Linearization of system with one variable :

1. Turbulent flow
2. Chemical reaction at nth order $\neq 1$
3. Radiation heat transfer

   Consider the following nonlinear differential equation , modeling a given process :$\frac{dx}{dt} = f(x)$

x_o :a known value operating point expand the nonlinear function $f(\mathrm{x})$ into a Taylor series around the point x_o and take :

$$f(\mathrm{x}) = f(x_o) + \left(\frac{df}{dx}\right)_{x_o} \frac{x - x_o}{1!} + \left(\frac{d^2 f}{dx^2}\right)_{x_o} \frac{{(x - x_o)}^2}{2!} + \cdots + \left(\frac{d^n f}{dx_n}\right)_{x_o} \frac{{(x - x_o)}^n}{n!}$$

If we neglect all terms of order two and higher , we take the following approximation for the value of $f(\mathrm{x})$:

$$f(\mathrm{x}) = f(x_o) + \left(\frac{df}{dx}\right)_{x_o} (x - x_o)$$

<u>Example:</u>

Linearized the following function:

$f(H) = Q = k\sqrt{H}$ around H°

solution

$$f(H) = Q = k\sqrt{H^o} + \frac{k}{2\sqrt{H^2}}(H - H_o)$$

Linearization of system with many variable :

Consider the following dynamic system :

$$\frac{dx_1}{dt} = f_1(x, x_o), \frac{dx_2}{dt} = f_2(x_1, x_2)$$

Expand the nonlinear function $f_1(x, x_o) and f_2(x_1, x_2)$ into Taylor series a round the point (x_1^o, x_2^o) and take :

$$f_1(x_1,x_2)=f_1(x_1^o,x_2^o)+\left[\frac{\partial f_1}{\partial x_1}\right]_{(x_1^o,x_2^o)}\left(x_1-x_1^o\right)+\left[\frac{\partial f_1}{\partial x_2}\right]_{(x_1^o,x_2^o)}\left(x_2-x_2^o\right)$$
$$+\left[\frac{\partial^2 f_1}{\partial x_1}\right]_{(x_1^o,x_2^o)}\frac{\left(x_1-x_1^o\right)^2}{2!}+\left[\frac{\partial^2 f_1}{\partial {x_2}^2}\right]_{(x_{1,0},x_2^o)}\frac{\left(x_2-x_2^o\right)^2}{2!}$$
$$+\left[\frac{\partial^2 f_1}{\partial x_1\partial x_2}\right]_{(x_1^o,x_2^o)}\left(x_1-x_1^o\right)\left(x_2-x_2^o\right)+\cdots$$

$$f_2(x_1,x_2)=\cdots$$

neglect terms of order two and higher , we take the following approximation:

$$f_1(x_1,x_2)\approx f_1(x_1^o,x_2^o)+\left[\frac{\partial f_1}{\partial x_1}\right]_{(x_1^o,x_2^o)}\left(x_1-x_1^o\right)+\left[\frac{\partial f_1}{\partial x_2}\right]_{(x_1^o,x_2^o)}\left(x_2-x_2^o\right)$$

$$\frac{dx_1}{dt}=f_1(x_1^o,x_2^o)+\left[\frac{\partial f_1}{\partial x_1}\right]_{(x_1^o,x_2^o)}\left(x_1-x_1^o\right)+\left[\frac{\partial f_1}{\partial x_2}\right]_{(x_{1,0},x_2^o)}\left(x_2-x_2^o\right)$$

<u>Example:</u>

$f_1(x_1)=k\left(x_1+x_2\right)\ linear$

$f(x_1,x_2)=k\left(x_1^2+x_2\right)\ nonlinear$

$f(x_1,x_2)=k\sqrt{x_1+x_2}$ nonlinear

$=kx_1x_2 nonlinear$

<u>Example:</u>

Linearized the following function

$$f(x_1,x_2)=k\sqrt{x_1+x_2}$$

Nears $x_1^o\ and\ x_2^o$

$$f(x_1,x_2)=k\sqrt{x_1^o+x_2^o}+\frac{k}{2\sqrt{x_1^o+x_2^o}}\left(x_1-x_2^o\right)+\frac{k}{2\sqrt{x_1^o+x_2^o}}\left(x_2-x_2^o\right)$$