CIVE1400 Examination 1996. Answers
Question 6

6(a) Use the Bernoulli equation to show that the relationship between flow and depth over a sharp-edged triangular weir is given by $Q = C_d \frac{8}{15} \tan\frac{\theta}{2} \sqrt{2g} H^{5/2}$

(10 marks)

6(b) A rectangular weir and a V-notch weir are located in parallel channels of the same dimensions. Both weirs have an opening 0.3 m wide at the top and 0.3 m deep. Both have a C_d of 0.6. What head would be required over the rectangular weir to pass the same flow as over the V-notch weir when it has a head of 0.29 m?

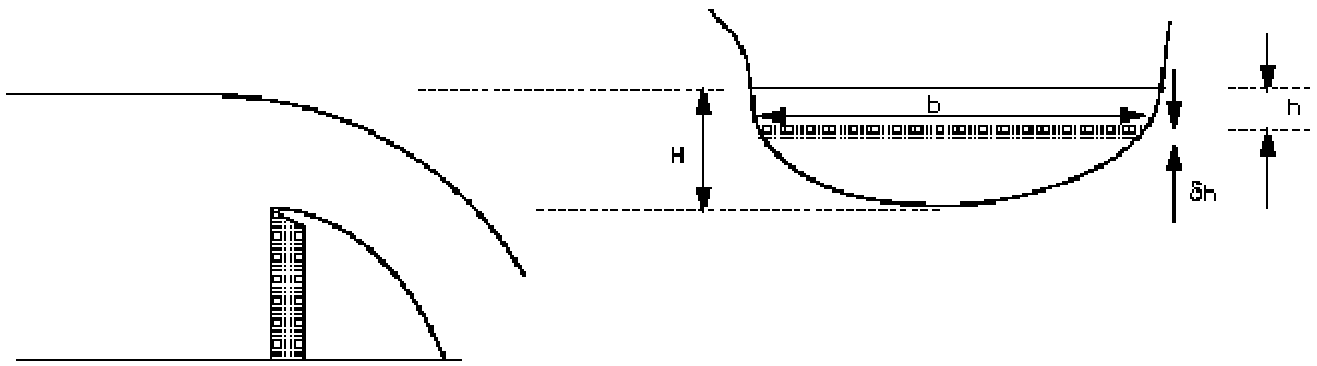
(For a rectangular weir $Q = C_d \frac{2}{3} b \sqrt{2g} H^{3/2}$)

(10 marks)

6(a):

A General Weir Equation

Consider a horizontal strip of width *b* and depth *h* below the free surface, as shown in the figure below.



Elemental strip of flow through a notch

Assuming the velocity is only due to the head.

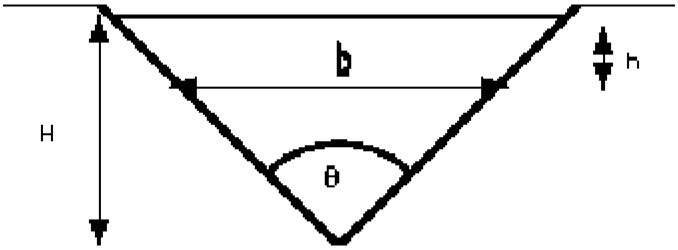
$$\text{velocity through the strip, } u = \sqrt{2gh}$$
$$\text{discharge through the strip, } \delta Q = Au = b\,\delta h\sqrt{2gh}$$

Integrating from the free surface, $h = 0$, to the weir crest, $h = H$ gives the expression for the total theoretical discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H b h^{1/2}\, dh$$

This will be different for every differently shaped weir or notch. To make further use of this equation we need an expression relating the width of flow across the weir to the depth below the free surface.

For the "V" notch weir the relationship between width and depth is dependent on the angle of the "V".



"V" notch, or triangular, weir geometry.

