Two lag in series

$$G(s)=\frac{k}{(\tau_1 s+1)(\tau_2 s+1)}$$

$$\varphi=-\tan^{-1}\omega\tau_1-\tan^{-1}\omega\tau_2$$

Analysis

$$\log|G|=\log k-\frac{1}{2}\log({\tau_1}^2\omega^2+1)-\frac{1}{2}\log({\tau_2}^2\omega^2+1)$$

$$L=20\log MR=20\log k-10\log({\tau_1}^2\omega^2+1)-10\log({\tau_2}^2\omega^2+1) \qquad (1)$$

$as\ \omega\to 0, L=20\log k$,(LFA)

$$({\tau_1}^2\omega^2+1)\gg({\tau_2}^2\omega^2+1)$$

$$L=20\log k-10\log({\tau_1}^2\omega^2+1)$$

$$L=20\log k-20\log\tau_1-20\log\omega\cdots(3)$$

Slope=-20 , $(HFA1)$

$x=10\log({\tau_1}^2\omega^2+1)$ at $\omega_c=\frac{1}{\tau_1}$ and $x=10\log 2=3.01$

$as\ \omega\to\infty,$

$$L=20\log k-10\log({\tau_1}^2\omega^2)-10\log({\tau_2}^2\omega^2)\cdots(4)$$

$$L=20\log k-20\log\tau_1\tau_2-40\log\omega\cdots(5)$$

HFA2, slope=40

$$\omega=\frac{1}{\tau_2}$$

$$x=20\log k-20\log k-10\log({\tau_1}^2\omega^2+1)-20\log k+10\log({\tau_1}^2\omega^2+1)+10\log({\tau_2}^2\omega^2+1)$$

$x=10\log({\tau_2}^2\omega^2+1)\ \ at\omega=\frac{1}{\tau_2}\to x=10\log(1+1)=3.01$

$$20\log k=20\log k-20\log\tau_1\tau_2-40\log\omega$$

$$\omega = \frac{1}{\sqrt{\tau_1 \tau_2}}$$

1. $\omega = 0 \quad \rightarrow \omega = \frac{1}{\tau_1}$ LFA
2. $\omega = \frac{1}{\tau_1} \quad \rightarrow \omega = \frac{1}{\tau_2}$ HFA1
3. $\omega = \frac{1}{\tau_2} \quad \rightarrow \omega = \infty$ HFA2

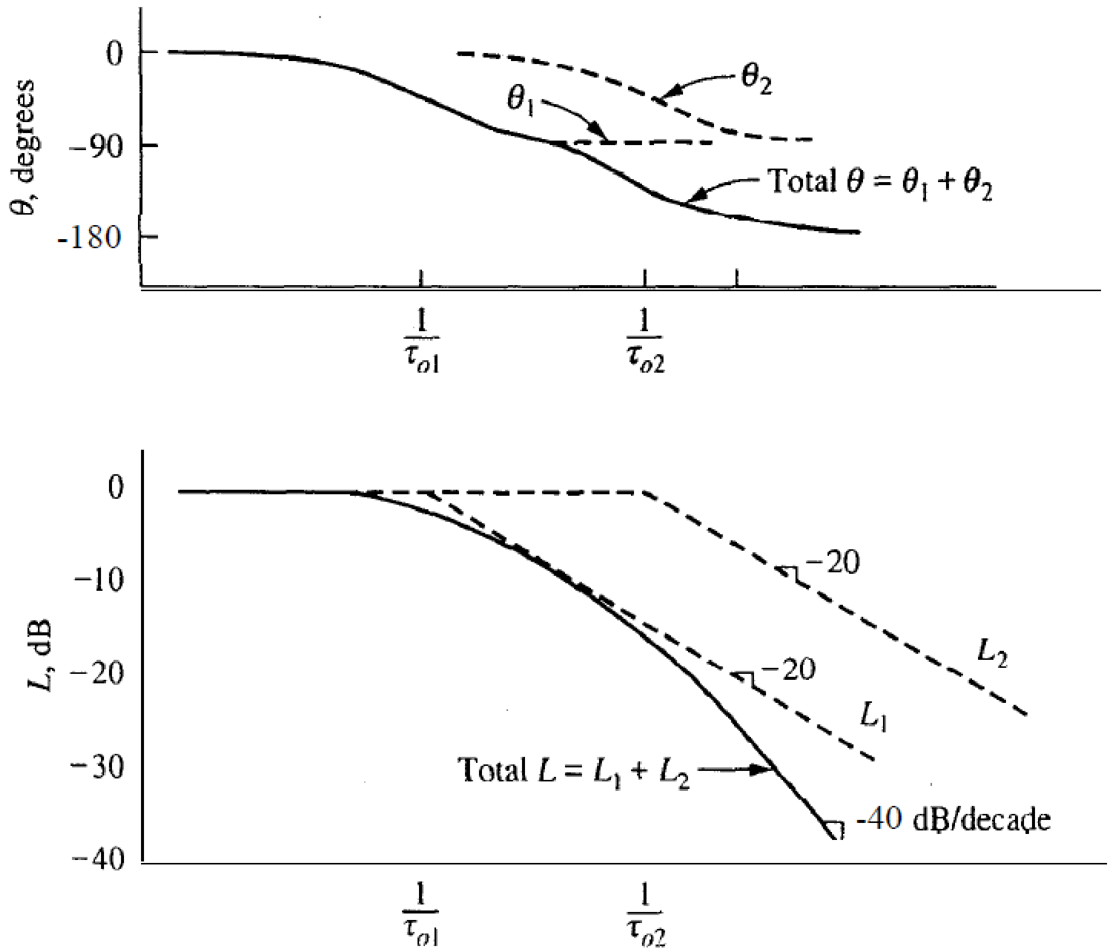

Fig.54 Bode plots for two lag in series