6-Dynamics of isothermal condition stirred tank reaction(CSTR):

It is important example in chemical eng. , continues stirred tank reactor :

1. 1[st] order reaction
2. Reaction is carried out isothermally $k \neq f(temp.)$

Rate of reaction under isothermal condition r=f(con.)

$r_A = kC_A$

1. Steady state mass balance on reactant A

$$QC_i^o - QC_o^o - VkC_o^o = V\frac{dC_o^o}{dt} = 0 \qquad (1)$$

Assuming at time t=0 , C_i is changed

2. Unsteady state mass balance on reactant A :

$$QC_i'(t) - QC_o'(t) - VkC_o'(t) = V\frac{dC_o'(t)}{dt} = 0 \qquad (2)$$

Subtracting eq.(1)from(2)

$$Q(C_i'(t) - C_i^o) - Q(C_o'(t) - C_o^o) - Vk(C_o'(t) - C_o^o) = V\frac{d(C_o'(t) - C_o^o)}{dt}$$

$$QC_i(t) - QC_o(t) - VkC_o(t) = V\frac{dC_o(t)}{dt} \qquad (3)$$

$$V\frac{dC_o(t)}{dt} + (Q + kV)C_o(t) = QC_i(t)$$

$$\left(\frac{V}{Q + kV}\right)\frac{dC_o(t)}{dt} + C_o(t) = \frac{Q}{Q + kV}C_i(t)$$

$$(\tau s + 1)C_o(s) = kC_i(s)$$

$$G(s) = \frac{k}{Qs + 1} \qquad \tau = \frac{V}{Q + kV} \qquad k = \frac{Q}{Q + kV}$$

<u>Response of 1st order system to an impulse function:</u>

$$G(s) = \frac{y(s)}{f(s)} = \frac{k}{\tau s + 1}$$

$$f(t) = w \quad f(s) = C$$

Response $y(t) = \mathcal{L}^{1} C \frac{k}{\tau s+1} = \frac{Ck}{\tau} \mathcal{L}^{1} \frac{1}{s+\frac{1}{\tau}} = \frac{Ck}{\tau} e^{-t/\tau}$

$$y(t) = \alpha e^{-t/\tau}$$

t=0 $\quad y(t) = \alpha$

t→∞ $\quad y(t) = 0$

at t=τ $\quad y(t) = \alpha e^{-1} = 0.37\alpha$

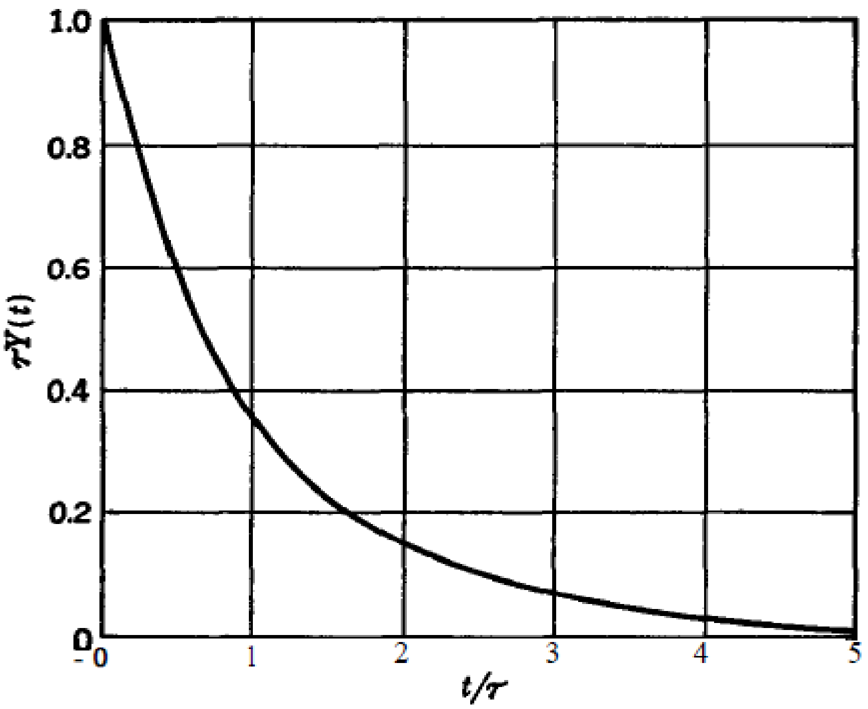

Fig.19 Response of first order system to impulse input