# Chapter 9

## Refrigeration and Liquefaction

Refrigeration is best known for its use in the air conditioning of buildings and in the treatment, transportation, and preservation of foods and beverages. It also finds large-scale industrial application, for example, in the manufacture of ice and the dehydration of gases. Applications in the petroleum industry include lubricating-oil purification, low-temperature reactions, and separation of volatile hydrocarbons. A closely related process is gas liquefaction, which has important commercial applications.

The purpose of this chapter is to present a thermodynamic analysis of refrigeration and liquefaction processes. However, the details of equipment design are left to specialized books.

The word refrigeration implies the maintenance of a temperature below that of the surroundings. This requires continuous absorption of heat at a low temperature level, usually accomplished by evaporation of a liquid in a steady-state flow process. The vapor formed may be returned to its original liquid state for reevaporation in either of two ways. Most commonly, it is simply compressed and then condensed. Alternatively, it may be absorbed by a liquid of low volatility, from which it is subsequently evaporated at higher pressure.

### 9.1 THE CARNOT REFRIGERATOR

In a continuous refrigeration process, the heat absorbed at a low temperature is continuously rejected to the surroundings at a higher temperature. Basically, a refrigeration cycle is a reversed heat-engine cycle.

Heat is transferred from a low temperature level to a higher one; according to the second law, this requires an external source of energy. The ideal refrigerator, like the ideal heat engine (Sec. 5.2), operates on a Carnot cycle, consisting in this case of two isothermal steps in which heat $|Qc|$ is absorbed at the lower temperature Tc and heat $|QH|$ is rejected at the higher temperature $T_H$, and two adiabatic steps. The cycle requires the addition of net work **W** to the system. Since **ΔU** of the working fluid is zero for the cycle, the first law is written:

$$W = |Q_H| - |Q_C| \tag{9.1}$$

The measure of the effectiveness of a refrigerator is its coefficient of performance $\omega$, or (COP) defined as:

$$\omega \equiv \frac{\text{heat absorbed at the lower temperature}}{\text{net work}} = \frac{|Q_C|}{\text{W}} \tag{9.2}$$

Equation (9.1) may be divided by $|Qc|$ :

$$\frac{W}{|Q_C|} = \frac{|Q_H|}{|Q_C|} - 1$$

Combination with Eq. (5.7) gives:

$$\frac{W}{|Q_C|} = \frac{T_H}{T_C} - 1 = \frac{T_H - T_C}{T_C}$$

and Eq. (9.2) becomes:

$$\omega = \frac{T_C}{T_H - T_C} \qquad (9.3)$$

This equation applies only to a refrigerator operating on a Carnot cycle, and it gives the maximum possible value of $\omega$ for any refrigerator operating between given values of $T_H$ and Tc. It shows clearly that the refrigeration effect per unit of work decreases as the temperature of heat absorption Tc decreases and as the temperature of heat rejection $T_H$ increases. For refrigeration at a temperature level of 278.15 K (5°C) in a surroundings at 303.15 K (30°C), the value of $\omega$ for a Carnot refrigerator is:

$$\omega = \frac{278.15}{(303.15 - 278.15)} = 11.13$$

## 9.2 THE VAPOR-COMPRESSION CYCLE

The vapor-compression refrigeration cycle is represented in Fig. 9.1. Shown on the TS diagram are the four steps of the process.

