Case : for 0<ζ<1

$$G(s)=\frac{y(s)}{x(s)}=\frac{k}{\tau^2 s^2+2\zeta\tau s+1}$$

$$x(s)=\frac{A}{s}$$

$$y(s)=\frac{A}{s}=\frac{k}{\tau^2 s^2+2\zeta\tau s+1}$$

$$y(s)=\frac{k.A/\tau^2}{s\left(s^2+\frac{2\zeta}{\tau}s+\frac{1}{\tau^2}\right)}$$

Solving by convolution integral:

$$g(s)=\frac{k.A/\tau^2}{s}\rightarrow g(t)=k.A/\tau^2$$

$$h(s)=\frac{1}{\left(s^2+\frac{2\zeta}{\tau}s+\frac{1}{\tau^2}\right)}=\frac{1}{\left(s^2+\frac{2\zeta}{\tau}s+\frac{\zeta^2}{\tau^2}+\frac{1}{\tau^2}-\frac{\zeta^2}{\tau^2}\right)}$$

$$=\frac{1}{\left(s+\frac{2\zeta}{\tau}\right)^2+\left(\frac{1-\zeta^2}{\tau^2}\right)}$$

$$h(t)=\frac{\tau}{\sqrt{1-\zeta^2}}e^{\frac{-\zeta}{\tau}t}\sin\frac{\sqrt{1-\zeta^2}}{\tau}t$$

$$y(t)=\int_0^t\left(\frac{kA}{\tau^2}\right)\left(\frac{\tau}{\sqrt{1-\zeta^2}}\right)e^{\frac{-\zeta}{\tau}m}\sin\frac{\sqrt{1-\zeta^2}}{\tau}mdm$$

$$=\left(\frac{kA}{\tau\sqrt{1-\zeta^2}}\right)\int_0^t e^{\frac{-\zeta}{\tau}m}\sin\frac{\sqrt{1-\zeta^2}}{\tau}mdm$$

$$\int e^{ax}\sin bx=\frac{e^{ax}}{b^2+a^2}(a\sin bx-b\cos bx)$$

$$a=\frac{-\zeta}{\tau}\ ,b=\frac{\sqrt{1-\zeta^2}}{\tau}\quad ,\rightarrow b^2+a^2=\frac{1}{\tau^2}$$

$$y(t)=\frac{kA\tau}{\sqrt{1-\zeta^2}}\left[-e^{\frac{-\zeta}{\tau}t}\left(\frac{\zeta}{\tau}\sin\frac{\sqrt{1-\zeta^2}}{\tau}t+\frac{\sqrt{1-\zeta^2}}{\tau}\cos\frac{\sqrt{1-\zeta^2}}{\tau}t\right)+\frac{\sqrt{1-\zeta^2}}{\tau}\right]$$

$$y(t)=kA\left[1-\frac{e^{\frac{-\zeta}{\tau}t}}{\sqrt{1-\zeta^2}}\left(\sin\frac{\sqrt{1-\zeta^2}}{\tau}t+\varphi\right)\right]$$

$$\varphi=\tan^{-1}\frac{\sqrt{1-\zeta^2}}{\tau}$$