Response of 1st order system to rectangular pulse:

$$G(s) = \frac{y(s)}{f(s)} = \frac{k}{\tau s + 1}$$

$$f(s) = \frac{b}{s} - \frac{b}{s} e^{-as}$$

$$y(t) = \mathfrak{L}^{1} \left(\frac{k}{\tau s + 1}\right)\left(\frac{b}{s} - \frac{b}{s} e^{-as}\right)$$

t=0 y(t)=0

0<t<a $y(t) = kb\left(1 - e^{-t/\tau}\right)$

t=a $y(t) = kb\left(1 - e^{-a/\tau}\right)$

a<t<∞ $y(t) = kb\left(1 - e^{-t/\tau}\right) - kb\left(1 - e^{-(t-a)/\tau}\right)$

$$= kb\left(e^{-a/\tau} - 1\right)e^{-t/\tau} = \alpha e^{-t/\tau} \quad at \quad t = a$$

Response of 1st order system to ramp function:

$$G(s) = \frac{k}{\tau s + 1}$$

$$y(t) = mt \qquad , f(s) = \frac{m}{s^2}$$

$$y(t) = \mathfrak{L}^{1} \left(\frac{k}{\tau s + 1}\right)\left(\frac{m}{s^2}\right)$$

$$y(t) = km\left(t - \tau + \tau e^{-\tau/\tau}\right)$$

t=0 y(t)=0

t→∞ $y(t) = km(t - \tau)$

$G(s) = \frac{k}{\tau s+1}$

$$y(t) = km\left(t - \tau + \tau e^{-\tau/\tau}\right)$$

$t \to \infty \;\; y(t) = km(t - \tau)$

Response of 1st order system to sinusoidal input:

$$G(s) = \frac{k}{\tau s + 1}$$

$$f(s) = \frac{a\omega}{s^2 + \omega^2}$$

$$y(t) = \mathcal{L}^{-1}\left(\frac{k}{\tau s + 1}\right)\left(\frac{a\omega}{s^2 + \omega^2}\right)$$

$$y(t) = \frac{ka\omega\tau}{\tau^2\omega^2 + 1} e^{-\tau/\tau} + \frac{ka}{\tau^2\omega^2 + 1}\sin\omega\tau - \frac{ka\omega\tau}{\tau^2\omega^2 + 1}\cos\omega\tau$$

Final response $y(t) = \frac{ka}{\tau^2\omega^2+1}\sin\omega\tau - \frac{ka\omega\tau}{\tau^2\omega^2+1}\cos\omega\tau$

$q = \frac{ka}{\tau^2\omega^2+1}$,$p = \frac{-ka\omega\tau}{\tau^2\omega^2+1}$,$\varphi = \tan^{-1} -\omega\tau$

$y(t) = \frac{ka}{\sqrt{\tau^2\omega^2+1}}\sin(\omega\tau - \varphi)$ frequency steady state response

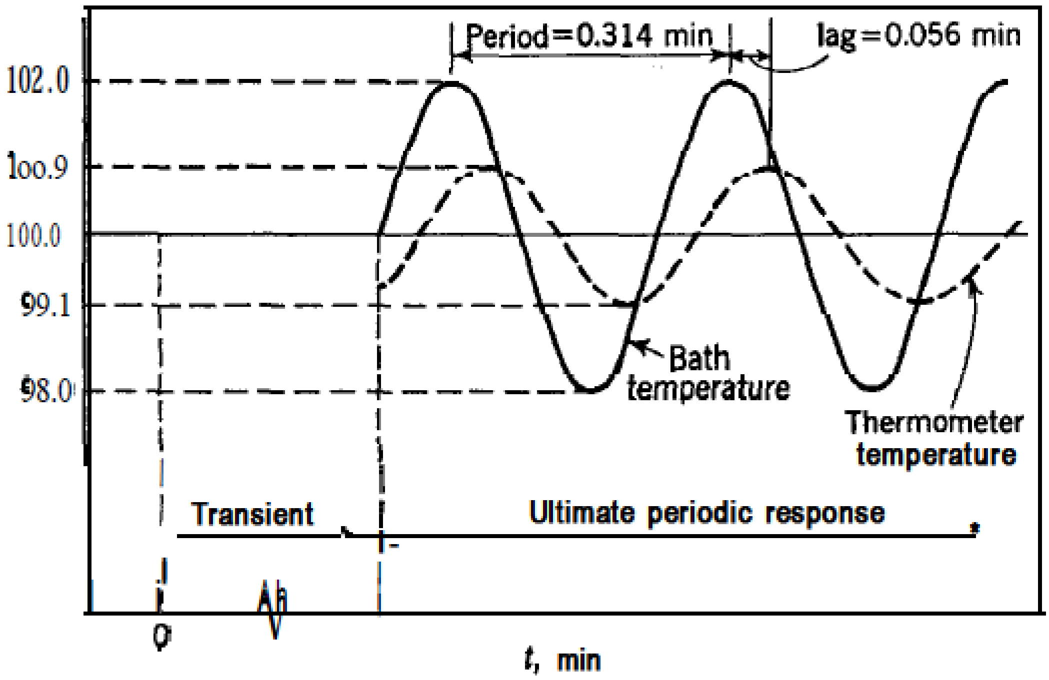

Fig.20 Response of thermometer to sinusoidal input