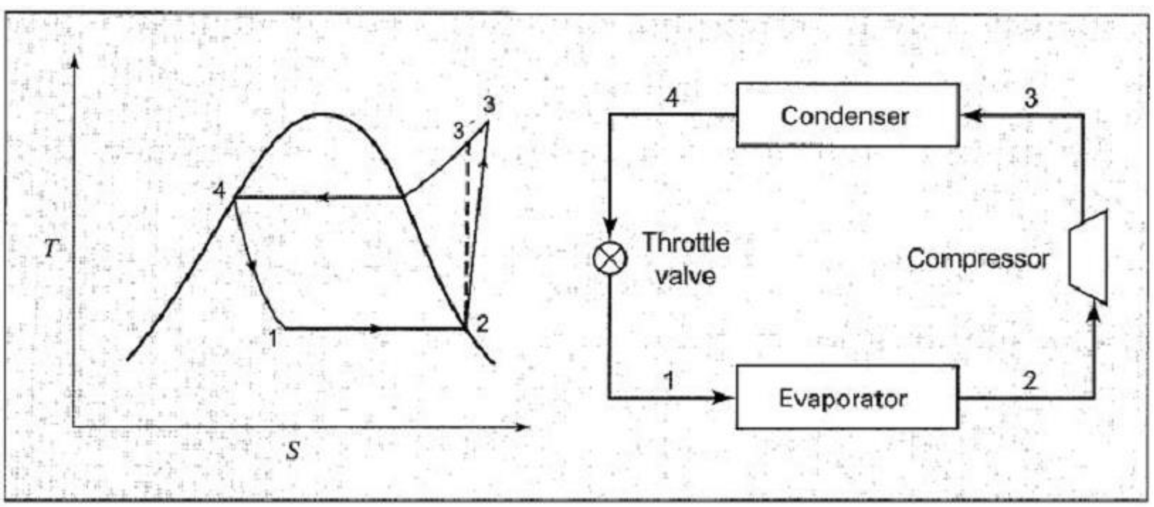


**Figure 9.1** Vapor-compression refrigeration cycle

A liquid evaporating at constant pressure (line 1- 2) provides a means for heat absorption at a low constant temperature. The vapor produced is compressed to a higher pressure, and is then cooled and condensed with rejection of heat at a higher temperature level. Liquid from the condenser returns to its original pressure by an expansion process. In principle, this can be carried out in an expander from which work is obtained, but for practical reasons is accomplished by throttling through a partly open valve. The pressure drop in this irreversible process results from fluid friction in the valve. As shown in Sec. 7.1, the throttling process occurs at constant enthalpy. In Fig. 9.1 line 4 - 1 represents this throttling process. The dashed line 2 - 3' is the path of isentropic compression (Fig. 7.6). Line 2 - 3, representing the actual compression process, slopes in the direction of increasing entropy, reflecting inherent irreversibilities.

On the basis of a unit mass of fluid, the equations for the heat absorbed in the evaporator and the heat rejected in the condenser are:

$$|Q_C| = H_2 - H_1 \qquad \text{and} \qquad |Q_H| = H_3 - H_4$$

These equations follow from Eq. (2.32) when the small changes in potential and kinetic energy are neglected. The work of compression is simply:

$$W = H_3 - H_2$$

and by Eq. (9.2), the coefficient of performance is:

$$\omega = \frac{H_2 - H_1}{H_3 - H_2} \tag{9.4}$$

To design the evaporator, compressor, condenser, and auxiliary equipment one must know the rate of circulation of refrigerant $\dot{m}$. This is determined from the rate of heat absorption in evaporate by the equation:

$$\dot{m} = \frac{|\dot{Q}_C|}{H_2 - H_1} \tag{9.5}$$

**In the United States refrigeration equipment is commonly rated in tons of refrigeration; a ton of refrigeration is defined as heat absorption at the rate of 12000 Btu $h^{-1}$ or 12652.2 kJ $h^{-1}$. This corresponds approximately to the rate of heat removal required to freeze 1 short ton [or 2000 (lb)] of water initially at 32 ($^{O}$F) per day or remove 3.5145 kW at 273.15 K (0°C).**

The vapor-compression cycle of Fig. 9.1 is shown on a PH diagram in Fig. 9.2. Such diagrams are more commonly used in the description of refrigeration processes than TS diagrams, because they show directly the required enthalpies. Although the evaporation and condensation processes are represented by constant-pressure paths, small pressure drops do occur because of fluid friction.

For given values of Tc and $T_H$, the highest possible value of $\omega$ is attained for Carnot- cycle refrigeration. The lower values for the vapor-compression cycle result from irreversible expansion in a throttle valve and irreversible compression. The following example provides an indication of typical values for coefficients of performance.

