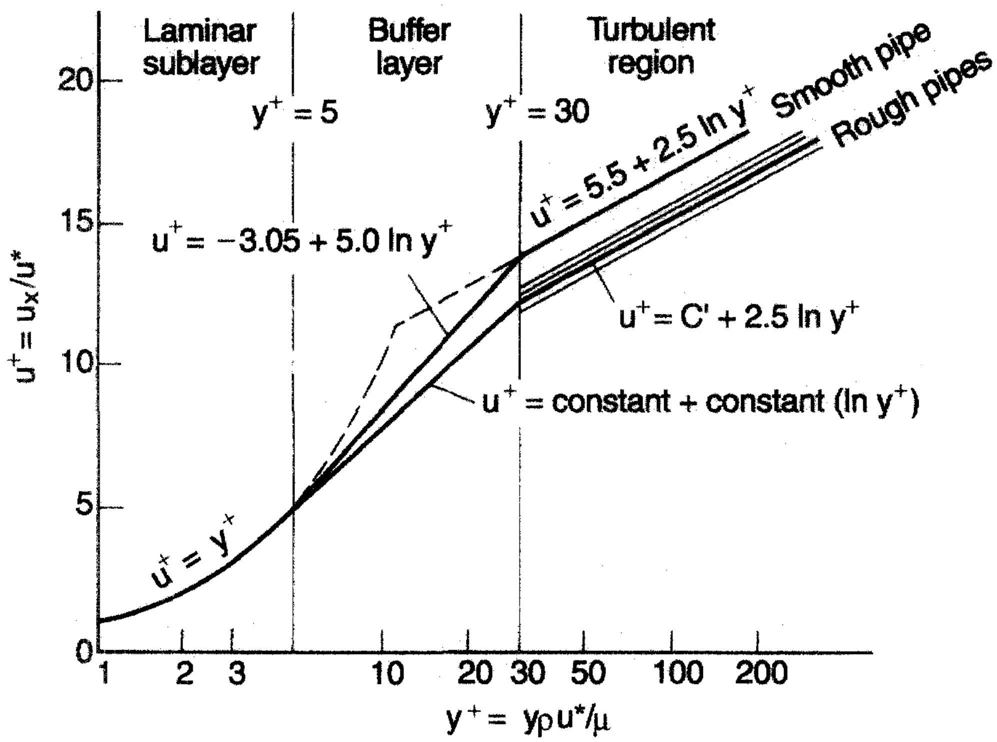
Use of universal velocity profile in Reynolds analogy



The universal velocity profile

u_0 : velocity of fluid of surface .
u_s : velocity of fluid outside boundary layer , or at pipe axis .
u_x : velocity in x-direction .

u^+ : shearing stress velocity $= \sqrt{\frac{R}{\rho}}$

$$u^+ = \frac{u_y}{u^*}$$

y = distance from surface .

y^+ = ratio of (y) to $(\frac{\mu}{\rho u^*})$

$$y^+ = y\frac{\rho u^*}{\mu}$$

Laminar sub-layer ; $0 < y^+ < 5$; $u^+ = y^+$.
Buffer layer ; $5 < y^+ < 30$; $u^+ = -3.05 + 5 \ln y^+$
Turbulent region ; $y^+ > 30$; $u^+ = 5.5 + 2.5 \ln y^+$

} for smooth Surface

For heat transfer :

$$\frac{R}{\rho u_s^2} = \frac{h}{\rho C_p u_s}[1 + 5\sqrt{\left(\frac{R}{\rho u_s^2}\right)}\left\{(p_r - 1) + ln\left(\frac{5}{6}p_r + \frac{1}{6}\right)\right\}]$$

For mass transfer :

$$\frac{R}{\rho u_s^2} = \frac{h_D}{u_s}[1 + 5\sqrt{\left(\frac{R}{\rho u_s^2}\right)}\left\{(s_c - 1) + ln\left(\frac{5}{6}s_c + \frac{1}{6}\right)\right\}]$$

Flow over a plane surface .

