

Nozzles

In a properly designed nozzle the area changes with length in such a way as to make the flow nearly frictionless. In the limit of reversible flow, the rate of entropy increase approaches zero, and $dS/dx = 0$. In this event Eqs. (7.9) and (7.10) become:

$$\frac{dP}{dx} = \frac{u^2}{VA} \left(\frac{1}{1-M^2} \right) \frac{dA}{dx} \quad \text{and} \quad \frac{du}{dx} = -\frac{u}{A} \left(\frac{1}{1-M^2} \right) \frac{dA}{dx}$$

The characteristics of flow depend on whether the flow is subsonic ($M < 1$) or supersonic ($M > 1$). The various cases are summarized in Table 7.2.

Table 7.2 Characteristics of Flow for a Nozzle

	Subsonic: $M < 1$		Supersonic: $M > 1$	
	Converging	Diverging	Converging	Diverging
$\frac{dA}{dx}$	—	+	—	+
$\frac{dP}{dx}$	—	+	+	—
$\frac{du}{dx}$	+	—	—	+

Thus, for subsonic flow in a converging nozzle, the velocity increases and the pressure decreases as the cross-sectional area diminishes. The maximum obtainable fluid velocity is the speed of sound, reached at the throat.

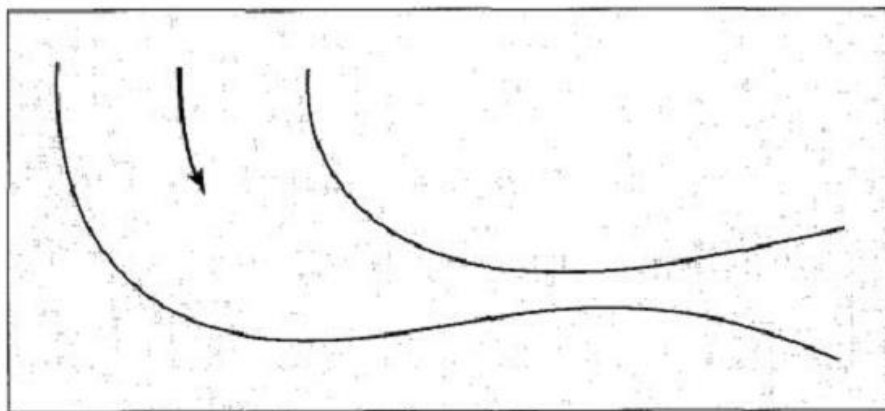


Figure 7.1 Converging/diverging nozzle

The relation of velocity to pressure in an isentropic nozzle can be expressed analytically if the fluid behaves as an ideal gas. Combination of Eqs. (6.8) and (7.3) for isentropic flow gives:

$$u du = -V dP$$

Integration, with nozzle entrance and exit conditions denoted by 1 and 2, yields:

$$u_2^2 - u_1^2 = -2 \int_{P_1}^{P_2} V dP = \frac{2\gamma P_1 V_1}{\gamma - 1} \left[1 - \left(\frac{P_2}{P_1} \right)^{(\gamma-1)/\gamma} \right] \quad (7.11)$$

where the final term is obtained upon elimination of V by Eq. (3.29c), $PV^\gamma = \text{const.}$ Equation (7.11) may be solved for the pressure ratio P_2/P_1 for which u_2 reaches the speed of sound, i.e., where

$$u_2^2 = c^2 = -V^2 \left(\frac{\partial P}{\partial V} \right)_s$$

The derivative is found by differentiation with respect to V of $PV^\gamma = \text{const.}$:

$$\left(\frac{\partial P}{\partial V} \right)_s = -\frac{\gamma P}{V}$$

These two equations together yield: $u_2^2 = \gamma P_2 V_2$

With this value for u_2^2 in Eq. (7.11) and with $u_1 = 0$, solution for the pressure ratio at the throat gives:

$$\frac{P_2}{P_1} = \left(\frac{2}{\gamma + 1} \right)^{\gamma/(\gamma-1)} \quad (7.12)$$

Example 7.2

A high-velocity nozzle is designed to operate with steam at 700 kPa and 300°C. At the nozzle inlet the velocity is 30 m s⁻¹. Calculate values of the ratio A/A_1 (where A_1 is the cross-sectional area of the nozzle inlet) for the sections where the pressure is 600, 500, 400, 300, and 200 kPa. Assume that the nozzle operates isentropically.