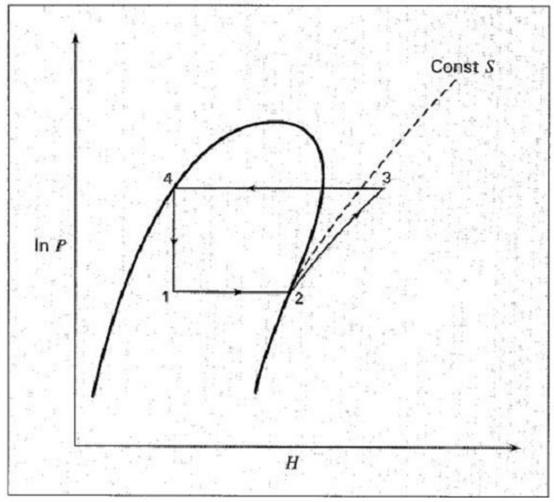


**Figure 9.2** Vapor-compression refrigeration cycle on a $P$H diagram

# Example 9.1

A refrigerated space is maintained at 10(°F), and cooling water is available at 70(°F). Refrigeration capacity is 120,000(Btu)(hr)$^{-1}$. The evaporator and condenser are of sufficient size that a 10(°F) minimum-temperature difference for heat transfer can be realized in each. The refrigerant is tetrafluoroethane (HFC-134a), for which data are given in Table 9.1 and Fig. G.2 (App. G).

(*a*) What is the value of $\omega$ for a Carnot refrigerator?

(*b*) Calculate $\omega$ and $\dot{m}$ for the vapor-compression cycle of Fig. 9.1 if the compressor efficiency is 0.80.

T **Solution 9.1** łi

(*a*) By Eq. (9.3) for a Carnot refrigerator,

$$\omega = \frac{0 + 459.67}{(80 + 459.67) - (0 + 459.67)} = 5.75$$

(*b*) Because HFC-134a is the refrigerant, the enthalpies for states 2 and 4 of Figs. 9.1 and 9.2 are read directly from Table 9.1. The entry at $10 - 10 = 0(^\circ\mathrm{F})$ indicates that HFC-134a vaporizes in the evaporator at a pressure of 21.162(psia). Its properties as a saturated vapor at these conditions are:

$$H_2 = 103.015(\mathrm{Btu})(\mathrm{lb_m})^{-1} \qquad S_2 = 0.22525(\mathrm{Btu})(\mathrm{lb_m})^{-1}(\mathrm{R})^{-1}$$

The entry at $70 + 10 = 80(^\circ\mathrm{F})$ in Table 9.1 shows that HFC-134a condenses at 101.37(psia); its enthalpy as a saturated liquid at these conditions is:

$$H_4 = 37.978(\mathrm{Btu})(\mathrm{lb_m})^{-1}$$

If the compression step is reversible and adiabatic (isentropic) from saturated vapor at state 2 to superheated vapor at state 3′,

$$S_3' = S_2 = 0.22525$$

The enthalpy from Fig. G.2 at this entropy and at a pressure of 101.37(psia) is about:

$$H_3' = 117(\mathrm{Btu})(\mathrm{lb_m})^{-1}$$

and the enthalpy change is:

$$(\Delta H)_S = H_3' - H_2 = 117 - 103.015 = 13.98(\mathrm{Btu})(\mathrm{lb_m})^{-1}$$

By Eq. (7.17) for a compressor efficiency of 0.80, the actual enthalpy change for step $2 \rightarrow 3$ is:

$$H_3 - H_2 = \frac{(\Delta H)_S}{\eta} = \frac{13.98}{0.80} = 17.48(\mathrm{Btu})(\mathrm{lb_m})^{-1}$$

Because the throttling process of step $1 \rightarrow 4$ is isenthalpic, $H_1 = H_4$. The coefficient of performance as given by Eq. (9.4) therefore becomes:

$$\omega = \frac{H_2 - H_4}{H_3 - H_2} = \frac{103.015 - 37.978}{17.48} = 3.72$$

and the HFC-134a circulation rate as given by Eq. (9.5) is:

$$\dot{m} = \frac{|\dot{Q}_C|}{H_2 - H_4} = \frac{120{,}000}{103.015 - 37.978} = 1{,}845(\mathrm{lb_m})(\mathrm{hr})^{-1}$$

### 9.5 THE HEAT PUMP

The heat pump, a reversed heat engine, is a device for heating houses and commercial buildings during the winter and cooling them during the summer. In the winter it operates so as to absorb heat from the surroundings and reject heat into the building.

The heat pump also serves for air conditioning during the summer. The flow of refrigerant is simply reversed, and heat is absorbed from the building and rejected through underground coils or to the outside air.

