

Chapter 7

Applications of Thermodynamics to Flow Processes

The equations of balance for open systems from Chaps. 2 and 5 are summarized here in Table 7.1 for easy reference. Included are Eqs. (7.1) and (7.2), restricted forms of the mass balance. These equations are the basis for the thermodynamic analysis of processes in this and the next two chapters. When combined with thermodynamic property statements, they allow calculation of process rates and system states.

Table 7.1 Equations of Balance

General Equations of Balance	Balance Equations for Steady-Flow Processes	Balance Equations for Single-Stream Steady-Flow Processes
$\frac{dm_{cv}}{dt} + \Delta(\dot{m})_{fs} = 0 \quad (2.25)$	$\Delta(\dot{m})_{fs} = 0 \quad (7.1)$	$\dot{m}_1 = \dot{m}_2 = \dot{m} \quad (7.2)$
$\frac{d(mU)_{cv}}{dt} + \Delta \left[\left(H + \frac{1}{2}u^2 + zg \right) \dot{m} \right]_{fs} = \dot{Q} + \dot{W} \quad (2.28)$	$\Delta \left[\left(H + \frac{1}{2}u^2 + zg \right) \dot{m} \right]_{fs} = \dot{Q} + \dot{W}_s \quad (2.30)$	$\Delta H + \frac{\Delta u^2}{2} + g\Delta z = Q + W_s \quad (2.32)$
$\frac{d(mS)_{cv}}{dt} + \Delta(S\dot{m})_{fs} - \sum_j \frac{Q_j}{T_{o,j}} = \dot{S}_G \geq 0 \quad (5.21)$	$\Delta(S\dot{m})_{fs} - \sum_j \frac{\dot{Q}_j}{T_{o,j}} = \dot{S}_G \geq 0 \quad (5.22)$	$\Delta S - \sum_j \frac{Q_j}{T_{o,j}} = S_G \geq 0 \quad (5.23)$

7.1 DUCT FLOW OF COMPRESSIBLE FLUIDS

Thermodynamics does provide equations that interrelate the changes occurring in pressure, velocity, cross-sectional area, enthalpy, entropy, and specific volume of a flowing stream. We consider here the adiabatic, steady-state, one-dimensional flow of a compressible fluid in the absence of shaft work and of changes in potential energy. The pertinent thermodynamic equations are first derived; they are then applied to flow in pipes and nozzles.

The appropriate energy balance is Eq. (2.32). With Q , W , and Δz all set equal to zero,

$$\Delta H + \frac{\Delta u^2}{2} = 0$$

In differential form,

$$dH = -u du \quad (7.3)$$

The continuity equation, Eq. (2.27), is also applicable. Because \dot{m} is constant, its differential form is:

$$d(uA/V) = 0$$

$$\frac{dV}{V} - \frac{du}{u} - \frac{dA}{A} = 0 \quad (7.4)$$

The fundamental property relation appropriate to this application is:

$$dH = TdS + VdP \quad (6.8)$$

In addition, the specific volume of the fluid may be considered a function of its entropy and pressure: $V = V(S, P)$. Then,

$$dV = \left(\frac{\partial V}{\partial S} \right)_P dS + \left(\frac{\partial V}{\partial P} \right)_S dP$$

This equation is put into more convenient form through the mathematical identity:

$$\left(\frac{\partial V}{\partial S} \right)_P = \left(\frac{\partial V}{\partial T} \right)_P \left(\frac{\partial T}{\partial S} \right)_P$$

Substituting for the two partial derivatives on the right by Eqs. (3.2) and (6.17) gives:

$$\left(\frac{\partial V}{\partial S} \right)_P = \frac{\beta VT}{C_P}$$

where β is the volume expansivity. The equation derived in physics for the speed of sound c in a fluid is:

$$c^2 = -V^2 \left(\frac{\partial P}{\partial V} \right)_S \quad \text{or} \quad \left(\frac{\partial V}{\partial P} \right)_S = -\frac{V^2}{c^2}$$