If the angle of the "V" is then the width, *b*, a depth *h* from the free surface is

$$b = 2(H-h)\tan\left(\frac{\theta}{2}\right)$$

So the discharge is

$$\begin{aligned} Q_{\text{theoretical}} &= 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\int_0^H (H-h)h^{1/2}\,dh \\ &= 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\left[\frac{2}{5}Hh^{3/2} - \frac{2}{5}h^{5/2}\right]_0^H \\ &= \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2} \end{aligned}$$

The actual discharge is obtained by introducing a coefficient of discharge

$$Q_{\text{actual}} = C_d\,\frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}$$

6(b):

Equating the two weir equations:

$$C_d\,\frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H_V^{5/2} = C_d\,\frac{2}{3}b\sqrt{2g}\,H_R^{3/2}$$

C_d is the same for both equations.
H_R = 0.29 m
b = 0.3m

Substituting these into the above equation gives.

$$\frac{8}{15}0.5(0.29)^{5/2} = 0.2\,H_R^{3/2}$$
$$H_R = 0.1539\,m$$

Back To June 1996 Questions Page

Back To June 1996 Questions Page

CIVE1400 Examination 1996. Answers

Question 7

7)
A plunger of diameter 0.1 m and length 0.15 m has five small holes of diameter 2 mm drilled through it in the direction of its length. The plunger fits closely inside a cylinder containing oil, such that no oil passes between the plunger and the cylinder. Calculate the force which must be applied to the plunger, in a downward vertical direction, to make the plunger fall with a speed of 0.0005 m/s. Assume that the upwards flow through the small holes is laminar and that the coefficient of viscosity of the oil is 0.2 kg/ms.

(20 marks)

Velocity, u = 0.0005 m/s
viscosity μ = 0.2 kg m^{-1} s^{-1}
length = 0.15m
hole diameter, d = 2mm = 0.002m
Plunger diameter D = 0.1m

The Hagen-Poiseuille equation for head loss during laminar flow in a pipe is:

$$h_f = \frac{32\mu L u}{\rho g d^2}$$

Pressure loss is given by

$$\Delta p = \rho g h_f = \frac{32\mu L u}{d^2}$$

Pressure difference between top and bottom of each hole is:

$$\Delta p = \frac{32 \times 0.2 \times 0.15 \times 0.0005}{0.002^2}$$
$$= 120\,N/m^2$$

So we need a pressure of 120 N/m^2 at the bottom of the cylinder.

Pressure = Force / area

$$Force = 120 \times \left(\frac{\pi\left(D^2 - 5d^2\right)}{4}\right)$$
$$= 0.94\,N$$

Back To June 1997 Questions Page

CIVE1400 Examination 1997. Answers

Question 1

1.a A differential "U"-tube manometer containing mercury of density $13000\,kg/m^3$ is used to measure the pressure drop along a horizontal pipe. If the fluid in the pipe is water and the manometer reading is $0.6\,m$ what is the pressure difference between the two tapping points.

(7 marks)

1.b A square tank of side length $1.3\,m$ is filled with oil to a depth of $0.8\,m$. If the density of the oil is $850\,kg/m^3$, find the resultant force and its point of action on one wall.

(6 marks)

1.c A jet of water of area $0.1m^2$ is being fired horizontally at a vertical wall. If the velocity of the jet is $25\,m/s$ estimate the force exerted on the wall.

(7 marks)

