3-Dynamics of liquid level system

Consider the system which consists of a tank of uniform cross –sectional area A to which is attached a flow resistance R such as a value ,a pipe ,or a weir.

Case 1 The volumetric flowrate "Q" through the resistance is related to the head "H" by linear relationship (R is linear resistance). The density of liquid is constant. Find the transfer function which relates the head to inlet flow.

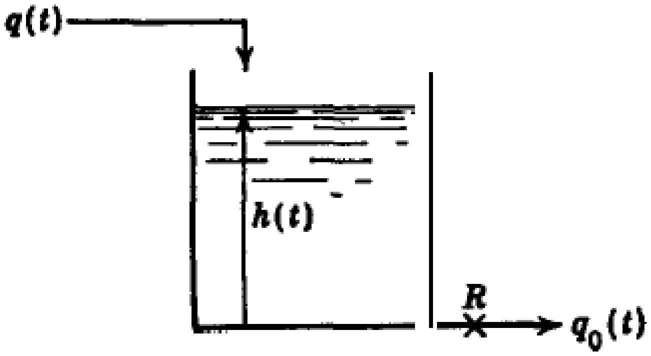

Fig.15 Liquid level system

Mass balance

Mass rate In – Mass rate Out= Mass accumulation in the tank

Steady state mass balance:

$$\rho Q_i^o - \rho Q_o^o = \rho A \frac{dH^o}{dt} = 0$$

$$\rho Q_i^o - \rho \frac{H^o}{R} = \rho A \frac{dH^o}{dt} = 0 \qquad (1)$$

Unsteady state mass balance:

$$\rho Q_i^{'}(t) - \rho \frac{H^{'}(t)}{R} = \rho A \frac{dH^{'}(t)}{dt} \qquad (2)$$

subtracting Eq. 1 from Eq.2:

$$Q_i(t) - \frac{H(t)}{R} = A \frac{dH(t)}{dt}$$

$$RA\frac{dH(t)}{dt} + H(t) = RQ_i(t)$$

$$\tau\frac{dH(t)}{dt}H(t) = KQ_i(t)$$

Linear first order differential equation and zero initial conditions then taking Laplace transform:

$$\tau sH(s) + H(s) = KQ_i(s)$$

$$G(s) = \frac{H(s)}{Q_i(s)} = \frac{K}{\tau s + 1}$$

Where τ=RA and K=R

At step change in $Q_i(t)$ of Q

$$H(t) = RQ_i\left(1 - e^{-t/\tau}\right)$$

<u>Case 2</u>

R is nonlinear and follows the square root relationship

$$Q_o = \frac{\sqrt{H}}{R} = k\sqrt{H} \;\; , k = \frac{1}{R}, \;\; \frac{m^3}{sec} = \frac{m^{0.5}}{R} \rightarrow R = \frac{sec}{m^{2.5}}$$

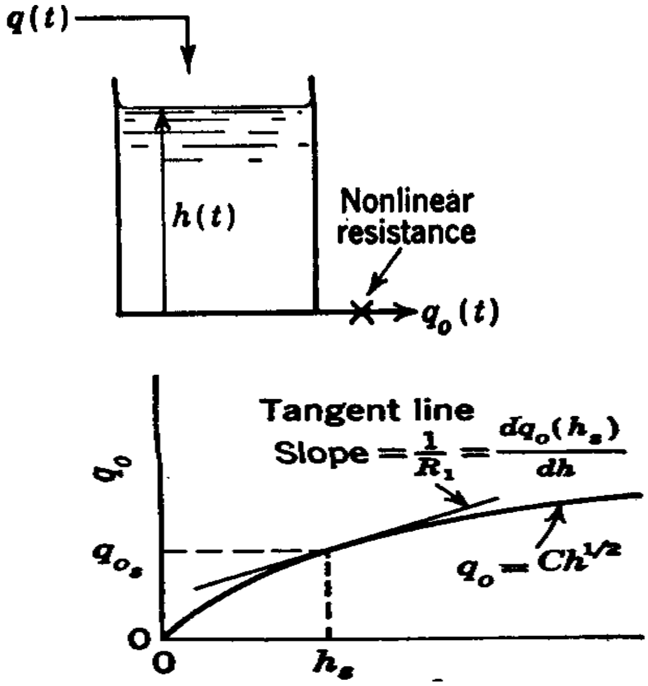

Fig.16 Liquid level system with nonlinear resistance

Steady state mass balance:

$$\rho Q_i^o - \rho Q_o^o = \rho A \frac{dH^o}{dt} = 0$$

$$\rho Q_i^o - \rho \frac{\sqrt{H^o}}{R} = \rho A \frac{dH^o}{dt} = 0 \qquad (1)$$

Transient mass balance :

$$\rho Q_i^{'}(t) - \rho \frac{\sqrt[2]{H^{'}(t)}}{R} = \rho A \frac{dH^{'}(t)}{dt} \qquad (2)$$

$$\rho Q_i^{'}(t) - \rho \left[\frac{\sqrt{H^o}}{R} + \frac{1}{2R\sqrt{H^o}}(H^{'} - H^o)\right] = \rho A \frac{dH^{'}(t)}{dt} \qquad (3)$$

subtracting Eq. 1 from Eq.3:

$$Q_i(t) - \frac{1}{2R\sqrt{H^o}}(H - H^o) = A\frac{dH(t)}{dt}$$

$$Q_i(t) - \frac{1}{2R\sqrt{H^o}}H(t) = A\frac{dH(t)}{dt}$$

$$2R\sqrt{H^o}A\frac{dH(t)}{dt} + H(t) = 2R\sqrt{H^o}Q_i(t)$$

$$\tau\frac{dH(t)}{dt}H(t) = KQ_i(t)$$

Where $\tau = 2R\sqrt{H^o}A$

$$k = 2R\sqrt{H^o}$$

$$k = \frac{sec}{m^{2.5}} \; m^{.5} = \frac{sec}{m^2}$$

1st order D.E. ,linear zero I.C. then taking Laplace transform

$$\tau s H(s) + H(s) = KQ_i(s)$$

$$Q(s) = \frac{H(s)}{Q_i(s)} = \frac{K}{\tau s + 1}$$