

Two-Phase Liquid/Vapor Systems

When a system consists of saturated-liquid and saturated-vapor phases coexisting in equilibrium, the total value of any extensive property of the two-phase system is the sum of the total properties of the phases. Written for the volume, this relation is:

$$nV = n^l V^l + n^v V^v$$

where V is the system volume on a molar basis and the total number of moles is $n = n^l + n^v$. Division by n gives:

$$V = x^l V^l + x^v V^v$$

where x^l and x^v represent the fractions of the total system that are liquid and vapor. Since $x^l = 1 - x^v$

$$V = (1 - x^v)V^l + x^v V^v$$

Analogous equations can be written for the other extensive thermodynamic properties. All of these relations are represented by the generic equation:

$$M = (1 - x^v)M^l + x^v M^v \quad (6.82a)$$

where M represents V , U , H , S , etc. An alternative form is sometimes useful:

$$M = M^l + x^v \Delta M^{lv} \quad (6.82b)$$

6.5 TABLES OF THERMODYNAMIC PROPERTIES

Thermodynamic tables for saturated steam from its normal freezing point to the critical point and for superheated steam over a substantial pressure range, in SI units, appear in App. F.

The first table for each system of units presents the equilibrium properties of saturated liquid and saturated vapor at even increments of temperature. The second table is for the gas region, and gives properties of superheated steam at temperatures higher than the saturation temperature for a given pressure.

Example 6.7

Superheated steam originally at P_1 and T_1 expands through a nozzle to an exhaust pressure P_2 . Assuming the process is reversible and adiabatic, determine the downstream state of the steam and ΔH for the following conditions:

- (a) $P_1 = 1,000 \text{ kPa}$, $t_1 = 250^\circ\text{C}$, and $P_2 = 200 \text{ kPa}$.
- (b) $P_1 = 150(\text{psia})$, $t_1 = 500(^{\circ}\text{F})$, and $P_2 = 50(\text{psia})$.

Solution 6.7

Because the process is both reversible and adiabatic, there is no change in the entropy of the steam.

(a) For the initial temperature of 250°C at 1,000 kPa, no entries appear in the SI tables for superheated steam. Interpolation between values for 240°C and 260°C yields, at 1000 kPa:

$$H_1 = 2,942.9 \text{ kJ kg}^{-1} \quad S_1 = 6.9252 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

For the final state at 200 kPa,

$$S_2 = S_1 = 6.9252 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

Because the entropy of saturated vapor at 200 kPa is greater than S_2 , the final state is in the two-phase liquid/vapor region. Thus t_2 is the saturation temperature at 200 kPa, given in the SI superheat tables as $t_2 = 120.23^\circ\text{C}$. Equation (6.82a) applied to the entropy becomes:

$$S_2 = (1 - x_2^v)S_2^l + x_2^v S_2^v$$

Whence,
$$6.9252 = 1.5301(1 - x_2^v) + 7.1268x_2^v$$

where the values 1.5301 and 7.1268 are entropies of saturated liquid and saturated vapor at 200 kPa. Solving,

$$x_2^v = 0.9640$$

On a mass basis, the mixture is 96.40% vapor and 3.60% liquid. Its enthalpy is obtained by further application of Eq. (6.82a):

$$H_2 = (0.0360)(504.7) + (0.9640)(2,706.3) = 2,627.0 \text{ kJ kg}^{-1}$$

Finally,
$$\Delta H = H_2 - H_1 = 2,627.0 - 2,942.9 = -315.9 \text{ kJ kg}^{-1}$$

(b) For the initial state at 150(psia) and 500(°F) data from Table F.4 for superheated steam in English units provide:

$$H_1 = 1,274.3 \text{ (Btu)(lb}_m\text{)}^{-1} \quad S_1 = 1.6602 \text{ (Btu)(lb}_m\text{)}^{-1} \text{ (R)}^{-1}$$

In the final state at 50(psia),

$$S_2 = S_1 = 1.6602 \text{ (Btu)(lb}_m\text{)}^{-1} \text{ (R)}^{-1}$$

Inspection of Table F.4 shows that S_2 is here greater than the entropy of saturated vapor at 50(psia). Hence the final state is in the superheat region. Interpolation on entropy at 50(psia) yields:

$$t_2 = 283.28(^\circ\text{F}) \quad H_2 = 1,175.3 \text{ (Btu)(lb}_m\text{)}^{-1}$$

and $\Delta H = H_2 - H_1 = 1,175.3 - 1,274.3 = -99.0(\text{Btu})(\text{lb}_m)^{-1}$

For a nozzle, under the stated assumptions the steady-flow energy balance, Eq. (2.32a), becomes:

$$\Delta H + \frac{1}{2} \Delta u^2 = 0$$

Thus the decrease in enthalpy found for parts (a) and (b) is exactly compensated by an increase in kinetic energy of the fluid. In other words, the velocity of a fluid increases as it flows through a nozzle, which is its usual purpose. Nozzles are treated in some detail in Sec. 7.1.