1.a

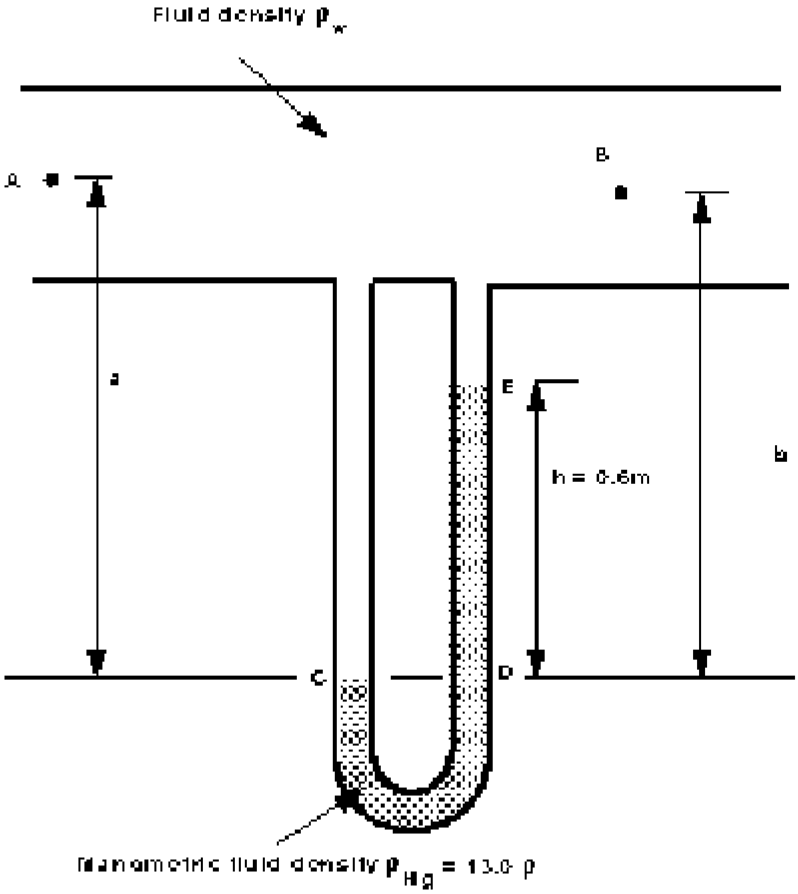


Figure of manometer setup

density of mercury $\rho = 13000\ kg/m^3$

pressure at C and D is equal:

$$p_C = p_D$$
$$p_A + \rho_w g\, a = p_B + \rho_w g\, (b - h) + \rho_{Hg}\, g\, h$$

$$p_A - p_B = \rho_w g\, b - \rho_w g\, h - \rho_w g\, a + \rho_{Hg}\, g\, h$$
$$= \rho_w g\, (b - a) + hg\, (\rho_{Hg} - \rho_w)$$

As horizontal a = b

$$p_A - p_B = hg\, (\rho_{Hg} - \rho_w)$$
$$= 0.6 \times 9.81 \times (13000 - 1000)$$
$$= 70\ 632\ N/m^2$$
$$= 70.6\ kN/m^2$$

1.b
Force per unit width R,

$$R = \rho\, g\, H \times H \times 0.5$$
$$= 850 \times 9.81 \times 0.8^2\ 0.5$$
$$= 2668 \text{ N/m}$$

Total force on wall = 2668 × 1.3 = 3468 N

Point of action , normal to wall through centroid of pressure diagram. 2/3 from surface.

Distance from surface to point of action of resultant = 2H/3 = 0.53 m

1.c
Force on the water:

$$\begin{aligned} F &= \dot{m}(u_2 - u_1) \\ &= \rho Q(u_2 - u_1) \\ &= \rho A u_1(u_2 - u_1) \\ &= 1000 \times 0.1 \times 25(0 - 25) \\ &= -62500\, N \text{ (in the opposite direction to the jet)} \end{aligned}$$

Force on the wall = R = -F = 62500 N (in the direction of the jet)

Back To June 1997 Questions Page

Back To June 1996 Questions Page

Back To June 1997 Questions Page

CIVE1400 Examination 1997. Answers

Question 2

2
Water flows ate a rate of 0.5m^3/s round a $50°$, contracting pipe bend which lies in a horizontal plane. The diameter at the bend entrance is $700\,mm$ and at the exit $500\,mm$ - as shown in Figure 1.

