Gm}$$

So we have:

$$\frac{\Delta H_m}{\Delta H_1} \quad \text{and} \quad \frac{\Delta H_m}{\Delta H_2}$$



Then from figure (13.17) we find f , then we can use the equation (*) to calculate the height of cooling tower (Z):

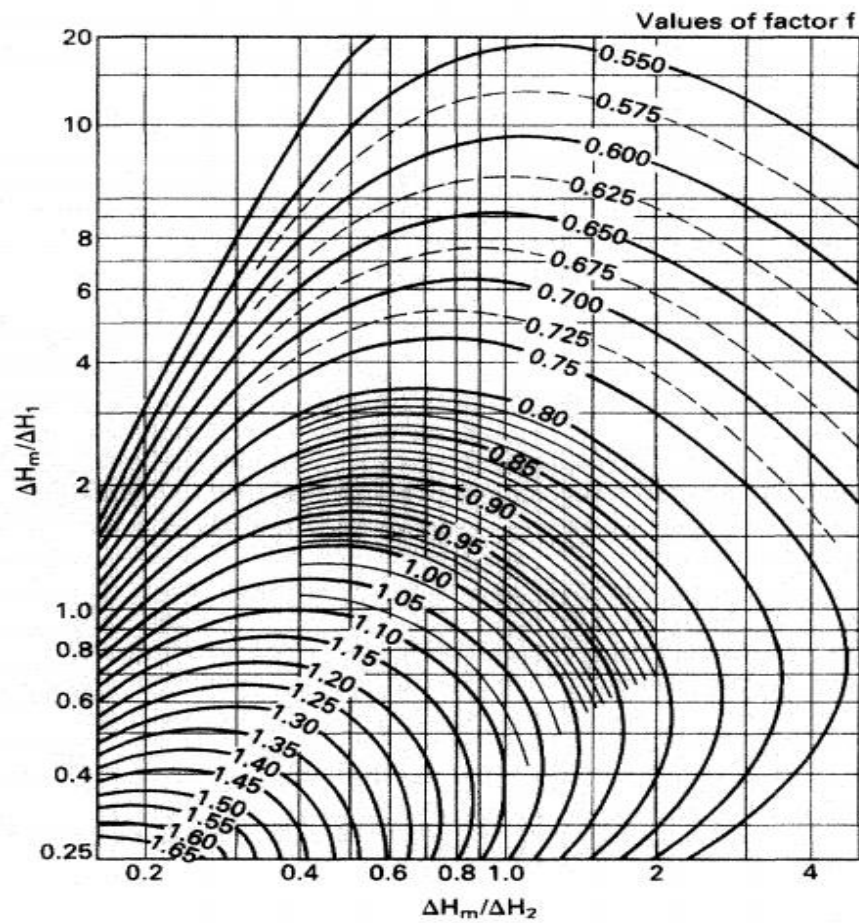


Figure 13.17. Correction factor f for obtaining the mean effective driving force in column

Example: Water is to be cooled from 328 to 293 K by means of a countercurrent air stream entering at 293 K with a relative humidity of 20 percent. The flow of air is $0.68 \text{ m}^3/\text{m}^2 \cdot \text{s}$ and the water throughput is $0.26 \text{ kg}/\text{m}^2 \cdot \text{s}$. The whole of the resistance to heat and mass transfer may be assumed to be in the gas phase and the product, $(h_D a)$, may be taken as $0.2 (\text{m/s})(\text{m}^2/\text{m}^3)$, that is 0.2 s^{-1} .

What is the required height of packing and the condition of the exit air stream?