Example 6.8

A 1.5-m³ tank contains 500 kg of liquid water in equilibrium with pure water vapor, which fills the remainder of the tank. The temperature and pressure are 100°C and 101.33 kPa. From a water line at a constant temperature of 70°C and a constant pressure somewhat above 101.33 kPa, 750 kg of liquid is bled into the tank. If the temperature and pressure in the tank are not to change as a result of the process, how much energy as heat must be transferred to the tank?

Solution 6.8

Choose the tank as the control volume. There is no work, and kinetic- and potential-energy changes are assumed negligible. Equation (2.29) therefore is written:

$$\frac{d(mU)_{\text{tank}}}{dt} - H' \dot{m}' = \dot{Q}$$

where the prime denotes the state of the inlet stream. The mass balance, $\dot{m}' = dm_{\text{tank}}/dt$, may be combined with the energy balance to yield:

$$\frac{d(mU)_{\text{tank}}}{dt} - H' \frac{dm_{\text{tank}}}{dt} = \dot{Q}$$

Multiplication by dt and integration over time (with H' constant) gives:

$$Q = \Delta(mU)_{\text{tank}} - H' \Delta m_{\text{tank}}$$

The definition of enthalpy may be applied to the entire contents of the tank to give:

$$\Delta(mU)_{\text{tank}} = \Delta(mH)_{\text{tank}} - \Delta(PmV)_{\text{tank}}$$

Because total tank volume mV and P are constant, $\Delta(PmV)_{\text{tank}} = 0$. Then, with $\Delta(mH)_{\text{tank}} = (m_2 H_2)_{\text{tank}} - (m_1 H_1)_{\text{tank}}$, the two preceding equations combine to yield:

$$Q = (m_2 H_2)_{\text{tank}} - (m_1 H_1)_{\text{tank}} - H' \Delta m_{\text{tank}} \quad (A)$$

where Δm_{tank} is the 750 kg of water bled into the tank, and subscripts 1 and 2 refer to conditions in the tank at the beginning and end of the process. At the end of the process the tank still contains saturated liquid and saturated vapor in equilibrium at 100°C and 101.33 kPa. Hence $m_1 H_1$ and $m_2 H_2$ each consist of two terms, one for the liquid phase and one for the vapor phase.

The numerical solution makes use of the following enthalpies taken from the steam tables:

$$\begin{aligned} H' &= 293.0 \text{ kJ kg}^{-1}; \text{ saturated liquid at } 70^\circ\text{C} \\ H_{\text{tank}}^l &= 419.1 \text{ kJ kg}^{-1}; \text{ saturated liquid at } 100^\circ\text{C} \\ H_{\text{tank}}^v &= 2,676.0 \text{ kJ kg}^{-1}; \text{ saturated vapor at } 100^\circ\text{C} \end{aligned}$$

The volume of vapor in the tank initially is 1.5 m^3 minus the volume occupied by the 500 kg of liquid water. Thus,

$$m_1^v = \frac{1.5 - (500)(0.001044)}{1.673} = 0.772 \text{ kg}$$

where 0.001044 and $1.673 \text{ m}^3 \text{ kg}^{-1}$ are the specific volumes of saturated liquid and saturated vapor at 100°C from the steam tables. Then,

$$(m_1 H_1)_{\text{tank}} = m_1^l H_1^l + m_1^v H_1^v = 500(419.1) + 0.772(2,676.0) = 211,616 \text{ kJ}$$

At the end of the process, the masses of liquid and vapor are determined by a mass balance and by the fact that the tank volume is still 1.5 m^3 :

$$\begin{aligned} m_2 &= 500 + 0.772 + 750 = m_2^v + m_2^l \\ 1.5 &= 1.673m_2^v + 0.001044m_2^l \end{aligned}$$

Whence, $m_2^l = 1,250.65 \text{ kg}$ and $m_2^v = 0.116 \text{ kg}$

Then, with $H_2^l = H_1^l$ and $H_2^v = H_1^v$,

$$(m_2 H_2)_{\text{tank}} = (1,250.65)(419.1) + (0.116)(2,676.0) = 524,458 \text{ kJ}$$

Substitution of appropriate values into Eq. (A) gives:

$$Q = 524,458 - 211,616 - (750)(293.0) = 93,092 \text{ kJ}$$