**4-Resistance and capacities**

a-**Resistance element (proportional element)**

**1.Laminar flow:**

$$Q = \left(\frac{\Delta p}{L}\right)\frac{\pi D^4}{128\mu}$$

$D: diameter\ of\ pipe\ (m)$

$L: lenght\ of\ pipe\ (m)$

$\mu: viscosity\ of\ the\ liquid\ (kg/m.sec)$

$\Delta p: difference\ of\ pressane\ between\ the\ ends\ of\ pipe\ (kg/sec^2m)$

$Q: volumetric\ rate\ (m^3/\text{sec})$

$$\Delta p = \rho H g$$

$$Q = H\left(\frac{\rho g \pi D^4}{128 L \mu}\right)$$

$$flowrate = \frac{driving\ force\ (pressure)}{resistance}$$

$$R = \frac{128 L\mu}{\rho g \pi D^4}\left(\frac{\text{sec}}{m^2}\right)$$

$$Q = \frac{H}{R}$$

$$G(s) = \frac{Q(s)}{H(s)} = \frac{1}{R} = K$$

**2.Turbulent flow:**

$$Q = C_D A\sqrt{2gH}$$

$$Q^2 = (C_D A)^2 2gH$$

$$2Q\frac{dQ}{dH} = (C_D A)^2 2g$$

$$\frac{dH}{dQ} = \frac{Q}{(C_D A)^2 g}$$

$$R = \frac{Q}{(C_D A)^2 g} \qquad R \neq constant\ , R = f(Q)\ variable\ resistance$$

1. Laminar flow:

$$Q = \frac{H}{R}$$

2. Turbulent flow:

$$Q = \frac{\sqrt{H}}{R}\ , generally\ Q = \frac{H^n}{R}\ \ n = 0.5 _{\rightarrow} 1$$

**<u>b. Capacitance element</u>**

If the flows of liquid into tank is independent of the level in the tank.

<u>Assumptions</u>

1- The flow is independent of the liquid level
2- No flow resistance
3- At t<0 the tank is empty
4- At t=0 open the valve and the level of liquid increase

Mass balance :

Mass flow rate in –mass flow out =rate of accumulation in the tank

Steady state mass balance :

$$\rho Q_i^o - 0 = \rho A \frac{dH^o}{dt} = 0 \qquad (1)$$

Transient state mass balance :

$$\rho Q_i'(t) - 0 = \rho A \frac{dH'(t)}{dt} \qquad (2)$$

subtracting Eq. 1 from Eq.2:

$$Q(t) = A\frac{dH(t)}{dt}$$

Laplace transform:

$$G(s) = \frac{Q(s)}{H(s)} = \frac{1}{As}$$

A: cross section area =vessel capacitance

$$H(t) = \mathcal{L}^{-1}\frac{Q}{s}.\frac{1}{As}$$

$$H(t) = \frac{Q}{A}.t$$