

## Time delay system

### 1. Pure Time delay system

Time delay : there is a time interval (short or long) during which no effect is observed on the outputs (response) of the system when an input variable of a system changed. This time interval is called time delay , or transportation lag or pure delay , or distance velocity lag. The delay time known as transportation time is basically the time required for a material to move a specific distant. However, time delay is an inherent property of any chemical process.

Sources:

- The use of chromatography to measure composition of liquids or gases is another source of dead time.
- Flow in a long pipe.
- Time lag produced from staged processes.

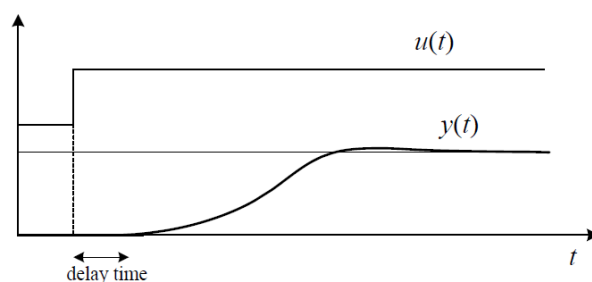


Fig.21 The response of time delay system to step change

$$\tau_D = \frac{\text{volume of pipe}}{\text{volume flow rate}} = \frac{A \cdot L}{A \cdot u_{av}} = \frac{L}{u_{av}}$$

Initially Water pure is flow in the pipe

$$\text{Time delay } \tau_D = \frac{L}{u}$$

$$C_o(t) = C_i(t - \tau_D)$$

Laplace Transformation

$$C_o(s) = C_i(s)e^{-\tau_D s}$$

$$G(s) = \frac{C_o(s)}{C_i(s)} = e^{-\tau_D s} \quad \text{steady state gain } k = 1$$

**Conveyor belt :**

Assume change is flow rate of mass from hopper . what response in mass flow rate in the end of the belt .

$$\tau_D = \frac{L}{u}, \quad G(s) = \frac{m_2(s)}{m_1(s)} = e^{-\tau_D s} = e^{-\frac{L}{u} s}$$

$$\text{At } t=0 \quad m_1 = m_1^0$$

$$\text{Step change} = 0.2m_1^0$$

$$m_2(s) = \frac{0.2m_1^0}{s} e^{-\frac{L}{u} s}$$

$$m_2(t) = 0.2m_1^0 U(t - \tau_D)$$

**Screw conveyer**

$$\tau_D = \frac{L}{u}$$

$$\theta = \frac{l}{R}, \omega = \frac{\theta}{t}, \theta = \omega t$$

$$u = \frac{l}{t}, u = \frac{\theta R}{t}$$

$$\tau_D = \frac{L}{u} = \frac{L}{2\pi NR}$$

Transfer function of pure time delay :

$$G(s) = \frac{y(s)}{f(s)} = \frac{\mathcal{F}(t - \tau_D)}{\mathcal{F}(t)}$$

Expressing  $\mathcal{F}(t - \tau_D)$  in terms of Taylor series:

$$f(t - \tau_D) = f(t) + \frac{\tau_D f'(t)}{1!} + \frac{\tau_D^2 f''(t)}{2!} + \dots, \quad t = 0$$

$$\begin{aligned} \text{L.T} \quad \mathcal{F}(t - \tau_D) &= f(s) + \frac{\tau_D s}{1!} + \frac{\tau_D^2 s^2}{2!} + \frac{\tau_D^3 s^3}{3!} + \dots, \\ &= f(s) e^{-\tau_D s} \end{aligned}$$

$$G(s) = \frac{y(s)}{f(s)} = \frac{f(s)e^{-\tau_D s}}{f(s)} = e^{-\tau_D s}$$