

## Reynolds Analogy

The analogy was first suggested by Reynolds to relate heat transfer rates to shear stress, but it's also applicable to mass transfer. It is assumed that elements of fluids are brought from remote regions to the surface by the action of the turbulent eddies; the elements do not undergo any mixing with the intermediate fluid through which they pass, and the instantaneous reach equilibrium on contact with the interfacial layer. An equal volume of fluid is at the same time, displaced in the reverse direction. Thus in a flowing fluid there is a transference of momentum and simultaneous transfer of heat if there is a temp. gradient, and of mass transfer if there is a concentration gradient. The turbulent fluid is assumed to have direct access to the surface and the existence of a buffer layer and laminar sub-layer is neglected. Modification of the model has been made by Taylor and Prandtl to take account of the laminar sub-layer. Subsequently, the effect of the buffer layer has been incorporated by applying the universal velocity profile.

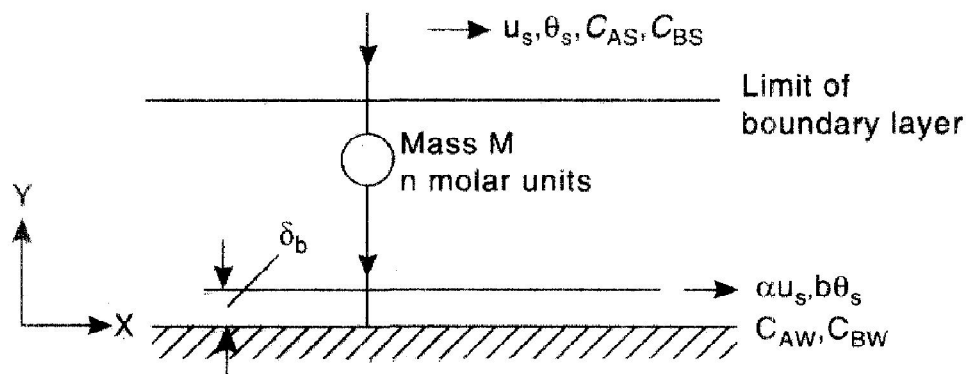


Fig.(1) The Reynolds analogy momentum, heat and mass transfer

## UNIT OPERATION

Suppose a mass  $M$  of fluid situated at a distance from the surface to be moving with a velocity  $u_s$  in the  $X$ -direction . If this element moves to the surface where the velocity is zero, it will give up its momentum  $Mu_s$  , in time  $t$  , say. If the temperature difference between the element and the surface is  $\theta_s$  and  $C_p$  is the specific heat of the fluid , the heat transferred to the surface will be  $M C_p \theta_s$  . If the surface is of area  $A$  , the rate of heat transfer is given by :

$$\frac{M C_p \theta_s}{t} = -q \cdot A$$

Where :

$q$  : is the heat transferred to the surface per unit area per unit time (the negative sign has been introduced as the positive direction is away from the surface) .

If the shear stress at the surface is  $R_0$  , the shearing force over the area  $A$  will equal the rate of change in momentum , i.e.

$$\frac{Mu_s}{t} = -R_0 A$$

Thus

$$\frac{C_p \theta_s}{u_s} = \frac{q}{R_0}$$

The shear stress (  $R_0$  ) in the fluid at the walls will be equal and opposite to the shear stress (  $R$  ) acting on the surface themselves . Thus , writing  $R = -R_0$  and (  $h$  ) as the heat transfer coefficient between the fluid and the surface :

$$-\frac{q}{\theta_s} = h = -\frac{R_0 C_p}{u_s} = \frac{RC_p}{u_s}$$

## UNIT OPERATION

OR

$$\frac{h}{\rho C_p u_s} = \frac{R}{\rho u_s^2} \quad \text{Reynolds analogy}$$

The dimensionless group  $(\frac{h}{\rho C_p u_s})$  is the **Stanton Group ( $S_t$ )**.

In this analysis, no allowance has been made for the variations in physical properties OR the fluid with temperature.

### *Mass transfer with bulk flow*

the movement of an element of fluid consisting of (n) molar units of a mixture of two constituents A and B from a region outside the boundary layer, where the molecular concentrations are  $C_{AS}$  and  $C_{BS}$ , to the surface where the corresponding concentrations are  $C_{Aw}$  and  $C_{Bw}$ . The total molar concentration is everywhere  $C_T$ . The transfer is effected in a time t and takes place at an area A of surface.

There is no net transference of the component B. When n molar units of material are transferred from outside the boundary layer to the surface:

$$\text{Transfer of A towards surface} = n \left( \frac{C_{AS}}{C_T} \right)$$

$$\text{Transfer of B towards surface} = n \left( \frac{C_{BS}}{C_T} \right)$$

In this case the molar rate of transfer of B away from the surface is equal to the transfer towards the surface.

$$\therefore \text{Transfer of B away from surface} = n \left( \frac{C_{BS}}{C_T} \right)$$

## UNIT OPERATION

Associated transfer of A away from surface =  
 $n \left( \frac{C_{Bs}}{C_T} \right) \left( \frac{C_{Aw}}{C_{Bw}} \right)$

Thus the net transfer of A towards the surface

$$\begin{aligned} -N_A \cdot A \cdot t &= n \left( \frac{C_{As}}{C_T} - \frac{C_{Bs}}{C_T} \cdot \frac{C_{Aw}}{C_{Bw}} \right) \\ &= n \left( \frac{C_T C_{As} - C_{Aw} C_{As} - C_{Aw} C_T + C_{Aw} C_{As}}{C_T (C_T - C_{Aw})} \right) \\ &= n \left( \frac{C_{As} - C_{Aw}}{C_{Bw}} \right) \end{aligned}$$

It is assumed that the total molar concentration is everywhere constant . Thus the rate of transfer per unit area and unit time is given by :

$$-N'_A = n \left( \frac{C_{As} - C_{Aw}}{C_{Bw}} \right) \quad (1)$$

The net transfer of momentum per unit time

$$= -R_0 A \frac{\left( \frac{n \rho u_s}{C_T} \right)}{t} \quad (2)$$

$\rho$  : is taken as the mean mass density of the fluid .

$$-R_0 = \frac{n \rho u_s}{C_T \cdot A \cdot t} \quad (3)$$

Dividing (1) and (3)

$$\frac{N'_A}{R_0} = \left( \frac{C_{As} - C_{Aw}}{\rho u_s} \right) \frac{C_T}{C_{Bw}} \quad (4)$$

Writing  $R_0 = -R$  and defining the mass transfer coefficient ( $h_D$ ) by the relation , then :

$$\frac{N'_A}{C_{As} - C_{Aw}} = -h_D \quad (5)$$

$$\therefore \left( \frac{h_D}{u_s} \right) \left( \frac{C_{Bw}}{C_T} \right) = \frac{R}{\rho u_s^2} \quad (6)$$

## UNIT OPERATION

If the concentration of the non diffusion component [B] is small , and in cases of equimolecular counter diffusion , eq.(6) are reduced to

$$\frac{h_D}{u_s} = \frac{R}{\rho u_s^2}$$