Substituting for the two partial derivatives in the equation for dV now yields:

$$\frac{dV}{V} = \frac{\beta T}{C_P} dS - \frac{V}{c^2} dP \quad (7.5)$$

Equations (7.3), (7.4), (6.8), and (7.5) relate the six differentials- dH , du , dV , dA , dS , and dP . With but four equations, we treat dS and dA as independent, and develop equations that express the remaining differentials as functions of these two. First, Eqs. (7.3) and (6.8) are combined:

$$T dS + V dP = -u du \quad (7.6)$$

Eliminating dV and du from Eq. (7.4) by Eqs. (7.5) and (7.6) gives upon rearrangement:

$$(1 - \mathbf{M}^2)V dP + \left(1 + \frac{\beta u^2}{C_P}\right) T dS - \frac{u^2}{A} dA = 0$$

where \mathbf{M} is the Mach number, defined as the ratio of the speed of the fluid in the duct to the speed of sound in the fluid, u/c . Equation (7.7) relates dP to dS and dA . Equations (7.6) and (7.7) are combined to eliminate $V dP$:

$$u du - \left(\frac{\frac{\beta u^2}{C_P} + \mathbf{M}^2}{1 - \mathbf{M}^2}\right) T dS + \left(\frac{1}{1 - \mathbf{M}^2}\right) \frac{u^2}{A} dA = 0 \quad (7.8)$$

This equation relates du to dS and dA . Combined with Eq. (7.3) it relates dH to dS and dA , and combined with (7.4) it relates dV to these same independent variables. The differentials in the preceding equations represent changes in the fluid as it traverses a differential length of its path. If this length is dx , then each of the equations of flow may be divided through by dx . Equations (7.7) and (7.8) then become:

$$V(1 - \mathbf{M}^2) \frac{dP}{dx} + T \left(1 + \frac{\beta u^2}{C_P}\right) \frac{dS}{dx} - \frac{u^2}{A} \frac{dA}{dx} = 0 \quad (7.9)$$

$$u \frac{du}{dx} - T \left(\frac{\frac{\beta u^2}{C_P} + \mathbf{M}^2}{1 - \mathbf{M}^2}\right) \frac{dS}{dx} + \left(\frac{1}{1 - \mathbf{M}^2}\right) \frac{u^2}{A} \frac{dA}{dx} = 0 \quad (7.10)$$

According to the second law, the irreversibilities due to fluid friction in adiabatic flow cause an entropy increase in the fluid in the direction of flow. In the limit as the flow approaches reversibility, this increase approaches zero. In general, then,

$$\frac{dS}{dx} \geq 0$$

Pipe Flow

For the case of steady-state adiabatic flow of compressible fluid in a horizontal pipe of constant cross-sectional area, $dA/dx = 0$, and Eqs. (7.9) and (7.10) reduce to:

$$\frac{dP}{dx} = -\frac{T}{V} \left(\frac{1 + \frac{\beta u^2}{C_P}}{1 - M^2} \right) \frac{dS}{dx} \qquad u \frac{du}{dx} = T \left(\frac{\frac{\beta u^2}{C_P} + M^2}{1 - M^2} \right) \frac{dS}{dx}$$

For subsonic flow, $M^2 < 1$, and all quantities on the right sides of these equations are positive;

whence,

$$\frac{dP}{dx} < 0 \qquad \text{and} \qquad \frac{du}{dx} > 0$$

The equations for pipe flow indicate that when flow is supersonic the pressure increases and the velocity decreases in the direction of flow. However, such a flow regime is unstable, and when a supersonic stream enters a pipe of constant cross section, a compression shock occurs, the result of which is an abrupt and finite increase in pressure and decrease in velocity to a subsonic value.

5.7 ENTROPY BALANCE FOR OPEN SYSTEMS

The statement of balance, expressed as rates, is therefore:

$$\left\{ \begin{array}{c} \text{Net rate of} \\ \text{change in} \\ \text{entropy of} \\ \text{flowing streams} \end{array} \right\} + \left\{ \begin{array}{c} \text{Time rate of} \\ \text{change of} \\ \text{entropy} \\ \text{in control} \\ \text{volume} \end{array} \right\} + \left\{ \begin{array}{c} \text{Time rate of} \\ \text{change of} \\ \text{entropy in} \\ \text{surroundings} \end{array} \right\} = \left\{ \begin{array}{c} \text{Total rate} \\ \text{of entropy} \\ \text{generation} \end{array} \right\}$$

The equivalent *equation of entropy balance* is

$$\Delta(\dot{m}S)_{fs} + \frac{d(mS)_{cv}}{dt} + \frac{dS'_{surr}}{dt} = \dot{S}_G \geq 0 \quad (5.20)$$

Where \dot{S}_G is the rate of entropy generation and $\dot{S}_G \geq 0$. This equation is the general rate form of the entropy balance, applicable at any instant. Each term can vary with time.

