7.3 COMPRESSION PROCESSES

Just as expansion processes result in pressure reductions in a flowing fluid, so compression processes bring about pressure increases. Compressors, pumps, fans, blowers, and vacuum pumps are all devices designed for this purpose.

Compressors

The compression of gases may be accomplished in equipment with rotating blades (like a turbine operating in reverse) or in cylinders with reciprocating pistons. Rotary equipment is used for high-volume flow where the discharge pressure is not too high. For high pressures, reciprocating compressors are required

In a compression process, the isentropic work, as given by Eq. (7.15), is the minimum shaft work required for compression of a gas from a given initial state to a given discharge pressure. Thus we define compressor efficiency as:

$$\eta \equiv \frac{W_s(\text{isentropic})}{W_s}$$

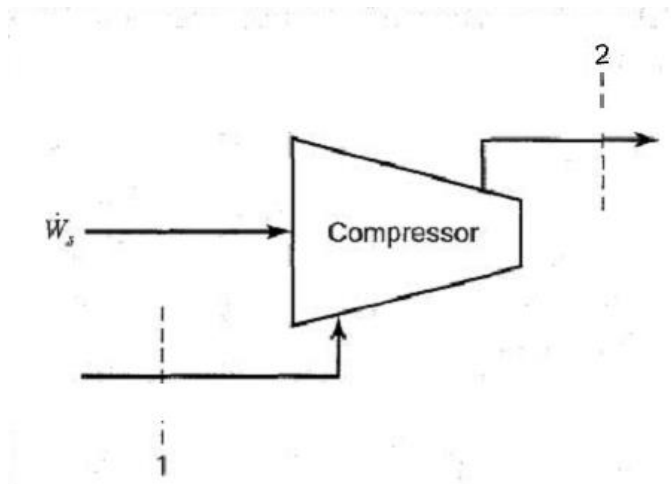

Figure *7.5* Steady-state compression process

In view of Eqs. (7.14) and (7.15), this is also given by:

$$\eta \equiv \frac{(\Delta H)_S}{\Delta H} \tag{7.17}$$

Compressor efficiencies are usually in the range of 0.7 to 0.8

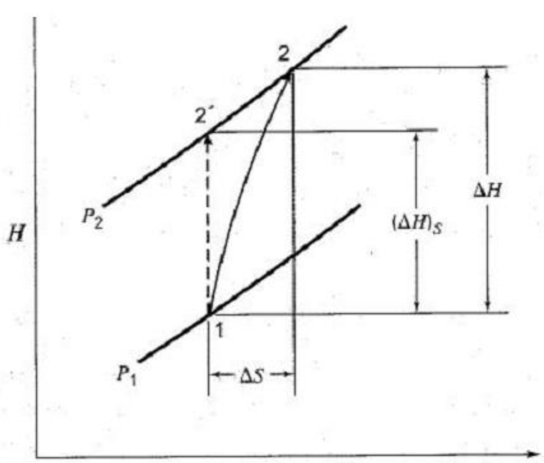

Figure *7.6* Adiabatic compression process

Example 7.8

Saturated-vapor steam at 100 kPa (t^{sat} = 99.63°C) is compressed adiabatically to 300 kPa. If the compressor efficiency is 0.75, what is the work required and what are the properties of the discharge stream?

For saturated steam at 100 kPa,

$$S_1 = 7.3598 \text{ kJ kg}^{-1} \text{ K}^{-1} \qquad H_1 = 2{,}675.4 \text{ kJ kg}^{-1}$$

For isentropic compression to 300 kPa, $S_2' = S_1 = 7.3598 \text{ kJ kg}^{-1} \text{ K}^{-1}$. Interpolation in the tables for superheated steam at 300 kPa shows that steam with this entropy has the enthalpy: $H_2' = 2{,}888.8 \text{ kJ kg}^{-1}$.

Thus, $$(\Delta H)_S = 2{,}888.8 - 2{,}675.4 = 213.4 \text{ kJ kg}^{-1}$$

By Eq. (7.17), $$\Delta H = \frac{(\Delta H)_S}{\eta} = \frac{213.4}{0.75} = 284.5 \text{ kJ kg}^{-1}$$

Whence, $$H_2 = H_1 + \Delta H = 2{,}675.4 + 284.5 = 2{,}959.9 \text{ kJ kg}^{-1}$$

For superheated steam with this enthalpy, interpolation yields:

$$T_2 = 246.1°\text{C} \qquad S_2 = 7.5019 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

