Reynolds analogy for heat transfer and mass transfer

The relation between momentum and heat transfer is given by the eq.

$$\frac{h}{\rho C_p u_s} = \frac{R}{\rho u_s^2}$$

The relation between momentum and equimolecular counter diffusion mass transfer is given by the eq.

$$\frac{h_D}{u_s} = \frac{R}{\rho u_s^2}$$

and when the second constituent is not transferred

$$\left(\frac{h_D}{u_s}\right)\left(\frac{C_{Bw}}{C_T}\right) = \frac{R}{\rho u_s^2}$$

Therefore for equimolar counter diffusion

$$h_D = \frac{h}{\rho C_p} \qquad (1)$$

and for conditions where bulk flow

$$h_D - \frac{C_{Bw}}{C_T} = \frac{h}{\rho C_p} \qquad (2)\ \text{lewis-relation}$$

It has been seen that for the flow through a tube the friction factor [R /ρu^2] is related approximately to the Reynolds Group by the Blasius eq.

[R /ρu^2] α $Re^{-0.25}$

So that the relation between heat transfer and the Reynolds group is obtained as :

$$\frac{h}{\rho C_p u} \ \alpha \ Re^{-0.25}$$

For mass transfer under conditions of equimolecular counter diffusion :

$$\frac{h_D}{u} \ \alpha \ Re^{-0.25}$$

For diffusion through a stationary gas :

$$\left(\frac{h_D}{u_s}\right)\left(\frac{C_{Bw}}{C_T}\right) \alpha \ Re^{-0.25}$$

[Since the mean velocity u is proportional to the velocity u_s remote from the surface] .

Modification of Reynolds analogy for heat transfer and mass transfer

The original Reynolds analogy involves a number of simplifying assumptions which are justifiable only in a limited range of conditions. Thus it was assumed that fluid was transferred from outside the boundary layer to the surface without mixing with the intervening fluid, that it was brought to rest at the surface, and that thermal equilibrium was established. Various modifications have been made to this simple theory to take account of the existence of the laminar sub-layer and the buffer layer close to the surface. TAYLOR and PRANDTL allowed for the existence of the laminar sub-layer but ignored the existence of the buffer layer in their treatment and assumed that the simple Reynolds analogy was applicable to the transfer of heat and momentum from the main stream to the edge of the laminar sub-layer of thickness δ_b. Transfer through the laminar sub-layer was then presumed to be attributable solely to molecular motion .

If αu_s and $b\theta_s$ are the velocity and temperature, respectively, at the edge of the laminar sub-layer , applying the Reynolds analogy for transfer across the turbulent region :

$$\frac{q}{R_0} = \frac{C_p(\theta_s - b\theta_s)}{u_s - \alpha u_s} \qquad (1)$$

The rate of transfer of heat by conduction through the laminar sub-layer from a surface of area A is given by :

$$qA = -kA\frac{b\theta_s}{\delta_b} \qquad (2)$$

The rate of transfer of momentum is equal to the shearing force and therefore :

$$R_0 A = -\mu A\frac{\alpha u_s}{\delta_b} = -RA \qquad (3)$$

Dividing eq.s (2) and (3)

$$\frac{q}{R_0} = \frac{k}{\mu}\frac{b\theta_s}{\alpha u_s} \qquad (4)$$

Thus from eq.s (1) and (4)

$$\frac{k}{\mu}\frac{b\theta_s}{\alpha u_s} = \frac{C_p(1-b)\theta_s}{(1-\alpha)u_s}$$

$$p_r\frac{(1-b)}{b} = \frac{(1-\alpha)}{\alpha}$$

$$\frac{b}{\alpha} = \frac{1}{\alpha + (1-\alpha)pr^{-1}}$$

Sub. :

$$\frac{q}{R_0} = \frac{C_p\theta_s}{u_s}\left[\frac{1}{1+\alpha(p_r-1)}\right] = \frac{h\theta_s}{R}$$

i.e. :

$$S_t = \frac{h}{\rho C_p u_s} = \frac{(R/\rho u_s^2)}{1+\alpha(p_r-1)} \qquad (5)$$

For flow over a plane surface
$\alpha = 2.1\ Re_x^{-0.1}$

Where Re_x is the Reynolds number $(\rho u_s x/\mu)$,

x being the distance from the leading edge of the surface .
For flow through a pipe of diameter d :

$\alpha = 2\ Re^{-1/8}$

Where Re is the Reynolds number $(\rho ud/\mu)$

For mass transfer to a surface, a similar relation to eq (5) . can be derived for equimolecular counter diffusion except that the Prandtl number is replaced by the Schmidt number. It follows that :

$$\frac{h_D}{u_s} = \frac{(R/\rho u_s^2)}{1+\alpha(s_c-1)} \qquad (6)$$

Where s_c is the Schmidt no.
$S_c = \frac{\mu}{\rho D}$

It is thus seen that by taking account of the existence of the laminar sub-layer, correction factors are introduced into the simple Reynolds analogy.
For heat transfer, the factor is $[1+\alpha(p_r-1]$ and for mass transfer it is $[1+\alpha(s_c-1)]$.

There are two sets of conditions under which the correction factor approaches unity:

(i) For gases both the Prandtl and Schmidt groups are approximately unity, and therefore the simple Reynolds analogy is closely followed .

(ii) When the fluid is highly turbulent, the laminar sub-layer will become very thin and the velocity at the edge of the laminar sub-layer will be small.