The third term accounts for entropy changes in the surroundings, the result of heat transfer between system and surroundings.

Let rate of heat transfer \dot{Q} , with respect to a particular part of the control surface be associated with $T_{\sigma,j}$ where subscript σ, j denotes a temperature in the surroundings. The rate of entropy change in the surroundings as a result of this transfer is then $-\dot{Q}_j/T_{\sigma,j}$. The minus sign converts \dot{Q}_j , defined with respect to the system, to a heat rate with respect to the surroundings. The third term in Eq. (5.20) is therefore the sum of all such quantities:

$$\frac{dS'_{surr}}{dt} = - \sum_j \frac{\dot{Q}_j}{T_{\sigma,j}}$$

Equation (5.20) is now written:

$$\Delta(\dot{m}S)_{fs} + \frac{d(mS)_{cv}}{dt} - \sum_j \frac{\dot{Q}_j}{T_{\sigma,j}} = \dot{S}_G \geq 0 \quad 5.21$$

The final term, representing the rate of entropy generation \dot{S}_G , reflects the second-law requirement that it be positive for irreversible processes. There are two sources of irreversibility: (a) those within the control volume, i.e., internal irreversibilities, and (b) those resulting from heat transfer across finite temperature differences between system and surroundings, i.e., external thermal irreversibilities. In the limiting case where $\dot{S}_G = 0$, the process must be completely reversible, implying:

- The process is internally reversible within the control volume.
- Heat transfer between the control volume and its surroundings is reversible.

For a steady-state flow process the mass and entropy of the fluid in the control volume are constant, and $d(mS)_{cv}/dt$ is zero. Equation (5.21) then becomes:

$$\Delta(S\dot{m})_{fs} - \sum_j \frac{\dot{Q}_j}{T_{\sigma,j}} = \dot{S}_G \geq 0 \quad (5.22)$$

If in addition there is but one entrance and one exit, with m the same for both streams, dividing through by \dot{m} yields:

$$\Delta S - \sum_j \frac{Q_j}{T_{\sigma,j}} = S_G \geq 0 \quad (5.23)$$

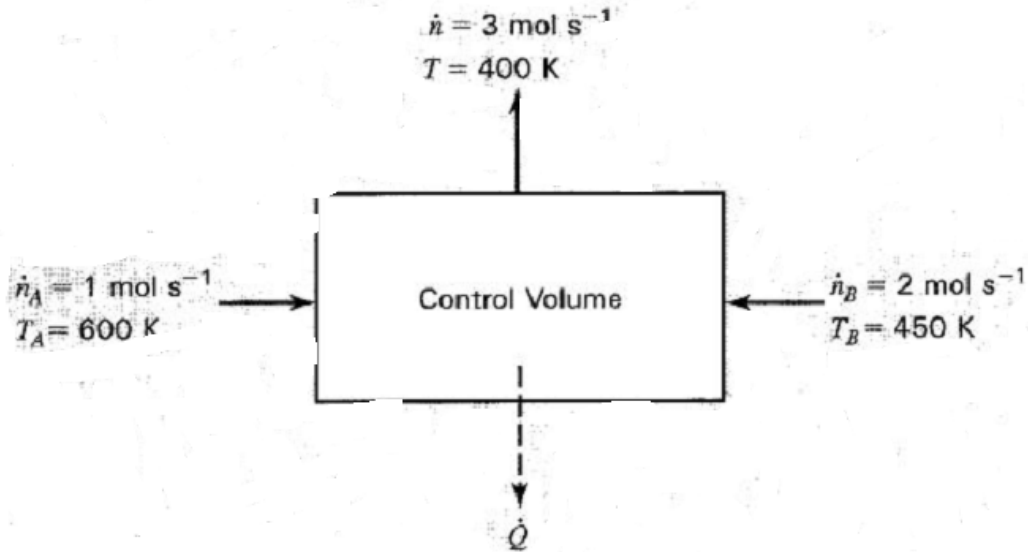
Example 5.5

In a steady-state flow process, 1 mol s^{-1} of air at 600 K and 1 atm is continuously mixed with 2 mol s^{-1} of air at 450 K and 1 atm. The product stream is at 400 K and 1 atm. A schematic representation of the process is shown in Fig. 5.7. Determine the rate of heat transfer and the rate of entropy generation for the process. Assume that air is an ideal gas with $C_P = (7/2)R$, that the surroundings are at 300 K, and that kinetic- and potential-energy changes are negligible.