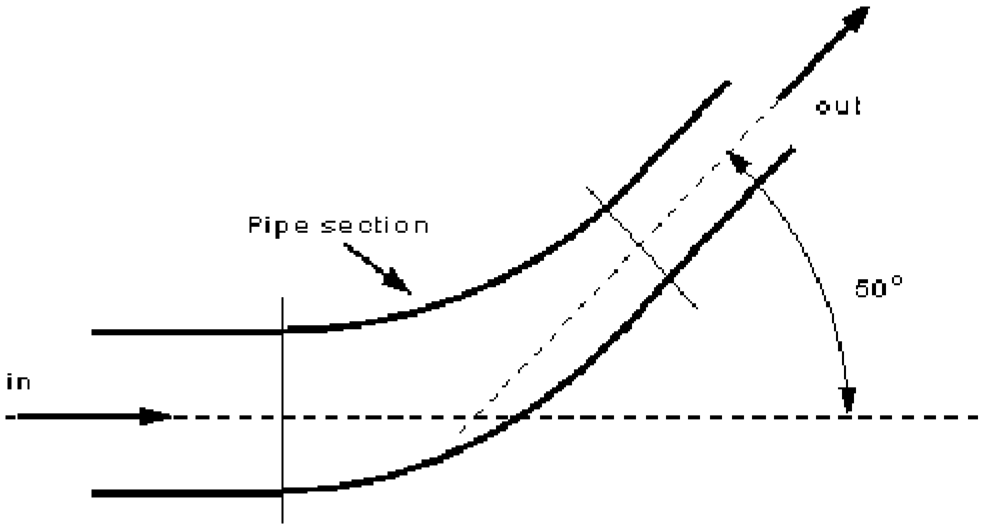


Figure 1.

If the pressure at the entrance to the bend is $200\,kN/m^2$, determine the magnitude and direction of the force exerted by the fluid on the bend.

(17 marks)

Comment on the reason why frictional losses may be neglected in this analysis.

(3 marks)

$A_1 = \pi d_1 / 4 = 0.3848\ m^2$
$A_2 = \pi d_2 / 4 = 0.1963\ m^2$
$u_1 = Q/A_1 = 0.5/0.3848 = 1.299\ m/s$
$u_2 = Q/A_2 = 0.5/0.1963 = 2.546\ m/s$
$p_1 = 200\ kN/m^2 = 200\ 000\ N/m^2$

<u>Calculate the total force</u>

In the x-direction:

$$
\begin{aligned}
F_{T_x} &= \rho Q\left(u_{2_x} - u_{1_x}\right) \\
u_{1_x} &= u_1 \\
u_{2_x} &= u_2 \cos\theta \\
F_{T_x} &= \rho Q\left(u_2 \cos\theta - u_1\right) \\
&= 1000 \times 0.5(2.546\cos 50 - 1.299) \\
&= 168.77\ N
\end{aligned}
$$

In the y-direction:

$$
\begin{aligned}
F_{T_y} &= \rho Q\left(u_{2_y} - u_{1_y}\right) \\
u_{1_y} &= u_1 \sin 0 = 0 \\
u_{2_y} &= u_2 \sin \theta \\
F_{T_y} &= \rho Q u_2 \sin \theta \\
&= 1000 \times 0.5 \times 2.546 \sin 50 \\
&= 975.17\ N
\end{aligned}
$$

Calculate the pressure force

Use Bernoulli to calculate force at exit, p_2

$$
\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f
$$

the friction loss h_f can be ignored, $h_f = 0$

As the pipe is in the horizontal plane, $z_1 = z_2$

By continuity, $Q = u_1 A_1 = u_2 A_2$

$$
\begin{aligned}
p_2 &= p_1 - \frac{\rho}{2}\left(u_2^2 - u_1^2\right) \\
&= 200000 - \frac{1000}{2}\left(2.546^2 - 1.299^2\right) \\
&= 200000 - 2397 \\
&= 197603\ N
\end{aligned}
$$

$$
F_p = \text{pressure force at 1 - pressure force at 2}
$$

$$
\begin{aligned}
F_{p_x} &= p_1 A_1 \cos 0 - p_2 A_2 \cos \theta = p_1 A_1 - p_2 A_2 \cos \theta \\
&= 200000 \times 0.3834 - 197603 \times 0.1963 \cos 50 \\
&= 52026\ N \\
F_{p_y} &= p_1 A_1 \sin 0 - p_2 A_2 \sin \theta = -p_2 A_2 \sin \theta \\
&= -197603 \times 0.1963 \sin 50 \\
&= -29714\ N
\end{aligned}
$$

Calculate the body force

There are no body forces as the pipe is in the horizontal plane.

