Properties of Transfer Function

Order: The order of the system is the highest derivative of the output variable in the defining differential equation. For Transfer Function, it is the highest power of s in the denominator.

Pole: is the root of the denominator of the transfer function, i.e. the root of the characteristic polynomial. It directly determines:

The stability of the system (positive poles)

The potential of periodic transient (imaginary poles)

Zero: is the root of the numerator of the transfer function. It determines an inverse response (positive zero).

A physical system is causal when the order of the denominator is greater than the numerator, and when the transfer function goes to 0 as s $\rightarrow\infty$, the system is hence strictly proper. If the transfer function contains $e^{\theta s}$ or the order of numerator is higher than the denominator, then the system is non-casual or not realizable because the current values of the system depends on the future values of the variables.

Steady state gain: is the steady state value of the transfer function, is evaluated by setting s = 0 in the stable transfer function.

Effect of poles and zeros

The poles and zeros of a transfer function affect the dynamic of a process.
Consider a particular transfer function:

$$G(s) = \frac{K}{s(\tau_1 s + 1)(\tau^2 s^2 + 2\zeta\tau s + 1)}$$

The poles, i.e. the roots of the characteristic equation are:

$$s_1 = 0$$

$$s_2 = -\frac{1}{\tau_1}$$

$$s_3 = -\frac{\zeta}{\tau_2} + j\frac{\sqrt{1-\zeta^2}}{\tau_2}$$

$$s_4 = -\frac{\zeta}{\tau_2} - j\frac{\sqrt{1-\zeta^2}}{\tau_2}$$

The poles can be represented in the complex plane as follows:

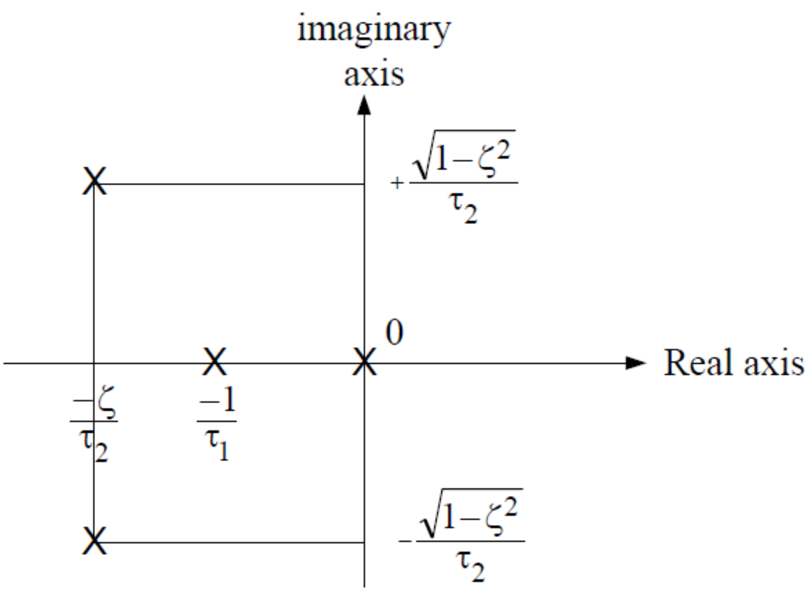

Fig.2 roots of the characteristic equation of G(s)

- Complex poles indicate the response will contain sine and cosine modes, i.e. will exhibit oscillation.
- Negative poles will result in a stable decaying response.

- Positive poles indicate that the response will have unstable mode.

The transfer function can be written as follows:

$$G(s) = \frac{b_m}{a_n}\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

- Positive zero leads to inverse response.
- Zero-pole cancellation occurs when a zero has exactly the same numerical value as a pole.
- A zero can exert a profound effect on the coefficient of response mode.