Assuming the latent heat of water at 273 K = 2495 kJ/kg

specific heat of air (C_a) = 1.003 kJ/kg K

and specific heat of water vapour (C_w) = 2.006 kJ/kg K

Solution:

$$z = \text{HTU} * \text{NTU}$$

$$Z = \frac{G'}{h_D a \rho} \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H_f - H_G}$$

The reference temperature = $\Theta_o = 273 \text{ K}$

The latent heat of vaporization of water = $\lambda_{\theta_o} = 2495 \text{ kJ/kg}$

To draw the saturation curve (Θ_f , H_f):

$$H_F = C_a(\theta_f - 273) + \lambda_o [C_w(\theta_f - 273) + \lambda] \text{ kJ/kg}$$

From humidity chart at RH % = 20%

Θ_f	\mathcal{H}	$\mathcal{H}_o = (\mathcal{H} / 0.2)$	H_f From enthalpy equation
293	0.003	0.015	57.7
294.5	0.004	0.02	65
302	0.005	0.025	98
309	0.009	0.045	137
316	0.0125	0.0625	190
323	0.018	0.09	265
328	0.020	0.1	355

To draw the operating line:

Point (1), (Θ_{L1} , H_{G1})

Point (2), (Θ_{L2} , H_{G2})

at $\theta_{L1} = 293 \text{ K}$, $\mathcal{H}_1 = 0.003$

$$H_{G1} = [1 + 1.9(0.003)](293 - 273) + (2495)(0.003) \\ = 27.7 \text{ kJ/kg}$$

Point (1), $(\theta_{L1} , H_{G1}) = (293, 27.7)$

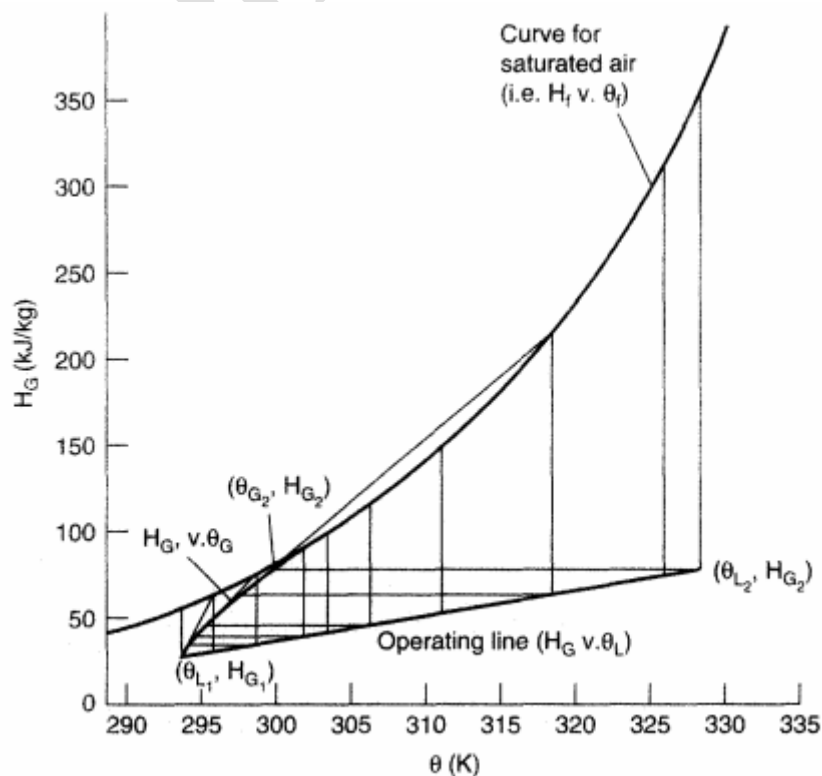
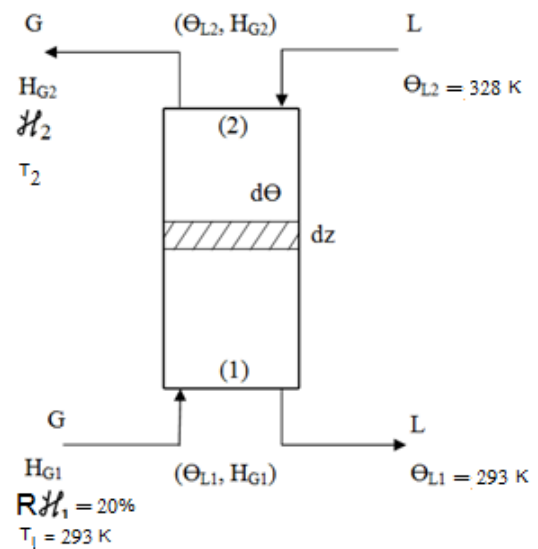
$$\rho_g = \frac{(P)(M. wt)}{R T} = \frac{101.3 * 29}{8.314 * 293} = 1.2 \text{ kg/m}^3$$

$$G = 0.68 \frac{\text{m}^3}{\text{m}^2 \cdot \text{s}} * 1.2 \frac{\text{kg}}{\text{m}^3} = 0.816 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}}$$