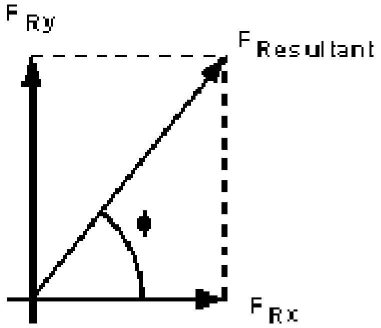
Calculate the resultant force

$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$
$$F_{T_y} = F_{R_y} + F_{P_y} + F_{B_y}$$

$$\begin{aligned}
F_{R_x} &= F_{T_x} - F_{P_x} - 0 \\
&= \rho Q(u_2 \cos\theta - u_1) - p_1 A_1 + p_2 A_2 \cos\theta \\
&= 1000 \times 0.5(2.546 \cos 50 - 1.299) - 200000 \times 0.3848 + 197603 \times 0.1963 \times \cos 50 \\
&= 168.7 - 52026 \\
&= -51858\ N
\end{aligned}$$

$$\begin{aligned}
F_{R_y} &= F_{T_y} - F_{P_y} - 0 \\
&= \rho Q u_2 \sin\theta + p_2 A_2 \sin\theta \\
&= 1000 \times 0.5 \times 2.546 \times \sin 50 + 197603 \times 0.1963 \sin 50 \\
&= 975 + 29714 \\
&= 30689\ N
\end{aligned}$$

