<u>Heating tank</u>

c. θ_o =f(m(s)):m is chanced while , θ_i and q are remained constant at their initial steady-state values

θ_o =f(m(s))

$$G_3(s) = \frac{\theta_o(s)}{m(s)}$$

1. **Steady state head balance :**

Heat in by flow + heat out by heat transfer - heat out by flow+ = Heat accumulation

$$m^o C_p \theta_i^o + q^o - m^o C_p \theta_o^o = MC_p \frac{d\theta_o^o}{dt} = 0 \qquad (1)$$

2. **Transient state**:

$$m'(t) C_p \theta_i^o + q^o - m'(t) C_p \theta_o'(t) = MC_p \frac{d\theta_o'(t)}{dt} \qquad (2)$$

Linearization :

Eq. (2) is a nonlinear D.E. due to the nonlinear form term $m' C_p Q_o^1$ .this form should be linear zed a round$m^o \, and \, Q_o^o$.

$$f(m', \theta_o') = f(m^o, \theta_o^o) + \left[\frac{\partial f}{\partial m'}\right]_{(m^o,\theta_o^o)} (m' - m^o) + \left[\frac{\partial f}{\partial Q_o'}\right]_{(m^o,\theta_o^o)} (\theta_o' - \theta_o^o)$$

$$f(m', \theta_o') = m'(t) C_p \theta_o'(t)$$

$\left[\frac{\partial f}{\partial m'}\right]_{(m^o,\theta_o^o)} = C_p \theta_o^o$ and $\left[\frac{\partial f}{\partial \theta_o'}\right]_{(m^o,\theta_o^o)} = C_p \theta_i^o$

$$f(m'(t), \theta_o'(t)) \approx m^o C_p \theta_i^o + C_p \theta_o^o (m'(t) - m^o) + C_p \theta_i^o (\theta_o'(t) - \theta_o^o)$$

After linearization, Eq.2 becomes:

$$m'(t) C_p \theta_i^o + q^o - \left(m^o C_p \theta_i^o + C_p \theta_o^o (m'(t) - m^o) + C_p \theta_i^o (\theta_o'(t) - \theta_o^o)\right) = MC_p \frac{d\theta_o'(t)}{dt} \quad (3)$$

Eq.4 is linear, first order differential equation and zero initial condition

Subtracting Eq.1 from Eq. 3 :

$$m(t) C_p \theta_i^o - m^o C_p \theta_o \, (t) - \theta_o^o C_p m(t) = MC_p \frac{d\theta_o(t)}{dt} \quad (4)$$

Eq.4 is linear, first order differential equation and zero initial condition

$$MC_p \frac{d\theta_o(t)}{dt} + m^o C_p \theta_o(t) = C_p(\theta_i^o - \theta_o^o)m(t)$$

$$\left(\frac{M}{m^o}\right)\frac{d\theta_o(t)}{dt} + \theta_o(t) = -\frac{(\theta_o^o - \theta_i^o)}{m^o}\, m(t)$$

$$\tau\frac{d\theta_o(t)}{dt} + \theta_o(t) = -K_P m(t)$$

$$G(s) = \frac{\theta_o(s)}{\theta_i(s)} = -\frac{K_P}{\tau s + 1}$$

$$\theta_o(t) = -K_P\left(1 - e^{-t/\tau}\right)$$

θ_i *and q are changed while m remained constant at their initial steady state*

Steady state heat balance:

$$m^o C_p \theta_i^o + q^o - m^o C_p \theta_o^o = MC_p \frac{d\theta_o^o}{dt} = 0 \qquad (1)$$

Unsteady state heat balance:

$$m^o C_p \theta_i'(t) + q'(t) - m^o C_p \theta_o'(t) = MC_p \frac{d\theta_o'(t)}{dt} \qquad (2)$$

Subtracting Eq.1 from Eq.2:

$$m^o C_p(\theta_i'(t) - \theta_i^o) + q^o - q'(t) - m^o C_p(\theta_i'(t) - \theta_i^o) = MC_p \frac{d\left(\theta_i'(t) - \theta_i^o\right)}{dt}$$

$$m^o C_p \theta_i(t) + q(t) - m^o C_p \theta_o(t) = MC_p \frac{d\theta_o(t)}{dt}$$

$$\tau\frac{d\theta_o(t)}{dt} + \theta_o(t) = \theta_i(t) + \frac{1}{m^o C_p} q(t) \qquad (3)$$

Taking Laplace transform of Eq.3:

(τs+1) $\theta_o(s) = K_1\theta_i(s) + K_2 q(s)$

$$\theta_o(s) = \frac{K_1}{\tau s + 1}\theta_i(s) + \frac{K_2}{\tau s + 1} q(s)$$

Step change in $\theta_i(t)$ of θ_i and $q(t)$of q

$$\theta_o(s) = \frac{K_1}{\tau s + 1}\frac{\theta_i}{s} + \frac{K_2}{\tau s + 1}\frac{q}{s}$$

$$\theta_o(t) = (K_1\theta_i + K_1 q)\left(1 - e^{-t/\tau}\right)$$