Size Reduction

Raw materials often occur in sizes that are too large to be used and, therefore, they must be reduced in size. This size-reduction operation can be divided into two major categories depending on whether the material is a solid or a liquid. All depend on the reaction to shearing forces.

1. The Mechanism of Size Reduction

In the grinding process, materials are reduced in size by fracturing them. In the process, the material is stressed by the action of mechanical moving parts in the grinding machine and initially the stress is absorbed internally by the material as strain energy.

Stress → Strain → Fracture in lines of Weakness → Released Heat

When the local strain energy exceeds a critical level , which is a function of the material, fracture occurs along lines of weakness and the stored energy is released. Some of the energy is taken up in the creation of new surface, but the greater part of it is dissipated as heat. Time also plays a part in the fracturing process and it appears that material will fracture at lower stress concentrations if these can be maintained for longer periods.
Grinding is achieved by mechanical stress followed by rupture and
the energy required depends upon:
1. the hardness of the material
2. the tendency of the material to crack (friability).
The force applied may be compression, impact, or shear, and both the magnitude of the force and the time of application affect the extent of grinding achieved.

2. Energy for Size reduction

For efficient grinding, the energy applied to the material should exceed, by as small a margin as possible, the minimum energy needed to rupture the material. Excess energy is lost as heat and this loss should be kept as low as practicable.
Grinding is a very inefficient process and it is important to use energy as efficiently as possible. A **three** theories depend upon the basic assumption that the energy required to produce a change *dL* in a particle of a typical size dimension *L* is a simple power function of *L*:

$$\frac{dE}{dL} = KL^n$$

where
dE is the differential energy required
dL is the change in a typical dimension
L is the magnitude of a typical length dimension
K, n are constants.

1) Kick : assumed that the energy required to reduce a material in size was directly proportional to the size reduction ratio dL/L.

This leads to n= -1 and $K=K_k f_C$, then ;

$$\frac{dE}{dL} = \frac{K_k f_c}{L}$$

Where
K_K is called Kick's constant
f_c is called the crushing strength of the material

which , on integration gives:

$$E = K_k f_c \ln\frac{L_1}{L_2} \qquad \textbf{\textit{Kick's Law(1)}}$$

2) **Rittinger** assumed that the energy required for size reduction is directly proportional, not to the change in length dimensions, but to the change in surface area .

This leads to ***n*= -2** and ***K*=*K_R f_c*,** *then* ;

$$\frac{dE}{dL} = \frac{K_R f_c}{L^2}$$

where K_R is called Rittinger's constant,
which, on integration gives:

$$E = K_R f_c \left(\frac{1}{L_2} - \frac{1}{L_1}\right)$$ ***Rittinger's Law (2)***

As the specific surface of a particle (the surface area per unit mass) is proportional to 1/L, equation (2) postulates that the energy required to reduce L for a mass of particles from 10 cm to 5 cm would be the same as that required to reduce, the same mass of 5 mm particles down to 4.7 mm. This is a very much smaller reduction in terms of energy per unit mass for the smaller particles, than that predicted by Kick's Law.
It has been found, experimentally, that for the grinding of coarse particles in which the increase in surface area per unit mass is relatively small, Kick's Law is a reasonable approximation. For the size reduction of fine powders, on the other hand, in which large areas of new surface are being created, Rittinger's Law fits the experimental data better.

3) **Bond** has suggested an intermediate course, in which he postulates that ***n* = -3/2** and this leads to:

$$E = E_i \sqrt{\frac{100}{L_2}} \left(1 - \frac{1}{\sqrt{q}}\right) \qquad (3)$$

Where:
L: is measured in microns
E_i: (the Work Index) is the amount of energy required to reduce unit mass of the material from an infinitely large particle size down to a particle size of 100 µm.
q is the reduction ratio where $q = L_1/L_2$.

Note that all of these equations (1), (2) and (3) are dimensional equations and so if quoted values are to be used for the various constants, the dimensions must be expressed in appropriate units.
The greatest use of these equations is in making comparisons between power requirements for various degrees of reduction .

Example 1 :

Sugar is ground from crystals of which it is acceptable that 80% pass a 500 µm sieve (Standard Sieve No.35), down to a size in which it is acceptable that 80% passes a 88 µm (Standard Sieve No. 170) sieve, and a 5 hp motor is found just sufficient for the required throughput. If the requirements are changed such that the grinding is only down to 80% through a 125 µm (No.120) sieve but the throughput is to be increased by 50% would the existing motor have sufficient power to operate the grinder? Assume Bond's equation.
Solution:

Letting m (kg/h) be the initial throughput
88 µm = 88×10^{-6} m
125 µm = 125×10^{-6} m
500 µm = 500×10^{-6} m

$$E = E_i \sqrt{\frac{100}{L_2}} \left(1 - \frac{1}{\sqrt{q}}\right)$$

First Case
E = 5/m
q=88/500=0.176
$5/m = E_i(100/88 \times 10\text{-}6)^{1/2} [1 - (0.176)^{1/2}]$
Ei=

Second Case

$E = x/1.5m = E_i(100/125x10\text{-}6)^{1/2}\ [1 - (125/500)^{1/2}]$

$x/7.5 = 0.84x(0.500/0.58)$

$x = 5.4$ hp

So the motor would be expected to have insufficient power to pass the 50% increased throughput, though it should be able to handle an increase of 40% .