Interacting system

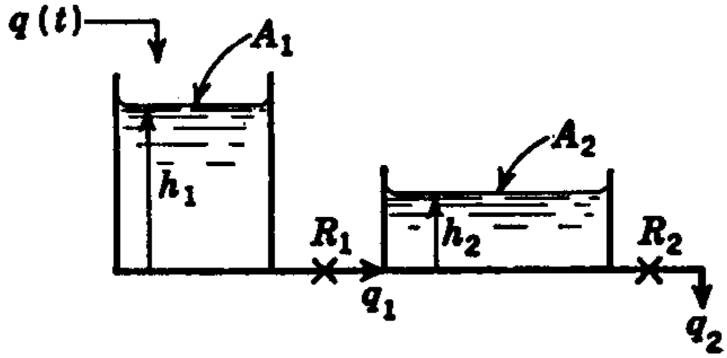

Fig.23 Interacting two –tank liquid level system

Tank one

mass balance around the tank one

Steady state balance

$$Q_i^o - Q_1^o = A_1 \frac{d{H_i}^o}{dt} = 0$$

assume the resistance of the value are linear

$$Q_i^o - \left(\frac{H_1^o - H_2^o}{R_1}\right) = A_1 \frac{d{H_1}^o}{dt} = 0 \qquad (1)$$

Transient (unsteady)state

$$Q_i^{'}(t) - \left(\frac{H_1^{'}(t) - H_2^{'}(t)}{R_1}\right) = A_1 \frac{d{H_1}^{'}(t)}{dt} \qquad (2)$$

Substrate Eq.(1) from (2):

$$(Q_i^{'}(t) - Q_i^o) - \left(\frac{H_1^{'}(t) - H_1^o}{R_1}\right) + \left(\frac{H_2^{'}(t) - H_2^o}{R_1}\right) = A_1 \frac{d({H_1}^{'}(t) - H_1^o)}{dt}$$

$$Q_i(t) - \frac{H_1(t)}{R_1} + \frac{H_2(t)}{R_1} = A_1 \frac{dH_1(t)}{dt}$$

$$R_1 A_1 \frac{dH_1(t)}{dt} + H_1(t) = H_2(t) + R_1 Q_i(t)$$

L.T.

$$(\tau_1 s + 1)H_1(s) = H_2(s) + R_1 Q_i(s) \qquad (2a)$$

$$\tau_1 = A_1 R_1$$

Tank two

mass balance Steady state balance

$$Q_1^o - Q_2^o = A_2 \frac{dH_2^o}{dt} = 0$$

$$\left(\frac{H_1^o - H_2^o}{R_1}\right) - \frac{H_2^o}{R_2} = A_2 \frac{dH_2{}^o}{dt} = 0 \qquad (3)$$

Transient (unsteady)state

$$\left(\frac{H_1^{'}(t) - H_2^{'}(t)}{R_1}\right) - \frac{H_2^{'}}{R_2} = A_2 \frac{dH_2{}^{'}(t)}{dt} \qquad (4)$$

Sub. Eq.(1) from (2):

$$\left(\frac{H_1^{'}(t) - H_1^o}{R_1}\right) - \left(\frac{H_2^{'}(t) - H_2^o}{R_1}\right) - \frac{H_2^{'}(t) - H_2^o}{R_2} = A_2 \frac{d(H_2{}^{'}(t) - H_2^o)}{dt}$$

$$\frac{H_1(t)}{R_1} - \frac{H_2(t)}{R_1} - \frac{H_2(t)}{R_2} = A_2 \frac{dH_2(t)}{dt}$$

$$R_2 A_2 \frac{dH_2(t)}{dt} + \left(\frac{R_2}{R_1} + 1\right) H_2(t) = \frac{R_2}{R_1} H_1(t)$$

$$\tau_2 \frac{dH_2(t)}{dt} + \left(\frac{R_2}{R_1} + 1\right) H_2(t) = \frac{R_2}{R_1} H_1(t) \qquad , where \quad \tau_2 = R_2 A_2$$

L.T.

$$\tau_2 sH_2(t) + \left(\frac{R_2}{R_1} + 1\right) H_2(s) = \frac{R_2}{R_1} H_1(s) \qquad (5)$$

By using eq.(2a) and (5) to eliminate $H_1(s)$

$$\tau_2 sH_2(s) + \left(\frac{R_2}{R_1} + 1\right) H_2(s) = \frac{R_2}{R_1}\left(\frac{H_2(s)}{\tau_1 s + 1} + \frac{R_1 Q_i(s)}{\tau_1 s + 1}\right)$$

$$G(s) = \frac{H_2(s)}{Q_i(s)} = \frac{R_2}{(\tau_1 s + 1)\left(\tau_2 s + \frac{R_2}{R_1} + 1 - \frac{R_2}{R_1(\tau_1 s + 1)}\right)} \qquad (6)$$

$$G(s) = \frac{R_2}{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2 + A_1R_2)s + 1}$$

$$G(s) = \frac{Q_2(s)}{Q_i(s)} = \frac{1}{\tau_1\tau_2 s^2 + (\tau_1 + \tau_2 + A_1R_2)s + 1} \qquad (7)$$

$$where\ \ Q_2(s) = \frac{H_2(s)}{R_2}$$

$$G(s) = \frac{Q_2(s)}{Q_i(s)} = \frac{1}{\tau^2 s^2 + 3\tau s + 1} \qquad (8)$$

Assume $A_2 = A_1$

$$\frac{Q_2(s)}{Q_i(s)} = \frac{1}{(0.38\tau s + 1)(2.62\tau s + 1)}$$

$$\frac{Q_2(s)}{Q_i(s)} = \frac{1}{(\tau_a s + 1)(\tau_b s + 1)} \qquad (9)$$

$$\tau_a = 0.38\tau, \tau_b = 2.62\tau$$

$$G(s) = \frac{1}{\tau_a\tau_b s^2 + (\tau_a + \tau_b)s + 1} \qquad (8)$$

$$\tau_a + \tau_b = \tau_1 + \tau_2 + A_1R_2$$

$$\tau_a\tau_b = \tau_1\tau_2$$

Step change in

$$Q_i(t) = Q_i$$

$$H_2(t) = R_2 Q_i \left(1 + \frac{\tau_b e^{-\frac{t}{\tau_b}} - \tau_a e^{-\frac{t}{\tau_a}}}{\tau_a - \tau_b} \right)$$

Where $R_2 = R_1$, $A_1 = A_2$, $\tau_1 = \tau_2$

At steady state $H_1 = H_2 = R_1 Q_i = R_2 Q_2$