## Example 9.2

A house has a winter heating requirement of 30 kJ $s^{-1}$ and a summer cooling requirement of 60 kJ $s^{-1}$. Consider a heat-pump installation to maintain the house temperature at 20°C in winter and 25°C in summer. This requires circulation of the refrigerant through interior exchanger coils at 30°C in winter and 5°C in summer. Underground coils provide the heat source in winter and the heat sink in summer. For a year-round ground temperature of 15°C, the heat-transfer characteristics of the coils necessitate refrigerant temperatures of 10°C in winter and 25°C in summer. What are the minimum power requirements for winter heating and summer cooling?

### Solution 9.2

The minimum power requirements are provided by a Carnot heat pump. For winter heating, the house coils are at the higher-temperature level $T_H$, and the heat requirement is $|Q_H| = 30$ kJ $s^{-1}$. Application of Eq. (5.7) gives:

$$|Q_C| = |Q_H|\frac{T_C}{T_H} = 30\left(\frac{10 + 273.15}{30 + 273.15}\right) = 28.02 \text{ kJ s}^{-1}$$

This is the heat absorbed in the ground coils. By Eq. (9.1),

$$W = |Q_H| - |Q_C| = 30 - 28.02 = 1.98 \text{ kJ s}^{-1}$$

Thus the power requirement is 1.98 kW.

For summer cooling, $|Q_C| = 60$ kJ $s^{-1}$, and the house coils are at the lower-temperature level $T_C$. Combine Eqs. (9.2) and (9.3) and solve for $W$:

$$W = |Q_C|\frac{T_H - T_C}{T_C} = 60\left(\frac{25 - 5}{5 + 273.15}\right) = 4.31 \text{ kJ s}^{-1}$$

The power requirement here is therefore 4.31 kW.

## 9.6 LIQUEFACTION PROCESSES

Liquefied gases are in common use for a variety of purposes. For example, liquid propane in cylinders serves as a domestic fuel, liquid oxygen is carried in rockets, natural gas is liquefied for ocean transport, and liquid nitrogen is used for low-temperature refrigeration. In addition, gas mixtures (e.g., air) are liquefied for separation into their component species by fractionation.

Liquefaction results when a gas is cooled to a temperature in the two-phase region. This may be accomplished in several ways:

1. By heat exchange at constant pressure.
2. By an expansion process from which work is obtained.
3. By a throttling process.

The Linde liquefaction process, which depends solely on throttling expansion, is shown in Fig. 9.6. After compression, the gas is precooled to ambient temperature. It may be even further cooled by refrigeration. The lower the temperature of the gas entering the throttle valve, the greater the fraction of gas that is liquefied. For example, a refrigerant evaporating in the cooler at 233.15 K (-40°C) provides a lower temperature at the valve than if water at 294.15 K (21°C) is the cooling medium.

