2-modified delay system

Double pipe heat exchanger

heat balance over an element (dx) at any time:

$$mC_p\theta_x(t) - mC_p\theta_{x+\Delta x}(t) - UA\big(\theta_x(t) - \theta_a(t)\big) = MC_p\frac{\partial\theta_x(t)}{\partial t}$$

θ_a: temperature of the pipe or surrounding (C°)

M: mass of fluid in the element Δx

m: mass flow rate input to pipe (kg/sec)

U: heat transfer coefficient(w/m^2.C°)

C_p: specific heat of fluid (kJ/kg. C°)

A: surface area of the element of pipe (m^2)

$$\theta_{x+\Delta x} = \theta_x + \frac{\Delta x}{1!}\frac{\partial\theta_x}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2\theta_x}{\partial x^2} + \cdots , \qquad \text{Taylor series}$$

$$mC_p\theta_x(t) - mC_p\left(\theta_x(t) + \frac{\partial\theta_x(t)}{\partial x}\Delta x\right) - UA\big(\theta_x(t) - \theta_a(t)\big) = MC_p\frac{\partial\theta_x(t)}{\partial t}$$

$$MC_p\frac{\partial\theta_x(t)}{\partial t} + UA\theta_x(t) = UA\theta_a(t) - mC_p\frac{\partial\theta_x(t)}{\partial x}\Delta x$$

$$A = \pi D\Delta x \quad ,M = \left(\frac{\pi}{4}D^2\Delta x\right)\rho,\ \ m = \frac{\pi}{4}D^2U\rho$$

$$C_p\frac{D}{4}\rho\frac{\partial\theta_x(t)}{\partial t} + U\theta_x(t) = U\theta_a(t) - \frac{1}{4}DU\rho\frac{\partial\theta_x(t)}{\partial x}C_p$$

$$C_p\frac{D}{4U}\rho\frac{\partial\theta_x(t)}{\partial t} + \theta_x(t) = \theta_a(t) - \frac{1}{4}DC_p\rho\frac{\partial\theta_x(t)}{\partial x}$$

$$\tau = \frac{MC_p}{UA} = \frac{\rho DC_p}{4U}$$

$$\tau\frac{\partial\theta_x(t)}{\partial t} + \theta_x(t) = \theta_a(t) - \tau U\frac{\partial\theta_x(t)}{\partial x}$$

$$\tau s\theta_x(s) + \theta_x(s) = \theta_a(s) - \tau U \frac{d\theta_x(s)}{dx}$$

$$\int_{\theta_i(s)}^{\theta_\circ(s)} \frac{d\theta_x(s)}{(\tau s+1)\theta_x(s) - \theta_a(s)} = \int_{x=0}^{x=l} -\frac{dx}{\tau U}$$

$$\ln \frac{(\tau s+1)\theta_o(s) - \theta_a(s)}{(\tau s+1)\theta_i(s) - \theta_a(s)} = \frac{-L}{\tau u}(\tau s+1) = \frac{-\tau_D \tau}{\tau} s - \frac{L}{\tau u}$$

$$\frac{(\tau s+1)\theta_o(s) - \theta_a(s)}{(\tau s+1)\theta_i(s) - \theta_a(s)} = e^{-\tau_D s} . e^{-\frac{L}{\tau u}} = k e^{-\tau_D s} \qquad (1)$$

$$e^{-\frac{L}{\tau u}} = k$$

If θ_a is constant all times $\theta_a(s) = 0$

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = k e^{-\tau_D s} \qquad (2)$$

$$\frac{\theta_o(s)}{\theta_a(s)} = \frac{1}{(\tau s+1)}\left(1 - e^{-\tau_D s} . e^{-\frac{\tau_D}{\tau}}\right)$$