Second order system

Sources of second order dynamics in the chemical industries come from a series of first-order systems, or a processing system with a controller.

For input step change of magnitude of *a*, *u*(*s*) = *a/s* we have:

$$y(s)=\frac{k_p a}{s(\tau^2 s^2+2\tau\zeta s+1)}\equiv\frac{k_p a/\tau^2}{s(s-p_1)(s-p_2)}$$

where $p_1=-\frac{\zeta}{\tau}+\frac{\sqrt{\zeta^2-1}}{\tau}$ $\qquad p_1=-\frac{\zeta}{\tau}-\frac{\sqrt{\zeta^2-1}}{\tau}$

Case A: (over-damped) when $\zeta > 1$, we have two distinct and real poles

$$y(t)=k_p a\left[1-e^{-\zeta t/\tau}\left(\cosh(\sqrt{\zeta^2-1}\,\frac{t}{\tau})+\frac{\zeta}{\sqrt{\zeta^2-1}}\sinh(\sqrt{\zeta^2-1}\,\frac{t}{\tau})\right)\right]$$

Case B: (critically-damped) when ζ = 1, we have two equal poles

$$y(t)=k_p a\left[1-\left(1+\frac{t}{\tau}\right)e^{-t/\tau}\right]$$

Case C: (under-damped) when $\zeta < 1$, we have two complex conjugate poles

$$y(t)=k_p a\left[1-\frac{1}{\sqrt{\zeta^2-1}}e^{-t\zeta/\tau}\sin(wt+\phi)\right]$$

2^{nd} order system is oscillate depending on value of damping factor ζ:

If ζ>1 then the system is not oscillation

If ζ=1 then the system is critical damped

If ζ<1 then the system is oscillation with damping

If ζ=0 then the system is continuous oscillation

And if ζ is -ve then the system is diverged oscillation

If ζ is +ve then the system is stable

The dynamic response for all cases is shown in Figure below

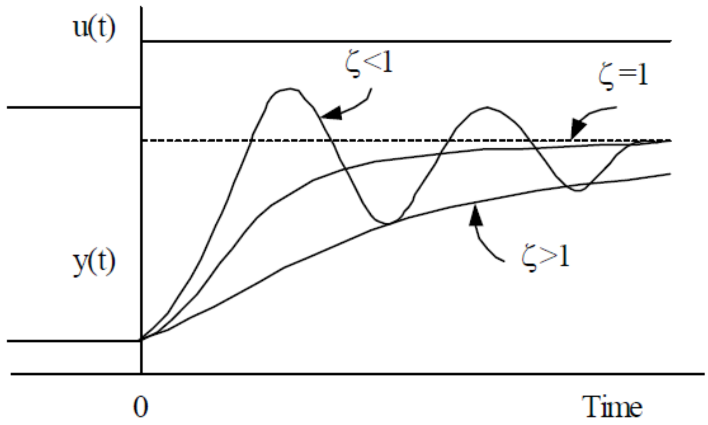

Fig.24 The response of second order system to step change

- The over-damped response is sluggish and resembles a little the response of a first-order system and becomes more sluggish with larger values of ζ.
- The critically damped response is faster than the over-damped one.
- The under-damped response is faster than the others, but oscillates. The oscillatory behavior becomes pronounced with smaller values ζ.

Typical 2nd order oscillatory system, simple manometer

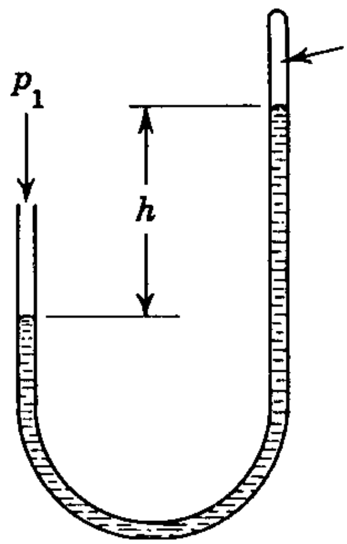

Fig.25 Manometer system

Manometer (momentum balance)

Balance of forces on the system=mass of system ×acceleration

Since acceleration =$\frac{d(velocity\ )}{dt}$

(Force due to pressure $p_1$on lag_1)–(force due to pressure $p_2$on lag_2) – (force due to liquid level difference in the two lags)-(force due to fluid friction) =(mass of liquid in the tube)×(acceleration)

$$p_1A_1 - p_2A_2 - (h)A_2\rho g - (\text{force due to fluid friction}) = m\frac{dv}{dt} \quad (1)$$

(force due to fluid friction)=$\Delta p\pi R^2 = A\frac{8\mu L}{R^2}\frac{dh}{dt}$ (2)

$$v = \frac{dh}{dt}\ and\ \frac{dv}{dt} = \frac{d^2h}{dt^2} \qquad (3)$$

Put eq.(2) and (3)in eq.(1)

$$\Delta pA - \rho gAh - \frac{8\mu LA}{R^2}\frac{dh}{dt} = \rho AL\frac{d^2h}{dt^2}$$

$$\left(\frac{L}{g}\right)\frac{d^2h}{dt^2} + \frac{8\mu L}{\rho gR^2}\frac{dh}{dt} + h = \frac{1}{\rho g}\Delta\rho$$

$$\tau^2 \frac{d^2h}{dt^2} + 2\zeta\tau\frac{dh}{dt} + h = k\rho$$

$$\frac{h(s)}{p(s)} = \frac{k}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$\tau^2 = \frac{L}{g}$ $\quad , \tau = \sqrt{\frac{L}{g}}$ $\quad ,2\zeta\tau = \frac{8\mu L}{\rho g R^2}$ $\quad , k = \frac{1}{\rho g}$

$$\zeta = \frac{4\mu}{\rho R^2}\sqrt{\frac{L}{g}}$$

If $\zeta \geq 1$

$\frac{4\mu}{\rho R^2}\sqrt{\frac{L}{g}} \geq 1$