5-Mercury in glass thermometer :

We shall develop the transfer function for a first order system by considering the unsteady state behavior of an ordinary mercury in glass thermometer . A cross sectional view of the bulb is shown in the figure.

1. It is assumed that the thermometer is(initially at steady state)

At t<0 $\theta_i^o = \theta_o^o = \theta_R$ room temperature

2. At t=0 the thermometer will be subjected to some change in the surrounding temperature $\theta_i(t)$

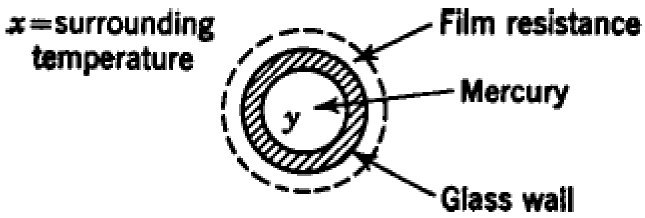

Fig.18 Cross -sectional view of thermometer

The following assumption will be used in this analysis:

1. All the resistance to heat transfer resides in the film surrounding the bulb (i.e., the resistance offered by the glass and mercury are neglected).
2. All the thermal capacity is in the mercury . furtherer , at any instant the mercury assume a uniform temperature throughout.
3. The glass will containing the mercury does not expand or contract during the transient response .

By applying the unsteady state energy balance on the mercury:

rate of heat input –rate of heat output = accumulation of heat in mercury .

$$hA\left(\theta_i'(t) - \theta_o'(t)\right) - 0 = MC_p \frac{d\theta_o'(t)}{dt} \qquad (1)$$

For the steady state condition eq. (1) may by written

$$hA(\theta_i^o - \theta_o^o) - 0 = MC_p \frac{d\theta_o^o}{dt} = 0$$

A:surface area of bulb for heat transfer m^2

C_p :heat capacity of mercury $(kJ/kg\,^oC)$

t :time (second)

M: mass of mercury in bulb ,(kg)

h : heat transfer coefficient of film $(kJ/sec.m^2.C^\circ)$

$$hA[(\theta_i'(t)-\theta_R)-(\theta_o'(t)-\theta_R)] = MC_p\frac{d(\theta_o'(t)-\theta_R)}{dt}$$

$$\theta_i'(t)-\theta_i' = \theta_i(t)$$

$$\theta_o'(t)-\theta_o^o = \theta_o(t)$$

$$hA\big(\theta_i(t)-\theta_o(t)\big) = MC_p\frac{d\theta_o(t)}{dt} \qquad (3)$$

$$MC_p\frac{d\theta_o(t)}{dt} + hA\theta_o(t) = hA\theta_i(t)$$

$$\frac{MC_p}{hA}\frac{d\theta_o(t)}{dt} + \theta_o(t) = \theta_i(t)$$

$$\tau\frac{d\theta_o(t)}{dt} + \theta_o(t) = \theta_i(t) \qquad (4)$$

$$\tau = \frac{MC_p}{hA}$$

Taking Laplace transform of eq.(4)

$$(\tau s+1)\theta_o(s) = \theta_i(s)$$

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{1}{\tau s+1}$$

<u>Response of thermometer to step change in temperature of surrounding :</u>

$$\theta_i(t) = \theta$$

$$\theta_i(s) = \frac{\theta}{s}$$

$$\theta_o(s) \ = \mathcal{L}^1 \frac{1}{\tau s+1}\frac{\theta}{s}$$

$$\theta_i(t) = \theta\left(1 - e^{-t/\tau}\right) \qquad (5)$$

t=0 $\quad \theta_o(t) = 0$

t→ ∞ $\quad \theta_o(t) = \theta_i$