Capacity and Capacitance:

1. Liquid Capacitance

$$Q = A\frac{dH}{dt}$$

$$AdH = Qdt = dv$$

$$A = \frac{v}{H} \qquad , capacity = \frac{capacity}{hieght}$$

$$C_E\frac{dv(t)}{dt} = I(t), \qquad C_E dv = Idt = dq, \qquad C_E = \frac{q}{v}$$

Discharge of Capacitance through resistance :

Mass flow rate in –mass flow out =rate of accumulation

$$0 - \rho\frac{H'(t)}{R} = \rho A\frac{dH'(t)}{dt} \qquad (1)$$

$$RA\frac{dH'(t)}{dt} + H'(t) = 0 \qquad (2)$$

$$\tau\frac{dH'(t)}{dt} + H'(t) = 0$$

Laplace transform of unsteady state

$$\tau sH'(s) - \tau H^o + H'(s) = 0$$

$$H(s) = \frac{H^o}{s + \frac{1}{\tau}}$$

$$H'(t) = H^o e^{-t/\tau}$$

t=0 $\quad H(t) = H^o$ initial level

t→0 $\quad H(t) = 0$ empty at t=t_D

Discharge of Capacitance through the positive displacement pump

t<0 :the pump is turn off t=0

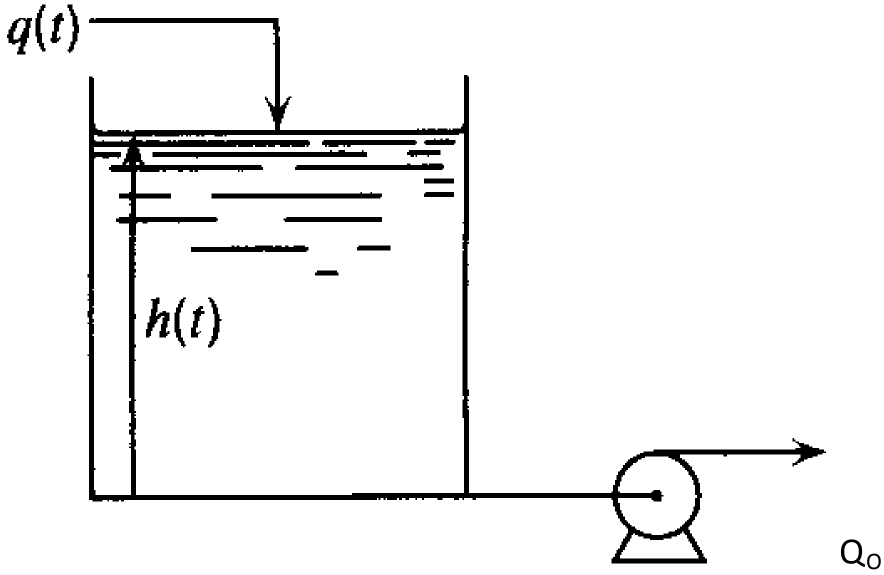

Fig.17 Discharge of Capacitance through the positive displacement pump

unsteady state mass balance:

$$0-\rho Q=\rho A\frac{dH(t)}{dt}$$

$$A\frac{dH(t)}{dt}=Q$$

$$AsH^{'}(s)-AH^{o}=\frac{-Q}{s}$$

$$AH^{'}(o)=H^{o}$$

$$H(s)=\frac{H^{o}}{s}-\frac{Q}{A}\frac{1}{s^{2}}$$

$$H(t)=H^{o}-\frac{Q}{A}\ t$$

t=0 $H(t)=H^{o}$

t=t_D empty $H(t)=0$

$$0 = H^o - \frac{Q^o}{A} t_D$$

$$t_D = \frac{AH^o}{Q^o}$$

Thermal Capacitance (energy storge):

Steady state heat balance:

$$q^o - 0 = MC_p \frac{d\theta^o}{dt} = 0 \qquad (1)$$

Unsteady state heat balance

$$q^{'}(t) - 0 = MC_p \frac{d\theta^{'}(t)}{dt} = 0 \qquad (2)$$

Subtract eq.(1) from(2)

$$q(t) - 0 = MC_p \frac{d\theta(t)}{dt}$$

$$q(t) = MC_p \frac{d\theta(t)}{dt}$$

Laplace transform:

$$q(s) = MC_p s\theta(s) \rightarrow G(s) = \frac{\theta(s)}{q(s)} = \frac{1}{MC_p s}$$

$C = MC_p$ *thermal capacitance*

$$\theta(t) = \mathfrak{L}^{1} \frac{q}{s} . \frac{1}{MC_p s} = \frac{q}{MC_p} t$$