General expressions for flow through beds in terms of Carman–Kozeny equations

Streamline flow—Carman–Kozeny equation

The equation for streamline flow through a circular tube is:

$$U = \frac{d_t^2}{32\mu} \cdot \frac{\Delta p}{L_t} \qquad \text{---------(1) Hagen –Poiseuille eq. for streamline flow in tubes}$$

where :

- μ : is the viscosity of the fluid
- U : is the mean velocity of the fluid
- dt : is the diameter of the tube
- Lt : is the length of the tube .

If the free space in the bed is assumed to consist of a series of tortuous channels, equation (1) may be rewritten for flow through a bed as:

$$U_1 = \frac{d_m'^2}{k'\mu} \cdot \frac{\Delta p}{L'} \qquad \text{-----------------(2)}$$

where:

d'_m : is same equivalent diameter of the pore channels .
k' : is a dimensionless constant whose value depends on the structure of the bed .
U_1 : is the average velocity through the pore channels .
L' : is the length of channel .

The average linear velocity through the pores, U_1, is given by:

$$U_1 = \frac{UX^2}{eX^2} = \frac{U}{e} \qquad \text{------------------(3)}$$

Kozeny proposed that d'_m may be taken as:

$$d'_m = \frac{e}{S_B} = \frac{e}{S(1-e)} \qquad \text{---(4)}$$

$$\frac{e}{S_B} = \frac{volume\ of\ voids\ \ filled\ with\ fluid}{wetted\ surface\ area\ of\ the\ bed}$$

$$= \frac{cross\ sectional\ area\ normal\ to\ flow}{wetted\ perimeter}$$

The hydraulic mean diameter for such a flow passage equal to

$$4 * \frac{cross\ sectional\ area}{wetted\ perimeter}$$

Then :

$$\frac{e}{S_B} = \frac{1}{4} * (hydroulic\ mean\ diameter)$$

Then taking $U_1 = U/e$ and $l' \propto l$, equation 2 becomes :

$$U = \frac{1}{k''} \cdot \frac{e^3}{S_B} \cdot \frac{1}{\mu} \cdot \frac{\Delta p}{L}$$

$$U = \frac{1}{k''} \cdot \frac{e^3}{S^2(1-e)^2} \cdot \frac{\Delta p}{\mu L} \qquad \text{---(5) Kozeny Eq.(for laminar flow)}$$

Or

Carman – Kozeny Eq.

Where :
K" : kozeny's constant = 5

And (B) the permeability coefficient is equal to

$$B = \frac{1}{k''} \cdot \frac{e^3}{S^2(1-e)^2} \qquad \text{---(6)}$$

Streamline and turbulent flow

Equation (5) applies to streamline flow conditions, but Carman and others have extended the analogy with pipe flow to cover both streamline and turbulent flow conditions through packed beds. In this treatment a modified friction factor ($R_1/\rho u^2{}_1$) is plotted against a modified Reynolds number Re_1.

For both streamline and turbulent flow

$$Re_1 = \frac{U}{e} \cdot \frac{e}{S(1-e)} \cdot \frac{\rho}{\mu}$$

$$Re_1 = \frac{\rho U}{S(1-e)\mu}$$ Reynolds number through packed bed

Force Balance

$$(\Delta p).e = R_1 SL(1-e)$$

$$R_1 = \frac{e}{S(1-e)} \cdot \frac{\Delta p}{L}$$

$$\frac{R_1}{\rho U_1^2} = \frac{e^3}{S(1-e)} \cdot \frac{\Delta p}{\rho U_1^2 L}$$ Carman Eq.

used to calculate Δp through packed beds for turbulent and streamline flow after knowing ($\frac{R_1}{\rho U_1^2}$)

