**Laplace inverse**

$\mathcal{L}^{-1}[f(s)] = f(t)$

1) Expand the Q(s)/P(s) into a series of function,
$$f(s) = \frac{Q(s)}{P(s)} = \frac{C_1}{r_1(s)} + \frac{C_2}{r_1(s)} + \cdots + \frac{C_n}{r_n(s)} \qquad (1)$$
2) Compute the value of the constants $C_1$ ,$C_2$, . . . . .,$C_n$ from eq.(1)
3) Find the inverse Laplace transformation of every partial fraction . then the unknown function x(t) is given by :
$$f(t) = \mathcal{L}^{-1}\left[\frac{C_1}{r_1(s)}\right] + \left[\frac{C_2}{r_1(s)}\right] + \cdots + \mathcal{L}^{-1}\left[\frac{C_n}{r_n(s)}\right]$$

**Dynamics process:**

The dynamic behavior of the process is described by an nth order linear (or linearized nonlinear )differential equation .

In general a linear differential equation is written as follow s :

$$a_n \frac{d^n y}{dt^n} + a_{n-1} \frac{d^{n-1} y}{dt^{n-1}} + \ldots\ldots\ldots + a_1 \frac{dy}{dt} + a_0\, y = b_m \frac{d^m y}{dt^m} + \ldots\ldots\ldots + b_1 \frac{df}{dt} + b_0 f(t)$$

If differential equation is first order therefore the system is the first order.

If differential equation is second order therefore the system is the second order.

If differential equation is zero order therefore the system is the zero order.

The differential equation for the first order system is :

$$a_1 \frac{dy}{dt} + a_0\, y(t) = b_0 f(t)$$

For the second order system.

$$a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0\, y(t) = b_0 f(t)$$

For the zero order sys.

$$a_0\, y(t) = b_0 f(t)$$

Table 1: Lapalce transformation of the functions

| | $f(t)$ | $F(s)$ |
|---|---|---|
| 1. | $\delta(t)$ (unit impulse) | $1$ |
| 2. | $S(t)$ (unit step) | $\frac{1}{s}$ |
| 3. | $t$ (ramp) | $\frac{1}{s^2}$ |
| 4. | $t^{n-1}$ | $\frac{(n-1)!}{s^n}$ |
| 5. | $e^{-bt}$ | $\frac{1}{s+b}$ |
| 6. | $\frac{1}{\tau}e^{-t/\tau}$ | $\frac{1}{\tau s+1}$ |
| 7. | $\frac{t^{n-1}e^{-bt}}{(n-1)!}$ $(n>0)$ | $\frac{1}{(s+b)^n}$ |
| 8. | $\frac{1}{\tau^n(n-1)!}t^{n-1}e^{-t/\tau}$ | $\frac{1}{(\tau s+1)^n}$ |
| 9. | $\frac{1}{b_1-b_2}(e^{-b_2t}-e^{-b_1t})$ | $\frac{1}{(s+b_1)(s+b_2)}$ |
| 10. | $\frac{1}{\tau_1-\tau_2}(e^{-t/\tau_1}-e^{-t/\tau_2})$ | $\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$ |
| 11. | $\frac{b_3-b_1}{b_2-b_1}e^{-b_1t}+\frac{b_3-b_2}{b_1-b_2}e^{-b_2t}$ | $\frac{s+b_3}{(s+b_1)(s+b_2)}$ |
| 12. | $\frac{1}{\tau_1}\frac{\tau_1-\tau_3}{\tau_1-\tau_2}e^{-t/\tau_1}+\frac{1}{\tau_2}\frac{\tau_2-\tau_3}{\tau_2-\tau_1}e^{-t/\tau_2}$ | $\frac{\tau_3 s+1}{(\tau_1 s+1)(\tau_2 s+1)}$ |
| 13. | $1-e^{-t/\tau}$ | $\frac{1}{s(\tau s+1)}$ |
| 14. | $\sin\omega t$ | $\frac{\omega}{s^2+\omega^2}$ |
| 15. | $\cos\omega t$ | $\frac{s}{s^2+\omega^2}$ |
| 16. | $\sin(\omega t+\phi)$ | $\frac{\omega\cos\phi+s\sin\phi}{s^2+\omega^2}$ |
| 17. | $e^{-bt}\sin\omega t$ ($b$, $\omega$ real) | $\frac{\omega}{(s+b)^2+\omega^2}$ |
| 18. | $e^{-bt}\cos\omega t$ ($b$, $\omega$ real) | $\frac{s+b}{(s+b)^2+\omega^2}$ |
| 19. | $\frac{1}{\tau\sqrt{1-\zeta^2}}e^{-\zeta t/\tau}\sin(\sqrt{1-\zeta^2}\,t/\tau)$ $(0\le\lvert\zeta\rvert<1)$ | $\frac{1}{\tau^2s^2+2\zeta\tau s+1}$ |
| 20. | $1+\frac{1}{\tau_2-\tau_1}(\tau_1e^{-t/\tau_1}-\tau_2e^{-t/\tau_2})$ $(\tau_1\neq\tau_2)$ | $\frac{1}{s(\tau_1 s+1)(\tau_2 s+1)}$ |

## Transfer function, G(s)

Transfer function is an algebraic expression for the dynamic relation between the input and the output of the process model both expression in s domain y(s),f(s) .

**Block diagrams**

Block diagram is a graphical representation of transfer functions and their interactions. Block diagram assist the engineer in determining the quantitative aspects of dynamic performance and in understanding the qualitative features of the system.

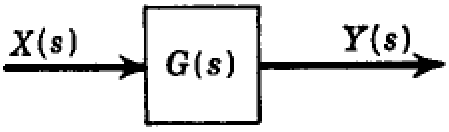


Fig.1 Block diagram

$$G(s) = \frac{Laplace\ transform\ of\ response\ function}{Laplace\ transform\ of\ input\ function}$$

$$G(s) = \frac{y(s)}{f(s)}$$