The simple Reynolds analogy can be used for calculating the point value of the heat transfer coefficient by substituting for $(\frac{R}{\rho u_s^2})$ in terms of the Reynolds group (Re_x) using the eq .

$$\frac{R}{\rho u_s^2} = 0.03\ Re_x^{-0.2}$$

$$\frac{h}{\rho C_p u_s} = \frac{R}{\rho u_s^2}$$ Reynolds analogy

Sub. for $\frac{R}{\rho u_s^2} = 0.03\ Re_x^{-0.2}$

Then : $$St = \frac{h}{\rho C_p u_s} = 0.03\ Re_x^{-0.2} \quad (1)$$

Gives the point value of (h)

The mean is

$$St = \frac{h}{\rho C_p u_s} = \frac{\int_0^x 0.03\ Re_x^{-0.2}\, dx}{x}$$

Then

$$St_{mean} = 0.037\ Re_x^{-0.2} \quad (2)$$

mean value of (h) according Reynolds analogy

The effect of the laminar sub-layer is allowed for by using the Taylor-Prandtl modification :

$$St = \frac{h}{\rho C_p u_s} = \frac{\frac{R}{\rho u_s^2}}{1+\alpha(p_r-1)}$$

Where :

$$\alpha = \frac{u_b}{u_s} = 2.1\ Re_x^{-0.1}$$

Thus

$$St = \frac{h}{\rho C_p u_s} = \frac{0.03\ Re_x^{-0.2}}{1+2.1\ Re_x^{-0.1}(p_r-1)} \qquad (3)$$

gives point value of (h)

- $St = \frac{Nu}{Re\ .\ Pr}$

The mean value over the whole surface is obtained by integration .
Similarly , substituting into eq. derived using universal velocity profile gives :

$$St = \frac{h}{\rho C_p u_s} = \frac{0.03\ Re_x^{-0.2}}{1+0.87\ Re_x^{-0.1}[(p_r-1)+ln\left(\frac{5}{6}p_r+\frac{1}{6}\right)]} \qquad (4)$$

gives the point value of (h)

Mean values may be obtained by graphical integration .

The same procedure may be used for obtaining relationships for mass transfer coefficients , for equimolecular counter diffusion

or where the concentration of the non-diffusing constituent is small:

$$\frac{h_D}{u_s} = \frac{R_1}{\rho u_s^2}$$

Sub. for

$$\frac{R}{\rho u_s^2} = 0.03\ Re_x^{-0.2}$$

$\therefore\ \frac{h_D}{u_s} = 0.03\ Re_x^{-0.2}$ gives point value of (h_D)

The mean value over the surface

$\frac{h_D}{u_s} = 0.037\ Re_x^{-0.2}$ gives (h_D) mean

For mass transfer through a stationary second component

$$\frac{h_D}{u_s} \cdot \frac{C_{Bw}}{C_T} = \frac{R}{\rho u_s^2}$$

The correction factor ($\frac{C_{Bw}}{C_T}$) must then be introduced into eqs. (5) and (6)

The resistance of the laminar sub-layer can be taken into account, however , for equirnolecular counter diffusion or for low concentration gradient

$$\frac{h_D}{u_s} = \frac{(R/\rho u_s^2)}{1+\alpha(s_c-1)}$$

Sub. for

$$\frac{R}{\rho u_s^2} = 0.03\ Re_x^{-0.2}$$

Gives

$$\frac{h_D}{u_s} = \frac{sh_x}{Re_x .S_c} = \frac{0.03\ Re_x^{-0.2}}{1+2.1\ Re_x^{-0.1}(s_c-1)}$$

$$sh = sherwood\ group = \frac{h_D.d}{D}$$

Flow in a pipe

For heat transfer

$$\frac{h}{\rho C_p u} = 0.032\ Re_x^{-\frac{1}{4}} \qquad (1)$$

gives (h) using simple Reynolds analogy

$$\frac{h}{\rho C_p u} = \frac{0.032\ Re^{-\frac{1}{4}}}{1+2\ Re^{-\frac{1}{8}}(p_r-1)} \qquad (2)$$

gives (h) using Taylar-Prandtl modification

$$St = \frac{0.032\ Re^{-\frac{1}{4}}}{1+0.82\ Re^{-\frac{1}{8}}[(p_r-1)+ln\left(\frac{5}{6}p_r+\frac{1}{6}\right)]}$$

gives (h) using universal velocity profile

For mass transfer :

$$\frac{h_D}{u} = 0.032\ Re^{-\frac{1}{4}}$$

equimolecular counter diffusion } using simple Reynolds analogy

For diffusion through a stationary gas

$$\frac{h_D}{u}.\frac{C_{Bw}}{C_T} = 0.032\ Re^{-\frac{1}{4}}$$

using simple Reynolds analogy

For equimolecular counter diffusion

$$\frac{h_D}{u} = \frac{0.032\ Re^{-\frac{1}{4}}}{1+2\ Re^{-\frac{1}{8}}(S_c-1)}$$ using Taylar-Prandtl modification