Solution 5.5

By Eq. (2.30), with \dot{m} replaced by \dot{n} ,

$$\begin{aligned} \dot{Q} &= \dot{n}H - \dot{n}_A H_A - \dot{n}_B H_B = \dot{n}_A (H - H_A) + \dot{n}_B (H - H_B) \\ &= \dot{n}_A C_P (T - T_A) + \dot{n}_B C_P (T - T_B) = C_P [\dot{n}_A (T - T_A) + \dot{n}_B (T - T_B)] \\ &= (7/2)(8.314) [(1)(400 - 600) + (2)(400 - 450)] = -8,729.7 \text{ J s}^{-1} \end{aligned}$$



By Eq. (5.22), again with \dot{m} replaced by \dot{n} ,

$$\begin{aligned}
 \dot{S}_G &= \dot{n}S - \dot{n}_A S_A - \dot{n}_B S_B - \frac{\dot{Q}}{T_\sigma} = \dot{n}_A(S - S_A) + \dot{n}_B(S - S_B) - \frac{\dot{Q}}{T_\sigma} \\
 &= \dot{n}_A C_P \ln \frac{T}{T_A} + \dot{n}_B C_P \ln \frac{T}{T_B} - \frac{\dot{Q}}{T_\sigma} = C_P \left(\dot{n}_A \ln \frac{T}{T_A} + \dot{n}_B \ln \frac{T}{T_B} \right) - \frac{\dot{Q}}{T_\sigma} \\
 &= (7/2)(8.314) \left[(1) \ln \frac{400}{600} + (2) \ln \frac{400}{450} \right] + \frac{8,729.7}{300} = 10.446 \text{ J K}^{-1} \text{ s}^{-1}
 \end{aligned}$$

The rate of entropy generation is positive, as it must be for any real process.

Example 7.1

Consider the steady-state, adiabatic, irreversible flow of an *incompressible* liquid in a horizontal pipe of constant cross-sectional area. Show that:

- (a) The velocity is constant.
- (b) The temperature increases in the direction of flow.
- (c) The pressure decreases in the direction of flow.

Solution 7.1

(a) The control volume here is simply a finite length of horizontal pipe, with entrance and exit sections identified as 1 and 2. By the continuity equation, Eq. (2.27),

$$\frac{u_2 A_2}{V_2} = \frac{u_1 A_1}{V_1}$$

However, $A_2 = A_1$ (constant cross-sectional area) and $V_2 = V_1$ (incompressible fluid). Hence, $u_2 = u_1$.

(b) The entropy balance of Eq. (5.23) here becomes simply $S_G = S_2 - S_1$. For an incompressible liquid with heat capacity C (see Ex. 6.2, pp. 203–204),

$$S_G = S_2 - S_1 = \int_{T_1}^{T_2} C \frac{dT}{T}$$

But S_G is positive (flow is irreversible) and hence, by the last equation, $T_2 > T_1$, and temperature increases in the direction of flow.

(c) As shown in (a), $u_2 = u_1$, and therefore the energy balance, Eq. (2.32), reduces for the stated conditions to $H_2 - H_1 = 0$. Combining this with the integrated form of Eq. (A) of Ex. 6.2 applied to an incompressible liquid yields:

$$H_2 - H_1 = \int_{T_1}^{T_2} C dT + V(P_2 - P_1) = 0$$

Whence,

$$V(P_2 - P_1) = - \int_{T_1}^{T_2} C dT$$

As shown in (b), $T_2 > T_1$; thus by the last equation, $P_2 < P_1$, and pressure decreases in the direction of flow.

Repeating this example for the case of *reversible* adiabatic flow is instructive. In this case $u_2 = u_1$ as before, but $S_G = 0$. The entropy balance then shows that $T_2 = T_1$, in which case the energy balance yields $P_2 = P_1$. We conclude that the temperature increase of (b) and the pressure decrease of (c) *originate* from flow irreversibilities, specifically from the irreversibilities associated with fluid friction. }