Solution 7.2

The required area ratios are given by Eq. (2.27), and the velocity u is found from the integrated form of Eq. (7.3):

$$\frac{A}{A_1} = \frac{u_1 V}{V_1 u} \quad \text{and} \quad u^2 = u_1^2 - 2(H - H_1)$$

For velocity units of m s⁻¹, u^2 has the units, m² s⁻². Units of J kg⁻¹ for H are consistent with these,³ because 1 J = 1 kg m² s⁻², whence 1 J kg⁻¹ = 1 m² s⁻².

Initial values for entropy, enthalpy, and specific volume from the steam tables:

$$S_1 = 7.2997 \text{ kJ kg}^{-1} \text{ K}^{-1} \quad H_1 = 3,059.8 \times 10^3 \text{ J kg}^{-1} \quad V_1 = 371.39 \text{ cm}^3 \text{ g}^{-1}$$

$$\text{Thus,} \quad \frac{A}{A_1} = \left(\frac{30}{371.39} \right) \frac{V}{u} \quad (A)$$

$$\text{and} \quad u^2 = 900 - 2(H - 3,059.8 \times 10^3) \quad (B)$$

Because the expansion is isentropic, $S = S_1$; steam-table values at 600 kPa are:

$$S = 7.2997 \text{ kJ kg}^{-1} \text{ K}^{-1} \quad H = 3,020.4 \times 10^3 \text{ J kg}^{-1} \quad V = 418.25 \text{ cm}^3 \text{ g}^{-1}$$

$$\text{From Eq. (B),} \quad u = 282.3 \text{ m s}^{-1}$$

$$\text{By Eq. (A),} \quad \frac{A}{A_1} = \left(\frac{30}{371.39} \right) \left(\frac{418.25}{282.3} \right) = 0.120$$

Area ratios for other pressures are evaluated the same way, and the results are summarized in the following table.

P/kPa	$V/\text{cm}^3 \text{ g}^{-1}$	$u/\text{m s}^{-1}$	A/A_1	P/kPa	$V/\text{cm}^3 \text{ g}^{-1}$	$u/\text{m s}^{-1}$	A/A_1
700	371.39	30	1.0	400	571.23	523.0	0.088
600	418.25	282.3	0.120	300	711.93	633.0	0.091
500	481.26	411.2	0.095	200	970.04	752.2	0.104

The pressure at the throat of the nozzle is about 380 kPa. At lower pressures, the nozzle clearly diverges.

³When u is in (ft)(s)⁻¹, H in (Btu)(lb_m)⁻¹ must be multiplied by 778.16(ft lb_f)(Btu)⁻¹ and by the dimensional constant $g_c = 32.174(\text{lb}_m)(\text{ft})(\text{lb}_f)^{-1}(\text{s})^{-2}$.

Example 7.3

Consider again the nozzle of Ex. 7.2, assuming now that steam behaves as an ideal gas. Calculate:

- The critical pressure ratio and the velocity at the throat.
- The discharge pressure for a Mach number of 2.0 at the nozzle exhaust.

