Spring –damper system

1. The force exerted by the spring (toward the left)of –ky where k is a positive constant ,called Hooke's constant.
2. The viscous friction force (acting to the left)of –c dy/dt ,where c is positive constant called the damping coefficient.
3. The external force F(t) (acting toward the right)

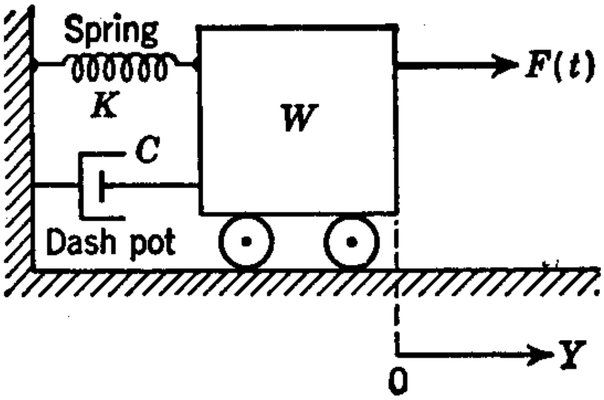

Fig.26 Spring –damper system

Force balance :

$F(t)-F_s-F_d=$ (mass ×acceleration)

$$F(t) - ky(t) - \frac{cdy(t)}{dt} = m\frac{d^2y}{dt^2}$$

$$m\frac{d^2y}{dt^2} + \frac{cdy\,(t)}{dt} + ky = F(t)$$

$$\frac{m}{k}\frac{d^2y}{dt^2} + \frac{c}{k}\frac{dy\,(t)}{dt} + y(t) = \frac{1}{k}F(t)$$

$$\tau = \sqrt{\frac{m}{k}}\ \ (\text{sec})\ \ ,\zeta = \frac{1}{2}\frac{c}{k}\sqrt{\frac{k}{m}} = \frac{c}{2}\sqrt{\frac{1}{mk}}$$

$$G(s) = \frac{y(s)}{F(s)} = \frac{1/k}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$\frac{y(s)}{x(s)} = \frac{1}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

$$s_1, s_2 = \frac{-2\zeta\tau \pm \sqrt{(2\zeta\tau)^2 - 4\tau^2}}{2\tau^2}$$

$$s_1, s_2 = \frac{-\zeta}{\tau} \pm \frac{\sqrt{\zeta^2 - 1}}{\tau}$$

Analysis of Second order system

$$\tau^2 = \frac{1}{\omega_n^2}$$

$$\omega_n = 2\pi f_n$$

ω_n : natural frequency rad/s

f_n : cyclical freq. (Hz)

ζ=damping factor

if a unit step in x(t) ,x(s)=$\frac{1}{s}$

$$y(s) = \frac{k}{s(\tau^2 s^2 + 2\zeta\tau s + 1)}$$

The characteristic eq. $\tau^2 s^2 + 2\zeta\tau s + 1 = 0$

$$s_1, s_2 = \frac{-2\zeta\tau \pm \sqrt{(2\zeta\tau)^2 - 4\tau^2}}{2\tau^2}$$

$$s_1, s_2 = \frac{-\zeta}{\tau} \pm \frac{\sqrt{\zeta^2 - 1}}{\tau}$$

1. If ζ >1 :unequal root ,-ve and real roots =-m_1,m_2

$$y(t) = e^{-m_1 t} + e^{-m_2 t}$$

The system is not oscillatory and over damped

2. If ζ =1 : equal root ,-ve and real roots =-m
   $y(t) = (\mathrm{A}t + \mathrm{B})e^{-tm}$

The system is critically damped response

3. 0<ζ<1 ,complex and -ve real part

$s_1 \text{and } s_2 = -\alpha \pm j\beta$

$y(t) = e^{-\alpha t} \sin \beta t$

The system is under damped response

4. ζ=0 ,pure imaginary

$s_1 \text{and } s_2 = \pm j\beta$

$y(t) = \sin \beta t$

The system is continuous oscillation

5. ζ<0 ,complex and +ve real part

$s_1 \text{and } s_2 = \alpha \pm j\beta$

$y(t) = e^{\alpha t} \sin \beta t$

The system is oscillation and unstable

In general for stable system the optimum of ζ=0.214