The -j- Factor of Chilton and Colburn

- For forced convection in tubes

$$Nu = 0.023\ Re^{0.8} Pr^{0.33}$$

If both sides are divided by the product (Re.Pr)

$$\frac{Nu}{Re.Pr} = St = \frac{h}{\rho C_p u} = 0.023\ Re^{-0.2}.Pr^{-0.67}$$

$$St.Pr^{0.67} = j_h = 0.023 Re^{-0.2}$$

i.e for heat transfer

$$j_h = St.Pr^{0.67}$$

Chilton and Colburn found that a plot of (j_h) against (Re) gave approximately the same curve as the friction chart for flow through tubes .

By analogy with the derivation gives above for heat transfer Chilton and Colburn have derived a factor for mass transfer (j_d) which they have expressed as :

$$j_d = \frac{h_D}{u}.\frac{C_{Bm}}{C_T}\left(\frac{\mu}{\rho D}\right)^{0.67}$$

Where :

C_{Bm} : The logarithmic mean of the conc. of B

$$\frac{C_{B2} - C_{B1}}{ln\frac{C_{B2}}{C_{B1}}}$$

C_T : total concentration

$\frac{\mu}{\rho D}$: Schmidt number

** Gillilond and Sherwood found the following relation for mass transfer (h_D) inside vertical tubes .

$$\frac{h_D}{u}.\frac{C_{Bm}}{C_T}Sc^{0.56} = 0.023\ Re^{-0.17} = j_d$$

For a plane surface $j_h \approx j_d$

$$St.Pr^{0.67} = \frac{h_D}{u}.\frac{C_{Bm}}{C_T}Sc^{0.67}$$

So that :

$$h_D = \frac{h}{\rho C_p}.\frac{C_T}{C_{Bm}}(\frac{Pr}{Sc})^{0.67}$$

** The general eq. for mass transfer in a wetted – wall column is :

$$\frac{h.d}{D_v}.\frac{P_{Bm}}{P_T} = 0.023\ Re^{0.83}.Sc^{0.44} \qquad (*)$$

Where :

D_v : vapor phase diffusivity

P_{Bm} : logarithmic mean of the press. of $B = \frac{P_{B2}-P_{B1}}{ln\frac{P_{B2}}{P_{B1}}}$

P_T : total press.

d : column diameter

eq.(*) is frequently re arranged as :

$$\frac{h_D}{u}.\frac{P_{Bm}}{P_T}Sc^{0.56} = 0.023\ Re^{-0.17} = j_d \qquad (a)$$

$K_G = \frac{h_D}{RT}.\frac{P_{Bm}}{P_T}$ or $h_D = K_G RT$

K_G : gas film transfer coefficient

** For most gases , we may write

$j_d \approx \frac{R}{\rho u^2}$ (the friction factor)

Gas phase

$$D_V = \frac{4.3*10^{-4}.\ T^{1.5}}{P\left(V_A^{1/3}+V_B^{1/3}\right)}\sqrt{\frac{1}{M_A}+\frac{1}{M_B}}$$

Liquid phase

$$D_L = \frac{7.7*10^{-16}\ T}{\mu\left(V^{1/3}+V_0^{1/3}\right)}$$

Ex : Calculate the rise in temperature of water which is passed at 3.5 m/s through a smooth 25 mm diameter pipe , 6 m long. The water enters at 300 K and the tube wall may be assumed constant at 330 K . The following methods may be used :

(a) the simple Reynolds analogy .

(b) the Taylor-Prandtl modification .

(c) the universal velocity profile .

(d) $Nu = 0.023\ Re^{0.8} Pr^{0.33}$.

Sol.

$$Re = \frac{0.025*3.5*1000}{0.7*10^{-3}} = 1.25 * 10^5$$

$$Pr = \frac{4.18*10^3*0.7*10^{-3}}{0.65} = 4.50$$

(a) Reynolds analogy

$$\frac{h}{\rho C_p u} = 0.032\ \mathrm{Re}^{-0.25}$$

$h = [4.18 \times 1000 \times 1000 \times 3.5 \times 0.032(1.25 \times 10^{5})^{-0.25}]$

$= 24{,}902\ \mathrm{W/m^2\,K}$ or $24.9\ \mathrm{kW/m^2\,K}$

Heat transferred per unit time in length dL of pipe = $h\,\pi\, 0.025\, dL\,(330-\theta)$ kW, where θ is the temperature at a distance L m from the inlet .

Rate of increase of heat content of fluid

$$= \left(\frac{\pi}{4}(0.025)^2 * 3.5 * 1000 * 4.18\right)d\theta \ \text{kw}$$

The outlet temperature θ' is then given by:

$$\int_{300}^{\theta'} \frac{d\theta}{(330-\theta)} = 0.0109h\int_0^6 dL$$

Where : h is in $\mathrm{kW/m^2\,K}$.

$$\log_{10}(330-\theta') = \log_{10}30 - \left(\frac{0.0654h}{2.303}\right) = 1.477 - 0.0283h$$

In this case :

$h = 24.9\ \mathrm{kW/m^2\,K}$

$$\log_{10}\ (330-\theta') = (1.477 - 0.705) = 0.772$$

and : $\underline{\theta' = 324.1\ \mathrm{k}}$

(b) Taylor-Prattdtl equation

$$\frac{\mathrm{h}}{\rho C_p u} = 0.032\ \mathrm{Re}^{-\frac{1}{4}}[1 + 2.0\ \mathrm{Re}^{-\frac{1}{8}}(\mathrm{Pr}-1)]^{-1}$$

$$h = \frac{24.9}{(1+2.0*3.5/4.34)} = 9.53\ \mathrm{kW/m^2\,k}$$

and :

$$\log_{10}\ (330-\theta')=1.477-(0.0283*9.53)=1.207$$

$\underline{\theta' = 313.9\ k}$

(c) Universal velocity profile equation

$$\frac{h}{\rho C_p u}=0.032\ Re^{-\frac{1}{4}}\{1+0.82\ Re^{-\frac{1}{8}}[(Pr-1)+\ln(0.83Pr+0.17)]\}^{-1}$$

$$=\frac{24.9}{1+(0.82/4.34)(3.5+2303\ x\ 0.591)}$$

$= 12.98\ \ kW/m^2\ K$

$$\log_{10}\ (330-\theta')=1.477-(0.0283*12.98)=1.110$$

and : $\underline{\theta' = 317.1\ k}$

d) $Nu = 0.023\ Re^{0.8}\,Pr^{0.33}$

$$h=\frac{0.023*0.65}{0.0250}(1.25*10^5)^{0.8}(4.5)^{0.33}$$

$= 0.596\ x\ 1.195\ x\ 10^4\ x\ 1.64$

$= 1.168\ x\ 10^4\ W/m^2\ K$ or $11.68\ kW/m^2\ K$

and :

$$\log_{10}\ (330-\theta')=1.477-(0.0283*11.68)=1.147$$

$\underline{\theta' = 316.0\ \ k}$

Lecture 8