Moreover, by Eq. (7.14), the work required is:

$$W_s = \Delta H = 284.5 \text{ kJ kg}^{-1}$$

Where such information is not available, the generalized correlations of Sec. 6.7 may be used in conjunction with Eqs. (6.84) and (6.85), exactly as illustrated in Ex. 7.7 for an expansion process. The assumption of ideal gases leads to equations of relative simplicity. By Eq. (5.18) for an ideal gas:

$$\Delta S = \langle C_P \rangle_S \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

where for simplicity the superscript "ig" has been omitted from the mean heat capacity. If the compression is isentropic, ΔS = 0, and this equation becomes:

$$T_2' = T_1 \left(\frac{P_2}{P_1}\right)^{R/\langle C_P' \rangle_S} \tag{7.18}$$

where $\grave{T}_2$ is the temperature that results when compression from T_1 and P_1 to P_2 is isentropic and where $(\grave{C}p)_s$ is the mean heat-capacity for the temperature range from T_1 to $\grave{T}_2$

Applied to isentropic compression, Eq. (4.9) here becomes:

$$(\Delta H)_S = \langle C'_P \rangle_H (T'_2 - T_1)$$

In accord with Eq. (7.15),

$$W_s(\text{isentropic}) = \langle C'_P \rangle_H (T'_2 - T_1) \tag{7.19}$$

This result may be combined with the compressor efficiency to give:

$$W_s = \frac{W_s(\text{isentropic})}{\eta} \tag{7.20}$$

The actual discharge temperature T_2 resulting from compression is also found from Eq. (4.9), rewritten as:

$$\Delta H = \langle C_P \rangle_H (T_2 - T_1)$$

Whence,

$$T_2 = T_1 + \frac{\Delta H}{\langle C_P \rangle_H} \tag{7.21}$$

where by Eq. (7.14) $\Delta H = W_s$, Here $(Cp)_H$ is the mean heat-capacity for the temperature range from T_1 to T_2

For the special case of an ideal gas with constant heat capacities,

$$\langle C'_P \rangle_H = \langle C_P \rangle_H = \langle C'_P \rangle_S = C_P$$

Equations (7.18) and (7.19) therefore become:

$$T'_2 = T_1 \left(\frac{P_2}{P_1} \right)^{R/C_P} \qquad \text{and} \qquad W_s(\text{isentropic}) = C_P (T'_2 - T_1)$$

Combining these equations gives:

$$W_s(\text{isentropic}) = C_P T_1 \left[\left(\frac{P_2}{P_1} \right)^{R/C_P} - 1 \right] \tag{7.22}$$

For monatomic gases, such as argon and helium, R/Cp = 2/5 = 0.4. For such diatomic gases as oxygen, nitrogen, and air at moderate temperatures, R/Cp ≅ 2/7 = 0.2857. For gases of greater molecular complexity the ideal-gas heat capacity depends more strongly on temperature, and Eq. (7.22) is less likely to be suitable. One can easily show that the assumption of constant heat capacities also leads to the result:

$$T_2 = T_1 + \frac{T_2' - T_1}{\eta} \tag{7.23}$$

Example 7.9

If methane (assumed to be an ideal gas) is compressed adiabatically from 20°C and 140 kPa to 560 kPa, estimate the work requirement and the discharge temperature of the methane. The compressor efficiency is 0.75.

Solution 7.9

Application of Eq. (7.18) requires evaluation of the exponent $R/\langle C'_P \rangle_S$. This is provided Eq. (5.17), which for the present computation is represented by:

$$\frac{\langle C'_P \rangle_S}{R} = \text{MCPS(293.15,T2;1.702,9.081E-3,-2.164E-6,0.0)}$$

where the constants for methane are from Table C.1. Choose a value for T_2' somewhat higher than the initial temperature $T_1 = 293.15$ K. The exponent in Eq. (7.18) is the reciprocal of $\langle C'_P \rangle_S / R$. With $P_2/P_1 = 560/140 = 4.0$ and $T_1 = 293.15$ K, find a new value of T_2'. The procedure is repeated until no further significant change occurs in the value of T_2'. This process produces the values:

$$\frac{\langle C'_P \rangle_S}{R} = 4.5574 \qquad \text{and} \qquad T_2' = 397.37 \text{ K}$$

For the same T_1 and T_2', evaluate $\langle C'_P \rangle_H / R$ by Eq. (4.8):

$$\frac{\langle C'_P \rangle_H}{R} = \text{MCPH(293.15,397.37;1.702,9.081E-3,-2.164E-6,0.0)} = 4.5774$$

Whence. $\qquad \langle C'_P \rangle_H = (4.5774)(8.314) = 38.056 \text{ J mol}^{-1} \text{ K}^{-1}$