Solution 7.3

(a) The ratio of specific heats for steam is about 1.3. Substituting in Eq. (7.12),

$$\frac{P_2}{P_1} = \left(\frac{2}{1.3 + 1} \right)^{1.3/(1.3-1)} = 0.55$$

The velocity at the throat, equal to the speed of sound, is found from Eq. (7.11), which contains the product $P_1 V_1$. For steam as an ideal gas:

$$P_1 V_1 = \frac{RT_1}{M} = \frac{(8,314)(573.15)}{18.015} = 264,511 \text{ m}^2 \text{ s}^{-2}$$

In this equation R/M has the units:

$$\frac{\text{J}}{\text{kg K}} = \frac{\text{N m}}{\text{kg K}} = \frac{\text{kg m s}^{-2} \text{ m}}{\text{kg K}} = \frac{\text{m}^2 \text{ s}^{-2}}{\text{K}}$$

Thus RT/M , and hence $P_1 V_1$, is in $\text{m}^2 \text{ s}^{-2}$, the units of velocity squared. Substitution in Eq. (7.11) gives:

$$u_{\text{throat}}^2 = (30)^2 + \frac{(2)(1.3)(264,511)}{1.3 - 1} \left[1 - (0.55)^{(1.3-1)/1.3} \right] = 296,322$$

$$u_{\text{throat}} = 544.35 \text{ m s}^{-1}$$

This result is in good agreement with the value obtained in Ex. 7.2, because steam at these conditions closely approximates an ideal gas.

(b) For a Mach number of 2.0 (based on the velocity of sound at the nozzle throat) the discharge velocity is:

$$2u_{\text{throat}} = (2)(544.35) = 1,088.7 \text{ m s}^{-1}$$

Substitution of this value in Eq. (7.11) allows calculation of the pressure ratio:

$$(1,088.7)^2 - (30)^2 = \frac{(2)(1.3)(264,511)}{1.3 - 1} \left[1 - \left(\frac{P_2}{P_1} \right)^{(1.3-1)/1.3} \right]$$

$$\text{Thus, } \left(\frac{P_2}{P_1} \right)^{(1.3-1)/1.3} = 0.4834 \text{ and } P_2 = (0.0428)(700) = 30.0 \text{ kPa}$$

Throttling Process

When a fluid flows through a restriction, such as an orifice, a partly closed valve, or a porous plug, without any appreciable change in kinetic or potential energy, the primary result of the process is a pressure drop in the fluid. Such a throttling process produces no shaft work, and in the absence of heat transfer, Eq. (2.32) reduces to

$$\Delta H = 0 \quad \text{or} \quad H_2 = H_1$$

For example, if steam at 1000 kPa and 573.15 K (300°C) is throttled to 101.325 kPa (atmospheric pressure),

$$H_2 = H_1 = 3052.1 \text{ kJ kg}^{-1}$$

Interpolation in the steam tables at this enthalpy and at a pressure of 101.325 Pa indicates a downstream temperature of 561.95 K (288.8°C). The temperature has decreased, but the effect is small.

Throttling of wet steam to sufficiently low pressure may cause the liquid to evaporate and the vapor to become superheated. Thus if wet steam at 1000 kPa ($T^{\text{sat}}/t^{\text{sat}}=453.03\text{K}/179.88^\circ\text{C}$) with a quality of 0.96 is throttled to 101.325 kPa,

$$H_2 = H_1 = (0.04)(762.6) + (0.96)(2776.2) = 2695.7 \text{ kJ kg}^{-1}$$

At 101.325 kPa steam with this enthalpy has a temperature of 382.95 K (109.8°C); it is therefore superheated ($T^{\text{sat}}/t^{\text{sat}} = 373.15\text{K}/100^\circ\text{C}$). The considerable temperature drop here results from evaporation of liquid.

If a saturated liquid is throttled to a lower pressure, some of the liquid vaporizes or flashes, producing a mixture of saturated liquid and saturated vapor at the lower pressure. Thus if saturated liquid water at 1000 kPa ($T^{\text{sat}}/t^{\text{sat}} = 453.03\text{K}/179.88^\circ\text{C}$) is flashed to 101.325 kPa ($T^{\text{sat}}/t^{\text{sat}} = 373.15 \text{ K}/100^\circ\text{C}$),

$$H_2 = H_1 = 762.6 \text{ kJ kg}^{-1}$$

At 101.325 kPa the quality of the resulting steam is found from Eq. (6.73a) with $M = H$:

$$\begin{aligned} 762.6 &= (1 - x)(419.1) + x(2676.0) \\ &= 419.1 + x(2676.0 - 419.1) \\ x &= 0.152 \end{aligned}$$

Thus 15.2% of the original liquid vaporizes in the process. Again, the large temperature drop results from evaporation of liquid.

