Pade approximation :

Simplest approach to approximate a time delay by rational function is to choose the quotient of two polynomials specifically designed to match the terms of truncated Taylor series expansion of e^{-as}

$$e^{-as} = 1 - as + \frac{a^2s^2}{2!} - \frac{a^3s^3}{3!} + \cdots$$

$$e^{-as} = \frac{e^{-\frac{a}{2}s}}{e^{\frac{a}{2}s}} = G_1(s) = \frac{1 - \frac{a}{2}s}{1 + \frac{a}{2}s}$$

$$e^{-as} = G_2(s) = \frac{a^2s^2 - 6as + 12}{a^2s^2 + 6as + 12}$$

Time delay with sinusoidal input :

$\varphi = f(\omega)$

$\omega=0 \quad \varphi=0$

$\omega \to \infty \quad \varphi \to -\infty$

$\varphi = -\omega\tau_D$

AR=1

$y(t) = kasin\omega(t - \tau_D)$

$\varphi = -\omega\tau_D$

AR$=\frac{ka}{a} = k$

First order system in series

$$G(s) = \frac{y(s)}{f(s)}$$

$$= \frac{y_1(s)}{f(s)} \times \frac{y_2(s)}{y_1(s)}$$

$$= G_1(s) \times G_2(s)$$

$$G(s) = \frac{k_1}{(\tau_1 s + 1)} \frac{k_2}{(\tau_2 s + 1)}.$$

$$G(s) = \frac{k}{\tau_1 \tau_2 s^2 + (\tau_1 + \tau_2) s + 1} \qquad (1)$$

$$G(s) = \frac{k}{\frac{1}{\omega_n^2} s^2 + \frac{2\varphi}{\omega_n} s + 1} \qquad (2)$$

$$\omega_n = \frac{1}{\sqrt{\tau_1 \tau_2}} \qquad , \varphi = \frac{1}{2}\left(\tau_1 + \tau_2 \frac{1}{\sqrt{\tau_1 \tau_2}}\right)$$