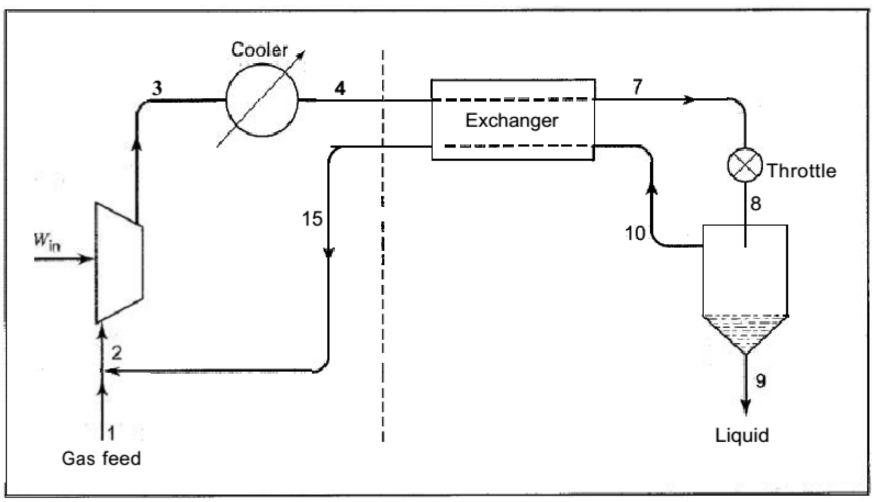


**Figure 9.6** Linde liquefaction process

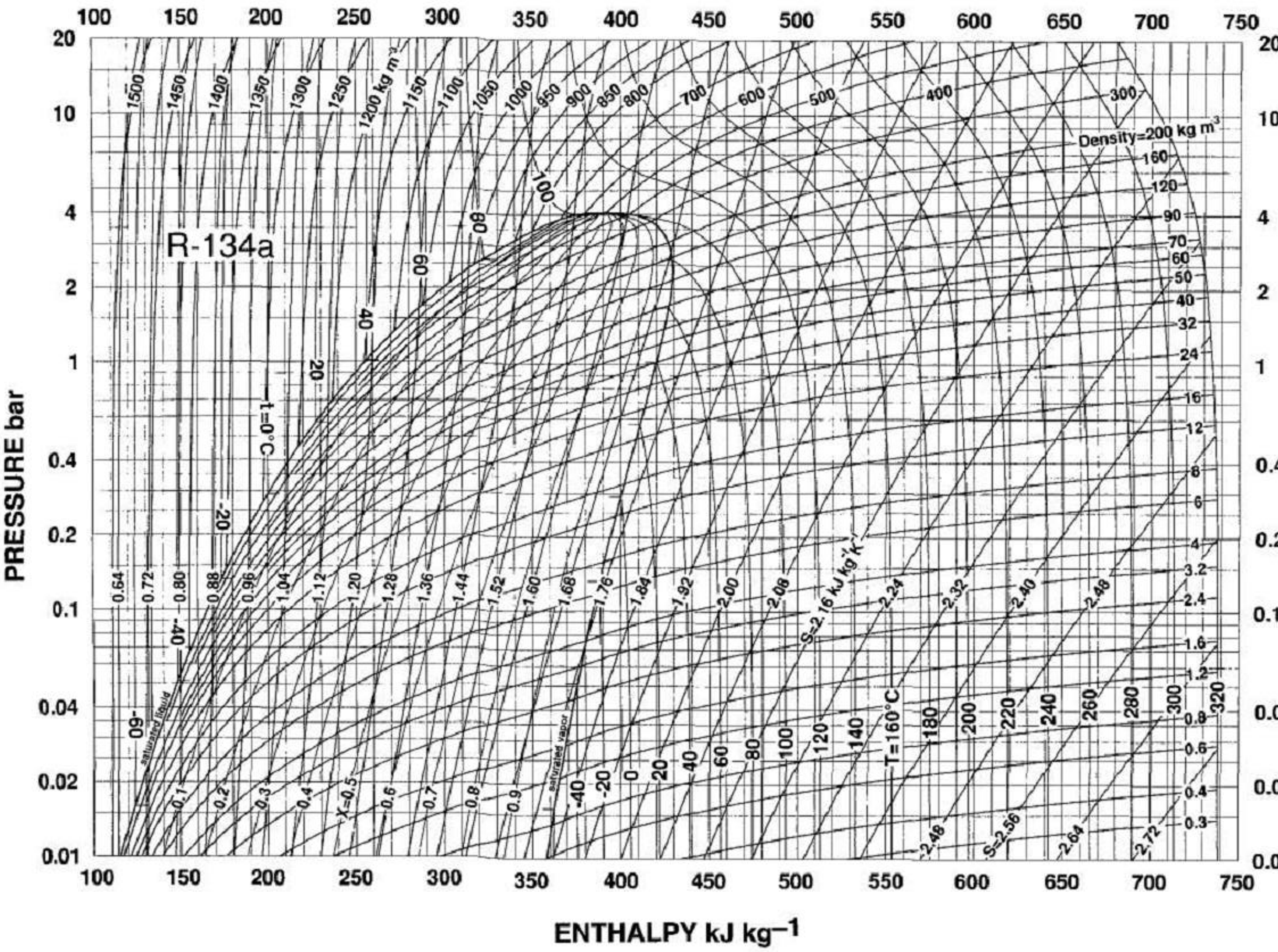


**Figure G.2:** *PH* diagram for tetrafluroethane (HFC-134a). (Reproduced by permission, ASHRAE Handbook : Fundamentals, p.17.28, American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., USA, 1993.