Interaction of heat transfer lags

Heat balance on the bulb and using deviation

$$M\,C_p \frac{d\theta_3(t)}{dt} = UA\big(\theta_2(t) - \theta_3(t)\big)$$

$$C_2 = M\,C_p\ , UA = \frac{1}{R_2}$$

$$C_2 \frac{d\theta_3(t)}{dt} = \frac{\theta_2(t) - \theta_3(t)}{R_2} \qquad (1)$$

Heat balance around shield :

heat rate in –out=heat accumulation

$$U_s A_s\big(\theta_1(t) - \theta_2(t)\big) - UA\big(\theta_2(t) - \theta_3(t)\big) = M_s\,C_{ps} \frac{d\theta_2(t)}{dt}$$

$$U_s A_s = \frac{1}{R_1}\ \ , M_s\,C_{ps} = C_1\ \ , C_2 = M\,C_p$$

$$C_1 \frac{d\theta_2(t)}{dt} = \frac{\theta_1(t) - \theta_2(t)}{R_1} - \frac{\theta_2(t) - \theta_3(t)}{R_2} \qquad (2)$$

L.T of the eq.(1),(2)

$$C_2 s\theta_3(s) = \frac{\theta_2(s) - \theta_3(s)}{R_2} \qquad (3)$$

$$C_1 s\theta_2(s) = \frac{\theta_1(s) - \theta_2(s)}{R_1} - \frac{\theta_2(s) - \theta_3(s)}{R_2} \qquad (4)$$

$$G(s) = \frac{\theta_3(s)}{\theta_2(s)} = \frac{1}{\tau_a\tau_b s^2 + (\tau_a + \tau_b)s + 1}$$

Step change in

$$Q_1(t) = {}^{Q_i}/_{s}$$

$$\theta_3(t) = \theta_i \left(1 + \frac{\tau_b e^{-\frac{t}{\tau_b}} - \tau_a e^{-\frac{t}{\tau_a}}}{\tau_a - \tau_b} \right)$$

Second order system

A 2nd order system is one whose output , y(t) ,is described by the solution of linear . a 2nd order differential eq. .for example ,the following eq. describes a 2nd order linear system:

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y(t) = bf(t) \quad if\ a_0 \neq 0$$

$$\tau^2 \frac{d^2y}{dt^2} + 2\zeta\tau \frac{dy}{dt} + a_0 y(t) = kf(t) \qquad (1)$$

Where $\tau^2 = {}^{a_2}/_{a_0}$, $2\zeta\tau = {}^{a_1}/_{a_0}$ $and\ k = {}^{b}/_{a_0}$

τ: natural period of oscillation of the system

$\tau = {}^{1}/_{\omega_n}$, $\omega_n = 2\pi f_n$

ω_n : natural freq. rad/s

f_n : cyclical freq. (Hz)

ζ: damping factor

k : steady state ,or static ,or simply gain of the system

$$G(s) = \frac{y(s)}{f(s)} = \frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1}$$