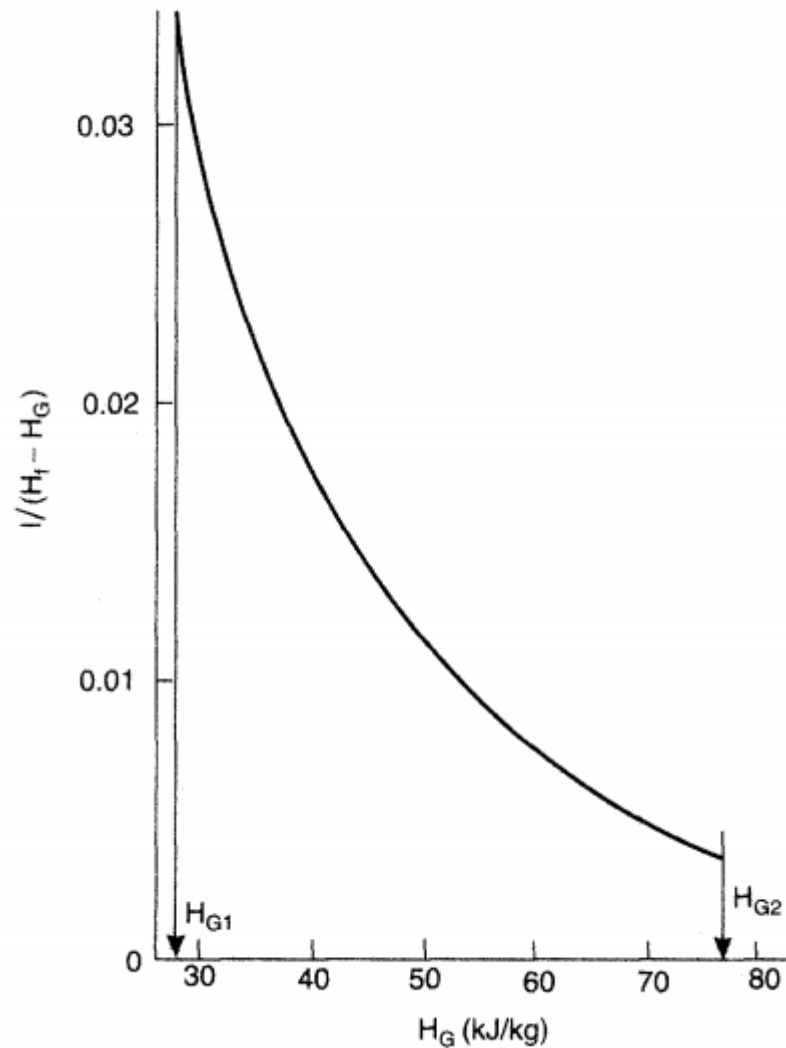
$$\text{Slope} = \frac{L C_L}{G} = \frac{(0.26)(4.18)}{0.816} = 1.332$$

$$H_{G2} = \frac{L' C_L}{G'} (\theta_{L2} - \theta_{L1}) + H_{G1} = 1.332 (328 - 293) + 27.7 = 74.22 \text{ kJ/kg}$$

Point (2), $(\theta_{L2} , H_{G2}) = (328, 74.22)$



H_G	$\theta = \theta_f$	H_f	$(H_f - H_G)$	$1/(H_f - H_G)$
27.7	293	57.7	30	0.0330
30	294.5	65	35	0.0285
40	302	98	58	0.0172
50	309	137	87	0.0114
60	316	190	130	0.0076
70	323	265	195	0.0051
76.5	328	355	279	0.0035



Using Merkel and Mickley method:

$$\text{Area under the curve} = \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H_f - H_G} = 0.65$$

$$Z = \frac{G'}{h_D a \rho} \int_{H_{G1}}^{H_{G2}} \frac{dH_G}{H_f - H_G} = \frac{0.68}{0.2} (0.65) = 2.2 \text{ m}$$