Example 7.4

Propane gas at 20 bar and 400 K is throttled in a steady-state flow process to 1 bar. Estimate the final temperature of the propane and its entropy change. Properties of propane can be found from suitable generalized correlations.

Solution 7.4

Apply Eq. (6.93) to this constant-enthalpy process:

$$\Delta H = \langle C_P^{ig} \rangle_H (T_2 - T_1) + H_2^R - H_1^R = 0$$

If propane in its final state at 1 bar is assumed to be an ideal gas, then $H_2^R = 0$, and the preceding equation, solved for T_2 , becomes

$$T_2 = \frac{H_1^R}{\langle C_P^{ig} \rangle_H} + T_1 \quad (A)$$

For propane, $T_c = 369.8 \text{ K}$ $P_c = 42.48 \text{ bar}$ $\omega = 0.152$

Thus for the initial state,

$$T_{r1} = \frac{400}{369.8} = 1.082 \quad P_{r1} = \frac{20}{42.48} = 0.471$$

At these conditions the generalized correlation based on second virial coefficients is satisfactory (Fig. 3.14), and calculation of H_1^R by Eqs. (6.87), (3.65), (6.89), (3.66), and (6.90) is represented by (Sec. 6.7):

$$\frac{H_1^R}{RT_c} = \text{HRB}(1.082, 0.471, 0.152) = -0.452$$

Whence, $H_1^R = (8.314)(369.8)(-0.452) = -1,390 \text{ J mol}^{-1}$

The only remaining quantity in Eq. (A) to be evaluated is $\langle C_P^{ig} \rangle_H$. Data for propane from Table C.1 provide the heat-capacity equation:

$$\frac{C_P^{ig}}{R} = 1.213 + 28.785 \times 10^{-3} T - 8.824 \times 10^{-6} T^2$$

For an initial calculation, assume that $\langle C_P^{ig} \rangle_H$ equals the value of C_P^{ig} at the initial temperature of 400 K, i.e., $\langle C_P^{ig} \rangle_H = 94.07 \text{ J mol}^{-1} \text{ K}^{-1}$.

From Eq. (A), $T_2 = \frac{-1,390}{94.07} + 400 = 385.2 \text{ K}$

Clearly, the temperature change is small, and $\langle C_P^{ig} \rangle_H$ is reevaluated to an excellent approximation as C_P^{ig} at the arithmetic mean temperature,

$$T_{\text{am}} = \frac{400 + 385.2}{2} = 392.6 \text{ K}$$

This gives: $\langle C_p^{ig} \rangle_H = 92.73 \text{ J mol}^{-1} \text{ K}^{-1}$

and recalculation of T_2 by Eq. (A) yields the final value: $T_2 = 385.0 \text{ K}$.

The entropy change of the propane is given by Eq. (6.94), which here becomes:

$$\Delta S = \langle C_p^{ig} \rangle_S \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} - S_1^R$$

Because the temperature change is so small, to an excellent approximation,

$$\langle C_p^{ig} \rangle_S = \langle C_p^{ig} \rangle_H = 92.73 \text{ J mol}^{-1} \text{ K}^{-1}$$

Calculation of S_1^R by Eqs. (6.88) through (6.90) is represented by:

$$\frac{S_1^R}{R} = \text{SRB}(1.082, 0.471, 0.152) = -0.2934$$

Whence, $S_1^R = (8.314)(-0.2934) = -2.439 \text{ J mol}^{-1} \text{ K}^{-1}$

and $\Delta S = 92.73 \ln \frac{385.0}{400} - 8.314 \ln \frac{1}{20} + 2.439 = 23.80 \text{ J mol}^{-1} \text{ K}^{-1}$

The positive value reflects the irreversibility of throttling processes.