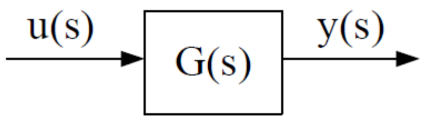
Transfer functions of the feedback control

Type of elements

1. Block



- $y(s) = G(s)u(s)$

2. summing junction

- $y(s) = A + B$

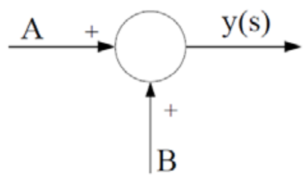


Fig.47 summing junction

3. take off point or bifurcation point

The following variable for control system are:

C(s) :output variable (controller variable)

C_m(s) :measured variable

R(s) :set point

E(s) :error

P(s):output controller variable

m(s) :final control element variable

L(s) :load variable

y_p(s) :process variable

The transfer function of feedback control system is found by divided the block diagram to many parts as the following:

1: The measuring element:

$C_m(s)=G_m(s)\ C(s)$ (1)

2: The comparator:

$E(s)=R(s)-C_m(s)$ (2)

3: The controller:

$P(s)=G_c(s)\ E(s)$ (3)

4: The final control element:

$m(s)= P(s)\ G_v(s)$ (4)

5: The process:

$y_p(s) =m(s)\ G_p(s)$ (5)

6: The controlled variable:

$y_L(s) =L(s)\ G_L(s)$ (6)

$C(s) = y_p(s) + L(s)$ (7)

Sub. Eq. 5 and 6 in Eq. 7 gives:

$C(s) = m(s)\ G_p(s) + L(s)\ G_L(s)$ (8)

sub. Eq. (3) in (4) and then sub Eq. 2 gives

$m_v(s)= E(s)\ G_c(s)\ G_v(s)$

$m(s)= (R(s)-c_m(s))\ G_c(s)\ G_v(s)$ (9)

Sub. Eq.(9) in (8)

$C(s) = (R(s)-C_m(s))\ G_c(s)\ G_v(s)G_p(s) + L(s)\ G_L(s)$ (10)

Let $G(s)= G_c(s)\ G_v(s)G_p(s)$:forward path transfer function

$C(s) = (R(s)-C_m(s))G(s) + L(s)\ G_L(s)$ (11)

Sub eq.(1) in (11)

$C(s) = R(s)G(s) - C(s)\, G_m(s)\, G(s) \quad + L(s)\, G_L(s)$

$C(s)\, [1+ G_m(s)\, G(s)] = R(s)G(s) \quad + L(s)\, G_L(s)$

$$C(s) = \frac{R(s)G(s)}{1+G_m(s)G(s)} + \frac{L(s)G_L(s)}{1+G_m(s)G(s)} \qquad (12)$$

The Eq. (12) is called the transfer function of the cascade control loop

If R(s) is fixed and called the regulator loop

$$C(s) = \frac{L(s)G_L(s)}{1+G_m(s)G(s)}$$

And if L(s) is fixed and called the servo loop

$$C(s)=\frac{R(s)G(s)}{1+G_m(s)G(s)}$$