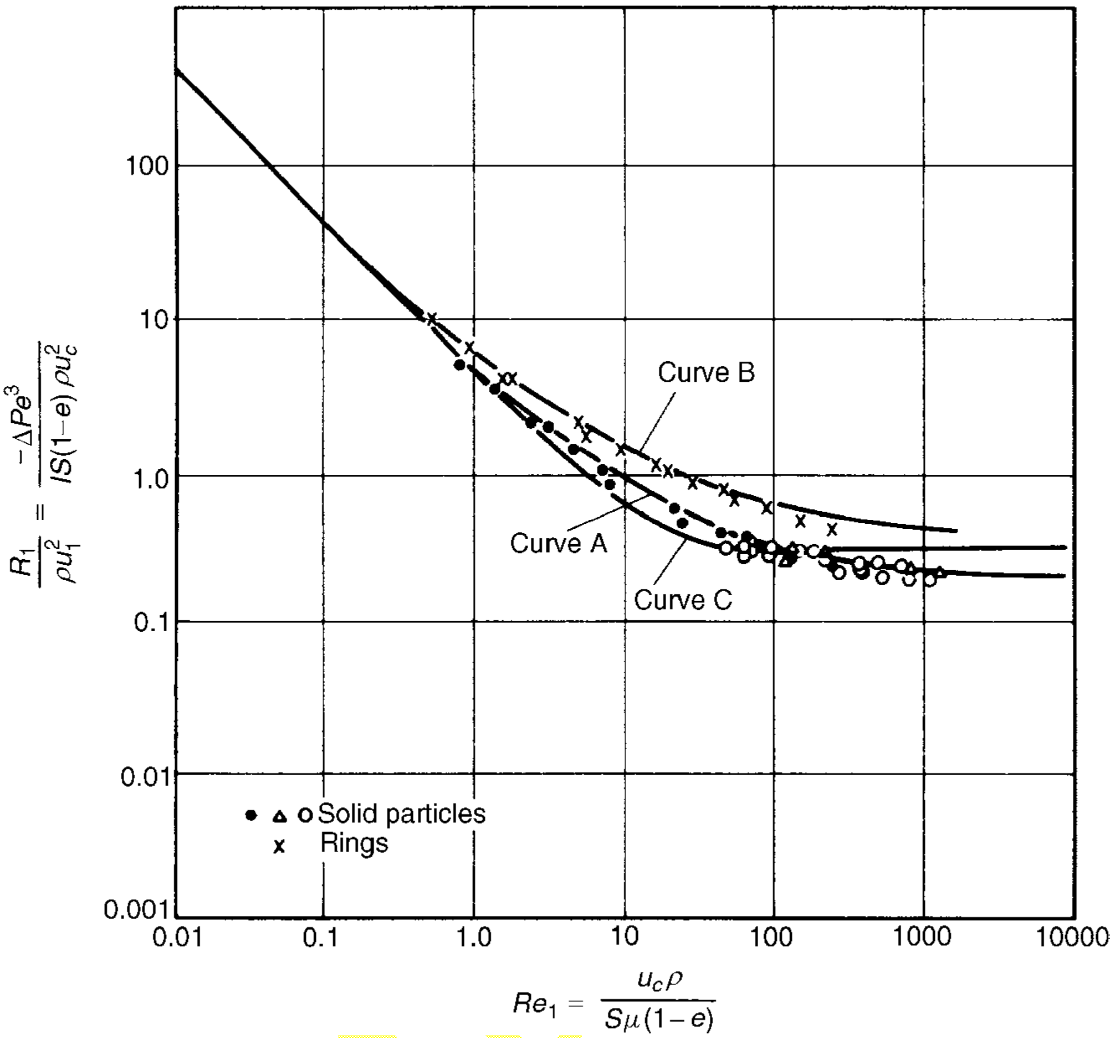

Figure . Carman's graph of $R_1/\rho u^2{}_1$ against Re_1

Curve B :Sawistowki Eq.

$$\frac{R_1}{\rho U_1^2} = 5Re_1^{-1} + Re_1^{-0.1}$$

Curve : carman Eq.

$$\frac{R_1}{\rho U_1^2} = 5Re_1^{-1} + 0.4Re_1^{-0.1}$$

Curve :

- For $Re_1 < 2$ the plot is straight line

(the second term is small and, approximately: $\frac{R_1}{\rho U_1^2} = 5Re_1^{-1}$

- For 2< Re_1 <100 , the slope of the plot gradually changes from (-1) to about (- 1/4)
- For Re_1 > 100 , the plot is approximately straight (horizontal)