Frequency response analysis

$G(s) = \frac{y(s)}{f(s)}$

$f(t) = a \sin \omega t \ , f(s) = \frac{a\omega}{s^2+\omega^2}$

$y(s) = G(s)\frac{a\omega}{s^2+\omega^2}$

$G(s) = G_1(s). G_2(s) \ldots = \frac{k_1}{s+\propto_1} . \frac{k_2}{s+\propto_2} . \frac{k_3}{s+\propto_3} \cdots$

$y(s) = G(s)\frac{a\omega}{s^2+\omega^2} = \frac{C_1\omega+C_2 s}{s^2+\omega^2} + \frac{k_1}{s-\propto_1} + \frac{k_2}{s-\propto_2} + \cdots \quad (1)$

$G(s) \, . \, a\omega = C_1\omega + C_2 s + (s^2+\omega^2)\left(\frac{k_1}{s+\propto_1} + \frac{k_2}{s+\propto_2} + \cdots\right)$

Set s=jω ,$s^2=-\omega^2$,$(s^2+\omega^2)=0$

G(jω) .aω =$c_1\omega+c_2 j\omega$

$G(j\omega) = \frac{c_1}{a} + j\frac{c_2}{a}$

$G(j\omega) = R + jI$

$|G| = \sqrt{I^2+R^2} = \frac{\sqrt{c_1^2+c_2^2}}{a}$

$\varphi = \tan^{-1}\frac{c_2}{c_1} =< G(phase\ lag)$

Laplace inverse of . eq.(1)

$y(t) = c_1 \sin \omega t + c_2 cos\omega t + k_1 e^{-\propto_1 t} + k_2 e^{-\propto_2 t} + \cdots$

$y(t) = c_1 \sin \omega t + c_2 cos\omega t$

$y(t) = \sqrt{c_1^2+c_2^2}\,\sin(\omega t + \varphi)$ the response function

$f(t) = a\sin \omega t$ force function

Amplitude of response =$\sqrt{c_1^2+c_2^2}$

Amplitude of force =a

φ phase lag

$|G| = |G_1|.|G_2|.|G_3| \ldots (MR = MR_1.MR_2 \ldots)$

$< G =< G_1 + < G_2 + < G_3 \quad (\varphi = \varphi_1 + \varphi_2 + \varphi_3 + \cdots)$

1st order lag system

$G(s) = \frac{k}{\tau s+1}$

Set s =jω

$G(j\omega) = \frac{k}{\tau j\omega + 1}$

$G(j\omega) = \frac{k}{\tau j\omega + 1} \times \frac{\tau j\omega - 1}{\tau j\omega - 1}$

$G(j\omega) = \frac{k}{\tau^2\omega^2+1} - j\frac{k\omega T}{\tau^2\omega^2+1}$

$G(j\omega) = R + jI$

$|G| = \sqrt{I^2 + R^2}$

$|G| = \frac{k}{\sqrt{\tau^2\omega^2+1}}$

$\varphi = \tan^{-1}\frac{I}{R}$

$< G = \tan^{-1} - \omega\tau = -\tan^{-1}\omega\tau$

Time delay

$G(s) = ke^{-\tau_D s}$

$G(j\omega) = ke^{-\tau_D j\omega} = k(cos\tau_D\omega - j\sin\tau_D\,\omega)$

$|G| = \sqrt{(cos\tau_D\omega)^2 - (\sin\tau_D\,\omega)^2} = k$

$< G = \tan^{-1}\frac{-\sin\tau_D\,\omega}{cos\,\tau_D\,\omega} = -\tau_D\omega$

Capacitance system

$G(s) = \frac{1}{cs}$

s =jω

$$G(j\omega) = \frac{1}{cj\omega} \cdot \frac{j}{j} = \frac{-j}{c\omega}$$

$$|G| = \sqrt{0 \quad + \left(\frac{-1}{c\omega}\right)^2} = \frac{1}{c\omega}$$

$$< G = \tan^{-1}\frac{0}{\frac{-1}{c\omega}} = \frac{-\pi}{2}$$

$$G(s) = k(\tau s + 1)$$

S=jω

$$|G| = k\sqrt{\tau^2\omega^2 + 1}$$

$$< G = \tan^{-1}\omega \mathrm{t}$$

Complex Transfer function

$$G(s) = \frac{ke^{-\tau_D s}}{\tau s + 1}$$

$$G(s) = G_1(s) \times G_2(s)$$

$$|G| = \frac{k}{\sqrt{\tau^2\omega^2+1}}$$

$$< G = \tan^{-1} - \omega\tau = -\tan^{-1}\omega\tau$$

Two lags in series

$$G(s) = \frac{k}{(\tau_1 s + 1)(\tau_2 s + 1)}$$

S=jω

$$|G| = \frac{k}{\sqrt{\tau_1{}^2\omega^2 + 1}\sqrt{\tau_2{}^2\omega^2 + 1}}$$

$$< G = -\tan^{-1}\omega\tau_1 - \tan^{-1}\omega\tau_2$$

Ex. Find $|G|$ and $< G$ for the following system:

$$G(s) = \frac{ke^{-\tau_D s}}{(\tau_1 s+1)(\tau_2 s+1)}$$

$G_1(s) = e^{-\tau_D s}, \quad |G_1| = 1 \ , < G_1 \ = -\tau_D \omega$

$G_2(s) = \frac{k}{\tau_1 s+1} \quad , |G_2| = \frac{k}{\sqrt{{\tau_1}^2 \omega^2+1}} \quad , < G_2 = -\tan^{-1} \omega \tau_1$

$G_3(s) = \frac{k}{\tau_2 s+1} \quad , |G_3| = \frac{k}{\sqrt{{\tau_2}^2 \omega^2+1}} \quad , < G_3 = -\tan^{-1} \omega \tau_2$

$|G| = \frac{k}{\sqrt{{\tau_1}^2 \omega^2+1}\sqrt{{\tau_2}^2 \omega^2+1}}$

$< G = -\tan^{-1} \omega \tau_1 - \tan^{-1} \omega \tau_2 - \tau_D \omega$