**Transfer function of a first order system:**

$$a_1 \frac{dy}{dt} + a_0\, y(t) = b_0 f(t)$$

$$\left(\frac{a_1}{a_0}\right)\frac{dy}{dt} + \ y(t) = \left(\frac{b_0}{a_0}\right) f(t)$$

Let $\tau_p = \frac{a_1}{a_0}$ and $k_p = \frac{b_0}{a_0}$

And take

$$\tau_p \frac{dy}{dt} + \ y(t) = k_p f(t)$$

Assuming zero initial condition.

Taking the Laplace transform of both sides given,

$$\tau_p\ (s\, y(s) - f(0)) + y(s) = k_p f(s)$$

$$\tau_p\ s\, y(s) + y(s) = k_p f(s)$$

$$\tau_p(s + 1)\ y(s) + y(s) = k_p f(s)$$

Where $\tau_p$ is time constant of the process (time) and $K_p$ is a steady state gain or static gain or simple the gain of the process . Unit of the $k_p = \frac{unit\ of\ output\ function}{unit\ of\ input\ function}$

At steady state G(s)=constant=$K_p$ .

**Second order system:**

$$a_2 \frac{d^2y}{dt^2} + a_1 \frac{dy}{dt} + a_0\, y(t) = b_0 f(t)$$

$$\frac{a_2}{a_0}\frac{d^2y}{dt^2} + \frac{a_1}{a_0}\frac{dy}{dt} + \ y(t) = \frac{b_0}{a_0} f(t)$$

If $a_0 \neq 0$ ,then above equation yield :

$$\tau_p^2 \frac{d^2y}{dt^2} + 2\zeta\tau_p \frac{dy}{dt} + \ y\tau = k_p f(t)$$

Where $\tau_p^2 = \frac{a_2}{a_0}$ and $2\zeta\tau_p = \frac{a_1}{a_0}$ and $k_p = \frac{b_0}{a_0}$

This equation in the standard form of a second order system

Where ζ =damping factor

$K_p$ :steady state ,or static , or simply gain of system .

$$\tau_p^2 \ \ (s^2y(s) - sy(0) - y(0)) + 2\zeta\tau_p(sy(s) - y(0)) + y(s) = k_p f(t)$$

$$G(s) = \frac{y(s)}{f(s)} = \frac{k_p}{\tau_p^2 s^2 + 2\zeta\tau_p s + 1}$$

**<u>Zero order system :</u>**

$$a_0\, y(t) = b_0 f(t)$$

$$G(s) = \frac{y(s)}{f(s)} = k_p$$

**<u>forcing function :</u>**

1. **Step function**

$$f(t) = \begin{Bmatrix} 0 & t < 0 \\ a & t \geq 0 \end{Bmatrix}$$

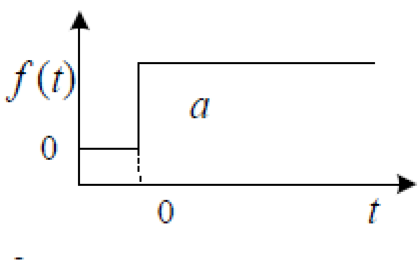


Fig.3 Step function

f(t)=a U(t)

$\mathcal{L}[f(t)] = f(s) = \frac{a}{S}$

A:magnitude of step +ve

Negative step change f(t)=-AU(t)

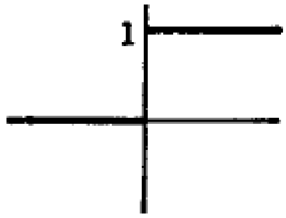


Fig.4 Unit step function