Using Carey and Williamson method:

$$Z = \frac{G'}{h_D a \rho} \cdot \frac{H_{G2} - H_{G1}}{f \Delta H_m}$$

$$\theta_m = \frac{\theta_{L1} + \theta_{L2}}{2} = \frac{293 + 328}{2} = 310.5$$

From figure we can get:

H_f	H_G	ΔH
$H_{f1} = 57.7$	$H_{G1} = 27.7$	$\Delta H_1 = 30$
$H_{f2} = 355$	$H_{G2} = 76.5$	$\Delta H_2 = 278.5$
$H_{fm} = 145$	$H_{Gm} = 52$	$\Delta H_m = 93$

To find f from figure (13.17):

$$\frac{\Delta H_m}{\Delta H_1} = \frac{93}{30} = 3.1 \quad \text{and} \quad \frac{\Delta H_m}{\Delta H_2} = \frac{93}{278.5} = 0.334$$

$$f = 0.79$$

$$Z = \frac{0.68}{0.2} \cdot \left[\frac{76.5 - 27.7}{0.79 * 93} \right] = 2.47 \text{ m}$$

The value of Θ_{G2} corresponding to $H_{G2} = 76.5$ kJ/kg is 300 K. From Figure 13.5, under these conditions, the exit air has a humidity of 0.019 kg/kg which from Figure 13.4 corresponds to a relative humidity of 83 percent.

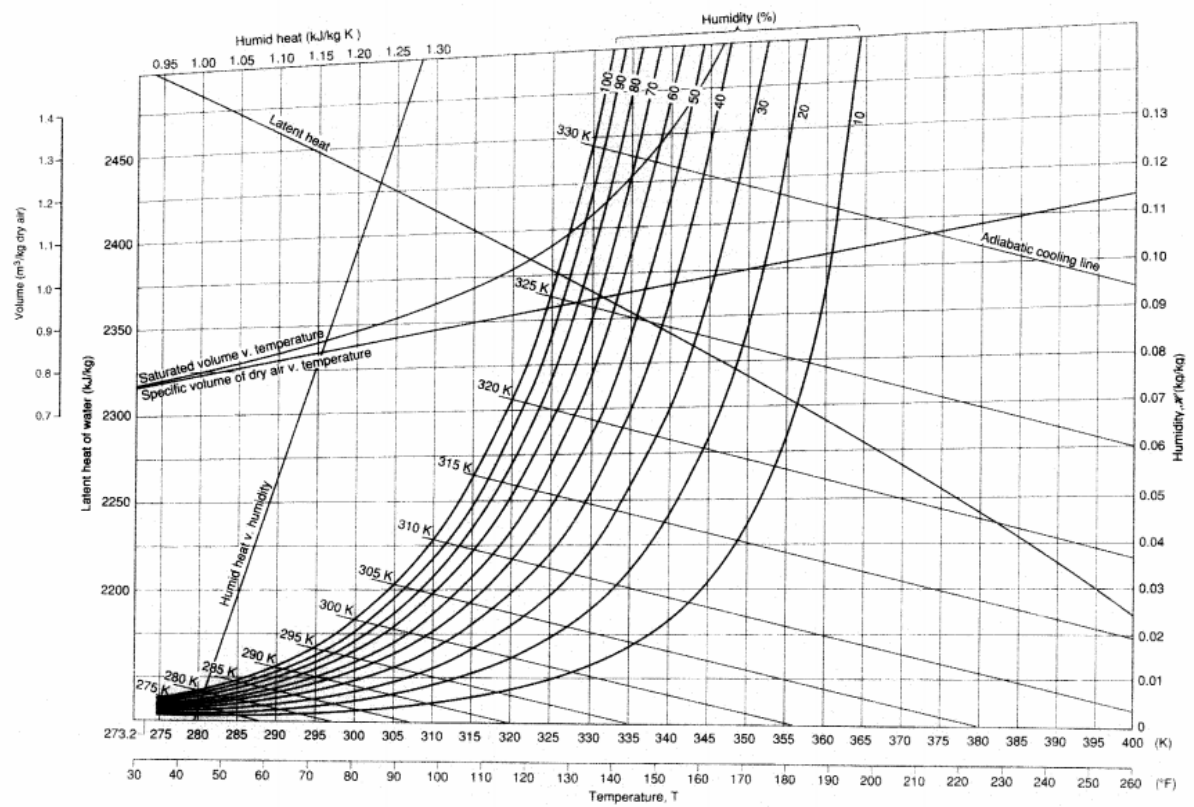


Figure 13.4. Humidity-temperature chart (See also the Appendix)

