Chapter 05.08
Why Do we Need Splines?

After reading this chapter, you should be able to:

1. *understand why we use splines for interpolation.*

Example

Peter: "Dr. Kaw, in class, you were talking about higher order interpolation being a bad idea and then telling us that taking more points is not going to get you a better approximation."
Kaw: "Yes, we were talking in class about the classic example taken by Runge. He took $f(x) = 1/(1+25x^2)$ in the domain [-1,1]. Choosing 20 equidistant points (Figure 1) on [-1,1] to approximate the function by a 19th order polynomial gave worse results than when we chose 6 equidistant points to approximate the function by a 5th order polynomial."
Peter: "Yes, it was wild. So what do we do? Accept this fact and roll over?"
Kaw: "Now, we do not have to do that. We can use interpolation such as cubic splines. Cubic splines approximate data between consecutive data points by cubic polynomials but at the same time use all the data to approximate the function. You can see from Figure 2 how cubic splines do a better job of approximating the data. The thin dash line is a 19^{th} order polynomial approximation of the function by choosing 20 equidistant data points in [-1, 1], while the thick dash line is the cubic spline approximation of the data. See how close the cubic splines are to the original function (continuous line)."

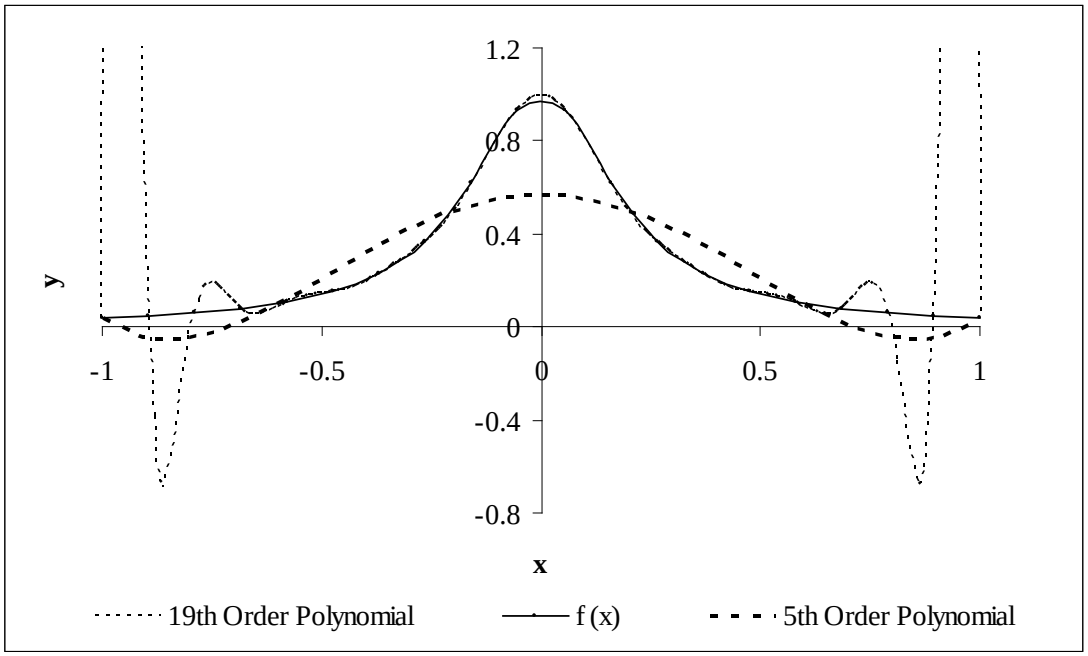

Figure 1 5th and 19th order polynomial approximations of Runge's function.

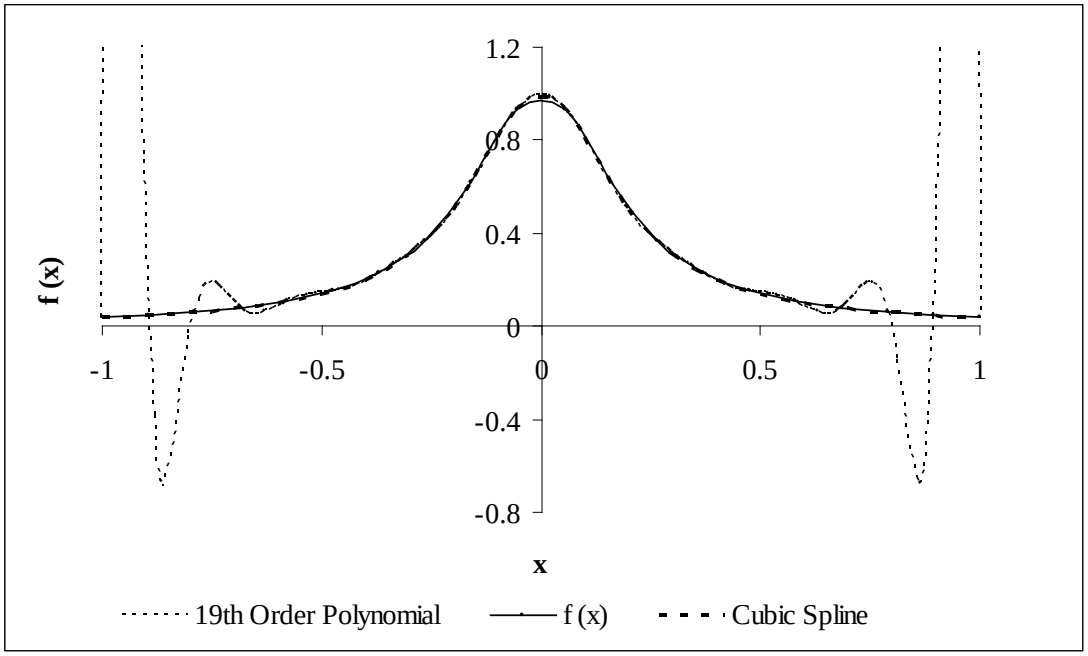

Figure 2 Approximating Runge's function by a 19th order polynomial and a cubic spline.