

1. Periodic function

a) Waves

1. Rectangular wave

$$f(t) = \begin{cases} 0 & t < 0 \\ k & 0 \leq t < b \\ 0 & b \leq t < 2b \\ k & 2b \leq t < 3b \\ 0 & 3b \leq t < 4b \end{cases}$$

$$f(s) = \frac{k}{s} - \frac{k}{s}e^{-s} + \frac{k}{s}e^{-2s} - \frac{k}{s}e^{-3s} + \frac{k}{s}e^{-4s} \dots \dots \dots$$

2. Triangular wave

This is a series of triangular pulse

$$f(t) = \begin{cases} 0 & t < 0 \\ kt & 0 \leq t < a \\ -kt & a \leq t < a + \frac{ak_1}{k_2} \\ kt & a + \frac{ak_1}{k_2} \leq t < 2a \end{cases}$$

$$f(s) = \frac{k_1}{s^2} - \left(\frac{k_1}{s^2} + \frac{k_2}{s^2}\right)e^{-as} + \left(\frac{k_1}{s^2} + \frac{k_2}{s^2}\right)e^{-\left(a + \frac{ak_1}{k_2}\right)s} - \left(\frac{k_1}{s^2} + \frac{k_2}{s^2}\right)e^{-\left(2a + \frac{ak_1}{k_2}\right)s}$$

b) Sinusoidal wave

1. Sine function

$$f(t) = \begin{cases} 0 & t < 0 \\ a \sin wt & t \geq 0 \end{cases}$$

$$f(s) = \frac{a\omega}{s^2 + \omega^2}$$

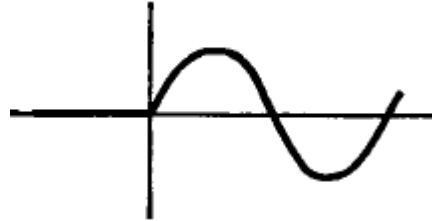
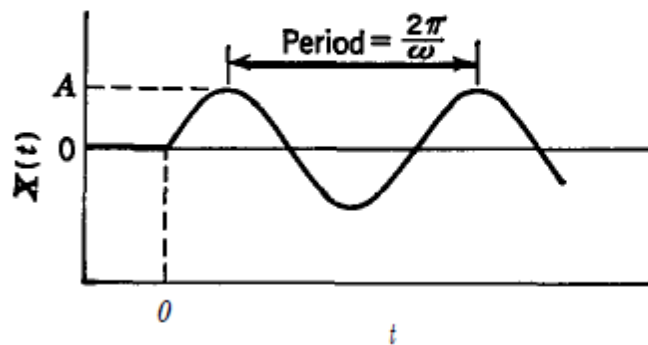


Fig.9 Sine function

RESPONSE 0:



$$\begin{aligned} X &= 0; t < 0 \\ X &= A \sin \omega t; t \geq 0 \\ X(s) &= \frac{A\omega}{s^2 + \omega^2} \end{aligned}$$

Fig.10 Sinusoidal input

2. Cos function

$$f(t) = \begin{cases} 0 & t < 0 \\ a \cos \omega t & t \geq 0 \end{cases}$$

$$f(s) = \frac{as}{s^2 + \omega^2}$$

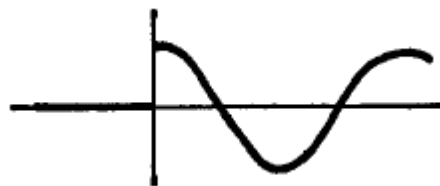


Fig.11 Cos function