Then by Eq. (7.19),

$$W_s(\text{isentropic}) = (38.056)(397.37 - 293.15) = 3{,}966.2 \text{ J mol}^{-1}$$

The actual work is found from Eq. (7.20):

$$W_s = \frac{3{,}966.2}{0.75} = 5{,}288.3 \text{ J mol}^{-1}$$

Application of Eq. (7.21) for the calculation of T_2 gives:

$$T_2 = 293.15 + \frac{5{,}288.3}{\langle C_P \rangle_H}$$

Because $\langle C_P \rangle_H$ depends on T_2, we again iterate. With T_2' as a starting value, this leads to the results:

$$T_2 = 428.65 \text{ K} \qquad \text{or} \qquad t_2 = 155.5°\text{C}$$

and $$\langle C_P \rangle_H = 39.027 \text{ J mol}^{-1} \text{ K}^{-1}$$

Pumps ∆

Liquids are usually moved by pumps, generally rotating equipment. The same equations apply to adiabatic pumps as to adiabatic compressors. Thus, Eqs. (7.13) through (7.15) and Eq. (7.17) are valid. However, application of Eq. (7.14) for the calculation of $W_s = \Delta H$ requires values of the enthalpy of compressed (subcooled) liquids, and these are seldom available. The fundamental property relation, Eq. (6.8), provides an alternative. For an isentropic process,

$$dH = V\,dP \qquad (\text{const } S)$$

Combining this with Eq. (7.15) yields:

$$W_s(\text{isentropic}) = (\Delta H)_S = \int_{P_1}^{P_2} V\,dP$$

The usual assumption for liquids (at conditions well removed from the critical point) is that V is independent of P. Integration then gives:

$$W_s(\text{isentropic}) = (\Delta H)_S = V(P_2 - P_1) \qquad (7.24)$$

Also useful are the following equations from Chap. 6:

$$dH = C_P\,dT + V(1 - \beta T)\,dP \qquad (6.28)$$

$$dS = C_P \frac{dT}{T} - \beta V\,dP \qquad (6.29)$$

where the volume expansivity β is defined by Eq. (3.2). Since temperature changes in the pumped fluid are very small and since the properties of liquids are insensitive to pressure (again at conditions not close to the critical point), these equations are usually integrated on the assumption that Cp, V, and β are constant, usually at initial values. Thus, to a good approximation

$$\Delta H = C_P\,\Delta T + V(1 - \beta T)\Delta P \qquad (7.25)$$

$$\Delta S = C_P \ln \frac{T_2}{T_1} - \beta V\,\Delta P \qquad (7.26)$$

Example 7.10

Water at 45°C and 10 kPa enters an adiabatic pump and is discharged at a pressure of 8,600 kPa. Assume the pump efficiency to be 0.75. Calculate the work of the pump, the temperature change of the water, and the entropy change of the water.

Solution 7.10

The following are properties for saturated liquid water at 45°C (318.15 K):

$$V = 1{,}010\ \text{cm}^3\ \text{kg}^{-1} \qquad \beta = 425 \times 10^{-6}\ \text{K}^{-1} \qquad C_P = 4.178\ \text{kJ kg}^{-1}\ \text{K}^{-1}$$

By Eq. (7.24),

$$W_s(\text{isentropic}) = (\Delta H)_S = (1{,}010)(8{,}600 - 10) = 8.676 \times 10^6\ \text{kPa cm}^3\ \text{kg}^{-1}$$

Because 1 kJ = 10^6 kPa cm^3.

$$W_s(\text{isentropic}) = (\Delta H)_S = 8.676 \text{ kJ kg}^{-1}$$

By Eq. (7.17), $$\Delta H = \frac{(\Delta H)_S}{\eta} = \frac{8.676}{0.75} = 11.57 \text{ kJ kg}^{-1}$$

and $$W_s = \Delta H = 11.57 \text{ kJ kg}^{-1}$$

The temperature change of the water during pumping, from Eq. (7.25):

$$11.57 = 4.178\, \Delta T + 1{,}010 \left[1 - (425 \times 10^{-6})(318.15)\right] \frac{8{,}590}{10^6}$$

Solution for ΔT gives:

$$\Delta T = 0.97 \text{ K} \qquad \text{or} \qquad 0.97^\circ\text{C}$$

The entropy change of the water is given by Eq. (7.26):

$$\Delta S = 4.178 \ln \frac{319.12}{318.15} - (425 \times 10^{-6})(1{,}010)\frac{8{,}590}{10^6} = 0.0090 \text{ kJ kg}^{-1} \text{ K}^{-1}$$