First order system:

First order system is described by a first order differential equation . thus in this case of linear (or linearized) system ,we have

$$a_1 \frac{dy}{dt} + a_0\, y(t) = b_0 f(t)$$

$$\frac{a_2}{a_0}\frac{d^2y}{dt^2} + \frac{a_1}{a_0}\frac{dy}{dt} + \; y(t) = \frac{b_0}{a_0} f(t)$$

Let $\tau_p = \frac{a_1}{a_0}$ and $k_p = \frac{b_0}{a_0}$

$$\tau_p \frac{dy}{dt} + \; y(t) = k_p f(t)$$

$$G(s) = \frac{y(s)}{f(s)} = \frac{k_p}{\tau_p s + 1}$$

Examples of first order system

1-Mixing tank (involving mass transfer operation)

Consider the mixing process which a stream of solution containing salt (KCL) flows at a constant volumetric flowrate Q into a tank of constant hold up volume V. The concentration of the salt in the entering stream ,C_i (kg/m^3), varies with time. Find the outlet concentration to step change in inlet concentration?

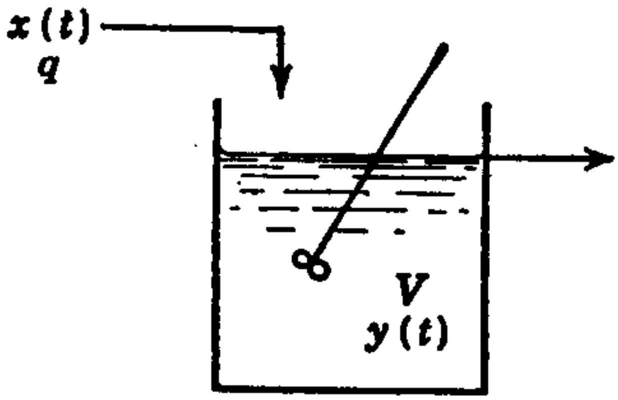

Fig.12 Mixing tank system

Solution

Assumptions:

1. The density of the solution is constant and the inlet flowrate is equal to the outlet flowrate. Since the holdup volume is constant.
2. Initially pure water is running through the tank.

Mass balance

Mass rate of salt In - Mass rate of salt Out = Mass rate of salt accumulation in the tank

Steady-state mass balance around the tank:

$$Q^o C_i^o - Q^o C_o^o = V\frac{dC_o^o}{dt} \qquad (1)$$

Unsteady-state mass balance around the tank:

$$Q^o C_i^{'}(t) - Q^o C_o^{'}(t) = V\frac{dC_o^{'}(t)}{dt} \qquad (2)$$

By subtracting the Eq. 1 from Eq.2 gives:

$$Q^o (C_i^{'}(t) - C_i^o) - Q^o (C_o^{'}(t) - C_o^o) = V\frac{d(C_o^{'}(t) - C_o^o)}{dt} \qquad (3)$$

Let $(C_i^{'}(t) - C_i^o) = C_i\,(t)$ and $(C_o^{'}(t) - C_o^o) = C_o(t)$

Where C_i(t) and C_o(t) are the concentration in deviation variables.

$$Q^o C_i(t) - Q^o C_o(t) = V\frac{dC_o(t)}{dt} \qquad (4)$$

$$\left(\frac{V}{Q}\right)\frac{dC_o(t)}{dt} + C_o(t) = C_i\,(t) \qquad (5)$$

Let $\tau = \frac{V}{Q}$

$$\tau\frac{dC_o(t)}{dt} + C_o(t) = C_i\,(t) \qquad (6)$$

Eq.6 is linear, first order differential equation and zero initial condition and taking Laplace transform of this expression and rearranging the results gives:

$$C_o(s)(\tau s + 1) = C_i\,(s)$$

$$G(s) = \frac{C_o(s)}{C_i\,(s)} = \frac{1}{\tau s + 1} \qquad (7)$$

Step change in C_i(t) of c_i and Laplace $C_i\,(s) = \frac{a}{s}$

$$G(s) = \frac{C_o(s)}{C_i\,(s)} = \frac{1}{\tau s + 1}$$

$$C_o(s) = C_i\,(s)\frac{1}{\tau s + 1}$$

$$C_o(s) = \frac{a}{s}\left(\frac{1}{\tau s + 1}\right) \qquad (8)$$

The Laplace inverse of Eq.8 gives:

$$C_o(t) = \mathcal{L}^{-1}[a \;\; \mathcal{L}\,\frac{1}{s(\tau s + 1)}]$$

$$C_o(t) = a\left(1 - e^{-t/\tau}\right) \qquad (9)$$

<u>Dynamic characteristics</u>**:**

- For a step change in the input, the process reaches a new steady state.
- The ultimate value of the output is $k_p\,a$, which can be found from the last Equation ,(Eq. 9) by setting t →∞
- The time constant, τ can be found be setting t = τ in the last equation, (Eq. 9) which gives y = 0.632 $k_p\,a$. Then from Figure2, the time needed for y to reach 0.632 $k_p\,a$ is τ.
- The smaller the value of τ, the steeper is the initial response of the output.
- The larger static gain of a process, the larger steady state value of its output for the same input change.

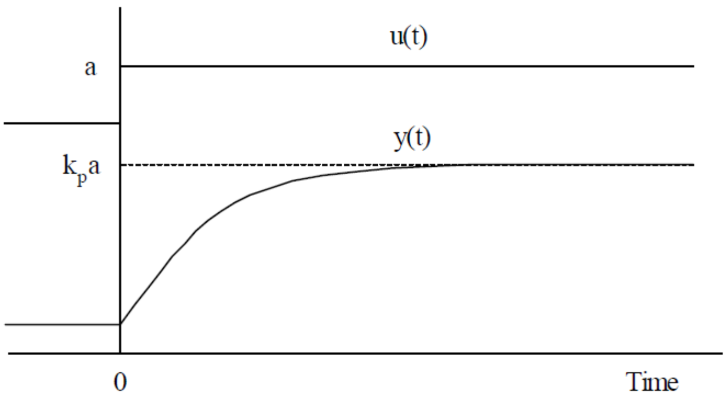

Fig.13 Response of first order system to a step input