Heat Transfer

The heat transferred per unit time through a unit area at a distance (y) from the surface is given by :

$$q_y = -k\frac{d\theta}{dy}$$

$$q_y = -\frac{k}{\rho C_p}\frac{d(\rho C_p\theta)}{dy}$$

Where :

C_p : is the specific heat of the fluid at constant pressure (J/kg . K) .

θ : is the temperature , (K)

k : is the thermal conductivity , (W/m .K)

$(\frac{k}{\rho C_p})$: is the thermal diffusivity , (m^2/s)

Mass Transfer

The rate of diffusion of a constituent A in a mixture is proportion to its concentration gradient .

$$N_A = -D\frac{dC_A}{dy}$$

Where :

N_A : is the molar rate of diffusion of constituent A per unit area , (Kmol / m^2 .s)

C_A :is the molar concentration of constituent A , (Kmol/m^3)

D : is the mass diffusivity , (m^3/sec.)

Viscosity

Consider the flow of a gas parallel to a solid surface and the movement of the molecules at right angles to this direction through a plane (a-a) of unit area , parallel to the surface and sufficiently close to it to be within the laminar sub layer .

Thus the rate at which momentum is transferred across the plane away from the surface [in the X-direction] = i . N . U_m .m . U_x

Where :

i.U_m : is the average velocity .

U_m : is the root mean square velocity , and (i) is some fraction of it .

N : is the numerical conc. of molecules close to the surface .

i . N . U_m : is the rate of passage of molecules .

i . N . U_m .dt : is the number of molecules passing .

U_x : is the mean velocity of molecules in the X-direction .

m : is the mass of each molecules .

By similar reasoning there must be an equivalent stream of molecules also passing through the plane in the opposite direction , otherwise there would be a resultant flow perpendicular to the surface .

It is now possible to give a physical interpretation to the **Reynolds number** .

$$Re = \frac{\rho . d . u}{\mu} = \frac{2}{u_m \lambda}\rho d = 2\frac{\rho d}{u_m \lambda}$$

Where :

λ : mean free path of molecules

From the kinetic theory

$$u_m = \sqrt{8RT/\pi\, m.wt}$$

And is independent of pressure , and $\rho\lambda$ is constant

Thus the velocity of a gas would be expected to be a function of temp. but not of pressure .

Thermal conductivity

Thermal diffusivity = k / (ρ C_p)

The prantle number (Pr) is defined as the ratio of the kinematic viscosity to the thermal diffusivity .

This :

$$\text{Pr} = \frac{\mu/\rho}{k/\rho C_p} = \frac{\mu C_p}{k}$$

$$= \frac{(4\gamma)}{(9\gamma - 5)} \quad \text{for gases}$$

Where :

$$\gamma = \frac{C_p}{C_v}$$

Values of (Pr) calculated from the above eq. are in close agreement with practical figures .

Diffusivity

The ratio of the kinematic viscosity to the diffusivity is the **Schmidt number (Sc)**

i.e

$$Sc = \frac{(\mu/\rho)}{D} = \frac{\mu}{\rho D}$$

It is thus seen that the kinematic viscosity , the thermal diffusivity , and the diffusivity for the mass transfer and all proportional product of the mean free path and the root mean square velocity of the molecules , and that the expressions for the transfer of momentum , heat , and mass are of the same form .

For liquids the same qualitative forms of relationships exist , but it is not possible to express the physical properties of the liquids in terms of molecular velocities and distances .