Noninteracting system

Assume:

1. The density of liquid is constant.
2. The tank to have uniform cross –section area.
3. The flow resistance is linear

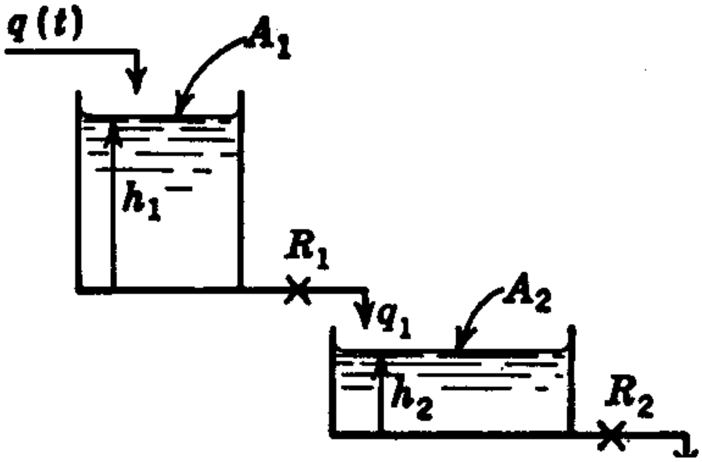

Fig.22 Noninteracting two –tank liquid level system

A mass balance on tank one gives:

$$Q\ (t) - Q_1(t) = A_1 \frac{dH_1(t)}{dt} \qquad (1)$$

A mass balance on tank two gives:

$$Q_1(t) - Q_2(t) = A_2 \frac{dH_2(t)}{dt} \qquad (2)$$

The flow head relationships for the two tanks are linear resistance and given by the expressions

$$Q_1(t) = \frac{H_1(t)}{R_1} \qquad (3)$$

$$Q_2(t) = \frac{H_2(t)}{R_2} \qquad (4)$$

Sub. Eq.(3) in eq.(4)

$$Q(t) - \frac{H_1(t)}{R_1} = A_1 \frac{dH_1(t)}{dt}$$

$$A_1 R_1 \frac{dH_1(t)}{dt} + H_1(t) = R_1 Q(t) \qquad (5)$$

$$(\tau_1 s + 1) H_1(s) = R_1 Q(s)$$

$$G(s) = \frac{H_1(s)}{Q(s)} = \frac{R_1}{(\tau_1 s + 1)} \qquad (6)$$

Sub. Eq. 3 in Eq. 6 to eliminate H_1(s) gives:

$$\frac{Q_1(s)}{Q(s)} = \frac{1}{(\tau_1 s + 1)} \qquad (7)$$

In the tank two:

$$Q_1(t) - \frac{H_2(t)}{R_2} = A_2 \frac{dH_2(t)}{dt}$$

$$A_2 R_2 \frac{dH_2(t)}{dt} + H_2(t) = R_2 Q_1(t)$$

$$(\tau_2 s + 1) H_2(s) = R_2 Q_1(s)$$

$$G_2(s) = \frac{H_2(s)}{Q_1(s)} = \frac{R_2}{(\tau_2 s + 1)} \qquad (8)$$

$$G(s) = \frac{Q_1(s)}{Q(s)} . \frac{H_2(s)}{Q_1(s)} = \frac{1}{(\tau_1 s + 1)} . \frac{R_2}{(\tau_2 s + 1)} \qquad (9)$$

<u>Tank one</u>

Eq(6) becomes:

$$H_1(s) = Q(s) \left(\frac{R_1}{(\tau_1 s + 1)} \right)$$

$$Q(s) = \frac{Q_i}{s}$$

$$H_1(t) = \mathfrak{L}^{1} \frac{Q_i}{s} \left(\frac{R_1}{(\tau_1 s + 1)} \right)$$

$$H_1(t) = R_1 Q_i \left(1 - e^{-\frac{t}{\tau_1}}\right)$$

t=0 $H_1(t) = 0$

$$\frac{dH_1(t)}{dt}\mid_{t=0} = \frac{R_1 Q_i}{\tau_1} \qquad max.$$

t→ ∞ $H_1(t) = R_1 Q_i \ final\ value$

$$\frac{dH_1(t)}{dt}\mid_{t=\infty} = 0 \quad steady\ state$$

<u>Tank two</u>

Eq.(9) becomes:

$$G(s) = \frac{H_2(s)}{Q(s)} . = \frac{R_2}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

Step change in Q(t)=Q_i

$$Q(s) = \frac{Q_i}{s}$$

$$H_2(s) = \frac{Q_i}{s} . \left(\frac{R_2}{(\tau_1 s + 1)(\tau_2 s + 1)}\right)$$

Using partial fraction

$$H_2(t) = R_2 Q_i \left(1 + \frac{\tau_2 e^{-\frac{t}{\tau_2}} - \tau_1 e^{-\frac{t}{\tau_1}}}{\tau_1 - \tau_2}\right) \qquad \tau_1 \neq \tau_2$$

t=0 $H_2(t) = 0$

$$\frac{dH_2(t)}{dt}\mid_{t=0} = 0$$

t→ ∞ $H_2(t) = R_2 Q_i$

$$\frac{dH_2(t)}{dt}\mid_{t=\infty} = 0$$