2-Dynamics of heating tank :

Consider a perfectly mixed stirred and perfectly insulated tank header ,with a single feed stream and single product stream, assuming that the flowrate and temperature of the Intel stream (and that the rate of heat added per unit time (q)) can vary , develop a model to find the tank temperature as a function of time.

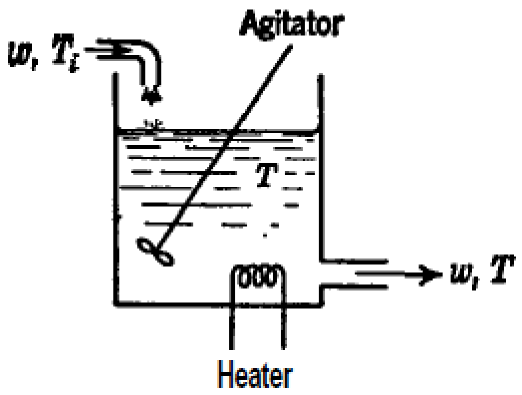

Fig.14 Heating tank

$Qo = f(\theta i\ , q\ , m)$

a. θ_i is chanced while ,q and m are remained constant at their initial steady-state values .

θ_o =f(θ_i)

$$G_1(s) = \frac{\theta_o(s)}{\theta_i(s)}$$

1. **Steady state heat balance :**

Heat in –heat out =heat accumulation in the tank

$$m^o C_p \theta_i^o + q^o - m^o C_p \theta_o^o = M C_p \frac{d\theta_o^o}{dt} = 0 \qquad (1)$$

2. **Transient (unsteady) state heat balance**

At t=0 ,Q_i is disturbed :

$$m^o C_p \theta_i^{'}(t) + q^o - m^o C_p \theta_o^{'}(t) = M C_p \frac{d\theta_o^{'}(t)}{dt} \qquad (2)$$

Subtracting eq.(1) form (2) to express in term of perturbation variable

$$m^o C_p (\theta_i^{'}(t) - \theta_i^o) + q^o - q^o - m^o C_p (\theta_i^{'}(t) - \theta_i^o) = M C_p \frac{d(\theta_i^{'}(t) - \theta_i^o)}{dt}$$

$$m^o C_p \theta_i(t) - m^o C_p \theta_o(t) = M C_p \frac{d\theta_o(t)}{dt} \qquad (3)$$

1st order d.E ., linear ,zero I.C.

$$\left(\frac{M}{m^o}\right)\frac{d\theta_o(t)}{dt} + \theta_o(t) = \theta_i(t)$$

Laplace transform :

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = \frac{1}{\tau s + 1}$$

Where $\tau = \frac{M}{m^o}[s]\,, k = 1[-]$

b. q is chanced while , θ_i and m are remained constant at their initial steady-state values

θ_o =f(q(s))

$$G_2(s) = \frac{\theta_o(s)}{q(s)}$$

1. **Steady state head balance :**

Heat in –heat out =heat accumulation

$$m^o C_p \theta_i^o + q^o - m^o C_p \theta_o^o = M C_p \frac{d\theta_o^o}{dt} = 0 \qquad (1)$$

2. **Transient (unsteady) state heat balance**:

$$m^o C_p \theta_i^o + q'(t) - m^o C_p \theta_o'(t) = M C_p \frac{d\theta_o'(t)}{dt} \qquad (2)$$

Subtracting eq.(1) form (2)

$$m^o C_p(\theta_i^o - \theta_i^o) + (q'(t) - q^o) - m^o C_p(\theta_o'(t) - \theta_o^o) = M C_p \frac{d(\theta_o'(t) - \theta_o^o)}{dt}$$

$$q(t) = (q'(t) - q^o)$$

$$\theta_o(t) = \theta_o'(t) - \theta_o^o$$

$$q(t) - m^o C_p \theta_o(t) = M C_p \frac{d\theta_o(t)}{dt}$$

1st order Q.E ., linear ,zero I.C.

$$\left(\frac{M}{m^o}\right)\frac{d\theta_o(t)}{dt} + \theta_o(t) = \frac{1}{m^o C_p} q(t)$$

Laplace transform :

$$G(s) = \frac{\theta_o(s)}{q(s)} = \frac{k}{\tau s + 1}$$

Where $\tau = \frac{M}{m^o}[s]$, $k = \frac{1}{m^o C_p}[C^o/w]$