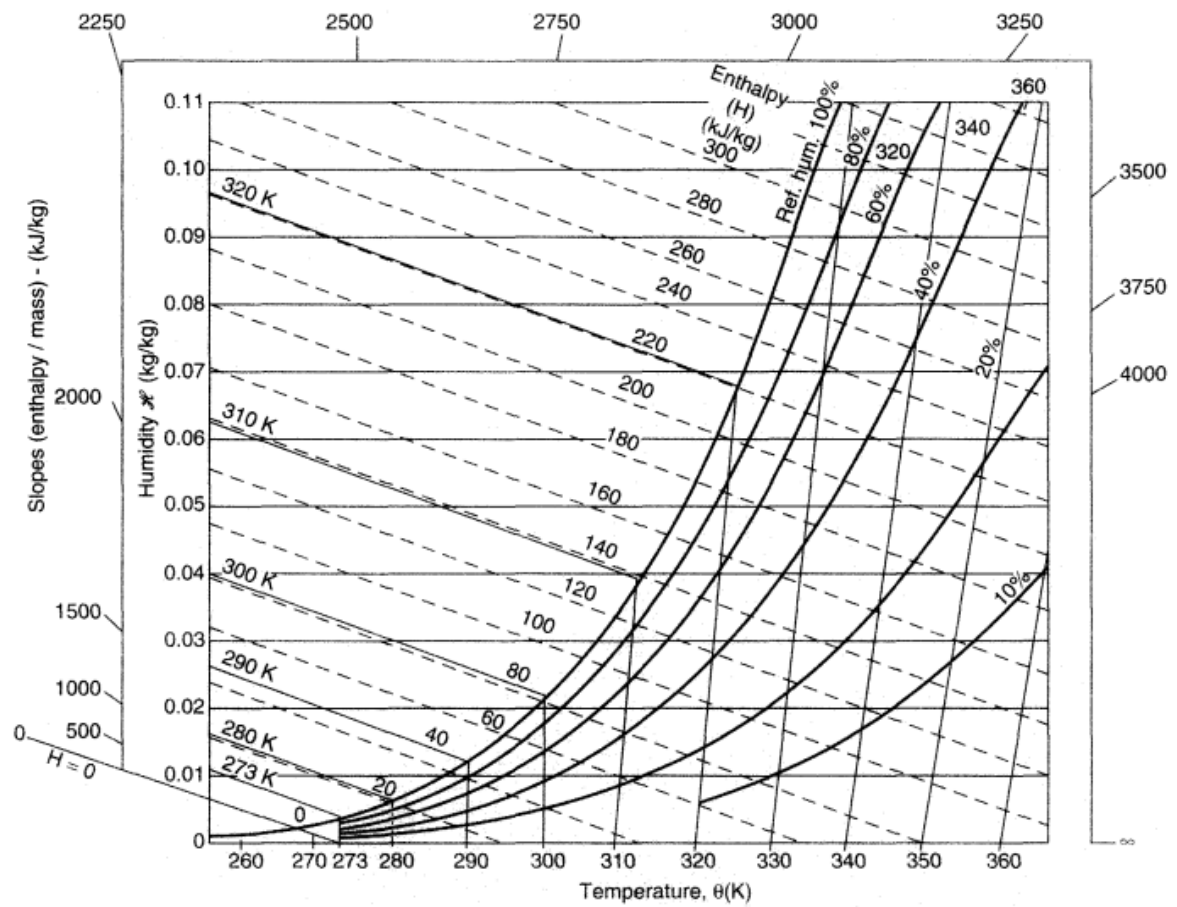


Figure 13.5. Humidity-enthalpy diagram for air-water vapour system at atmospheric pressure