And the resultant force on the fluid is given by



$$F_R = \sqrt{F_{R_x}^2 - F_{R_y}^2} = 62859\ N$$

And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = -30.6^\circ$$

the force on the bend is the same magnitude but in the opposite direction

$$R = -F_R$$

Back To June 1997 Questions Page

Back To June 1997 Questions Page

CIVE1400 Examination 1997. Answers
Question 3

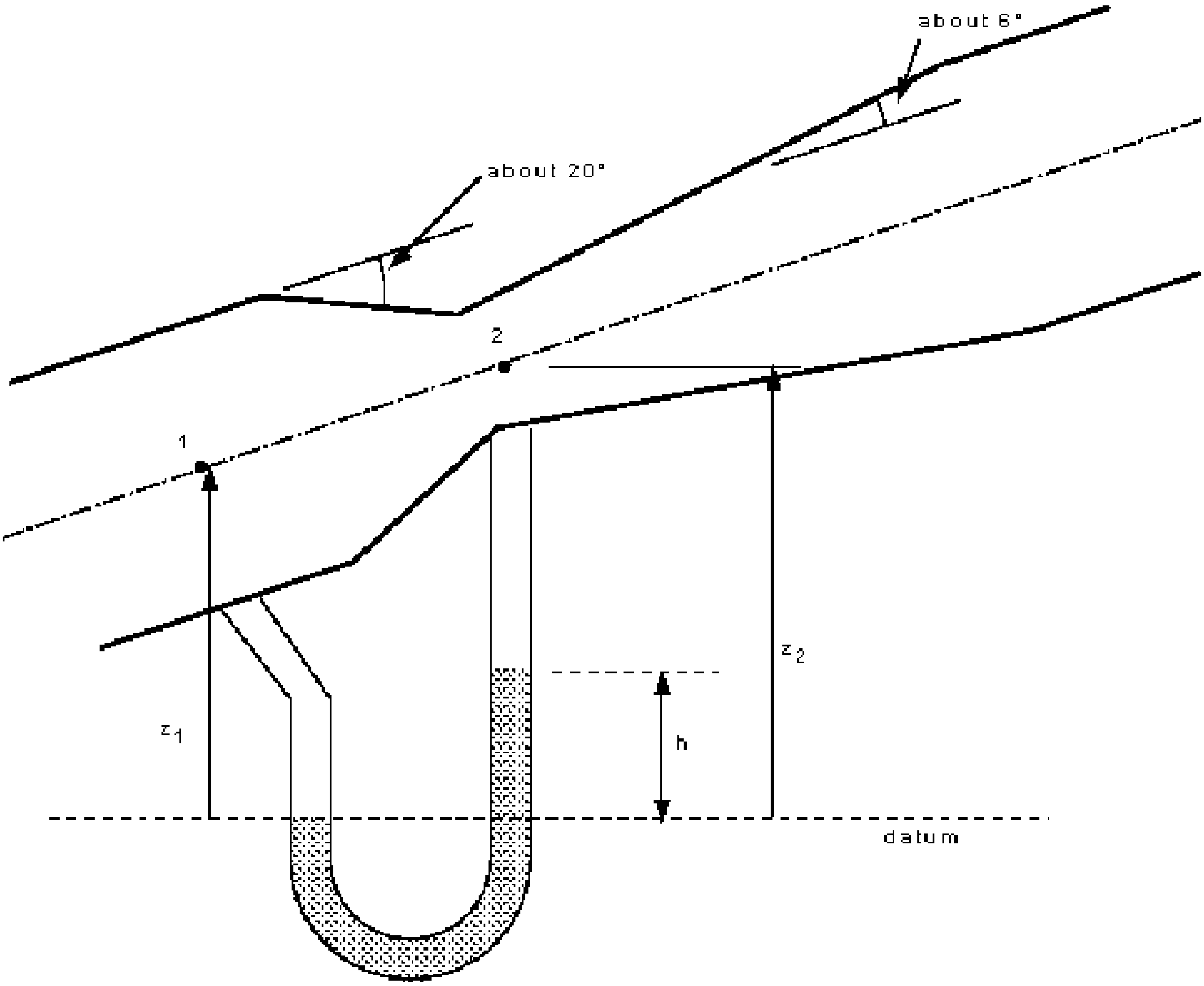
3.a Using continuity and the Bernoulli equation derive an expression which can be used to measure flow in a Venturi meter.

(15 marks)

3.b A Venturi meter is being used to measure flow in a pipeline of diameter $250\,mm$ which carries water. When the pressure difference between the throat and the entrance of the Venturi meter is $300\,mm$ on a mercury manometer, determine the flow in the pipeline.

The Venturi meter has a throat diameter of $80\,mm$ and a coefficient of discharge of 0.97. The relative density of mercury is 13.6.

(5 marks)



A Venturi meter

Applying Bernoulli along the streamline from point 1 to point 2 in the narrow *throat* of the Venturi meter we have

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

By the using the continuity equation we can eliminate the velocity u_2,

$$Q = u_1 A_1 = u_2 A_2$$

$$u_2 = \frac{u_1 A_1}{A_2}$$

Substituting this into and rearranging the Bernoulli equation we get

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = \frac{u_1^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$u_1 = \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$= A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 + A_2^2}}$$

To get the theoretical discharge this is multiplied by the area. To get the actual discharge taking in to account the losses due to friction, we include a coefficient of discharge

$$Q_{ideal} = u_1 A_1$$

$$Q_{actual} = C_d Q_{ideal} = C_d u_1 A_1$$

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}}$$

This can also be expressed in terms of the manometer readings

$$p_1 + \rho g z_1 = p_2 + \rho_{man} g h + \rho g (z_2 - h)$$

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

Thus the discharge can be expressed in terms of the manometer reading::

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}{A_1^2 - A_2^2}}$$

$A_1 = \pi\ 0.25^2 /4 = 0.049\ m^2$

$A_2 = \pi\ 0.08^2 /4 = 0.005\ m^2$

$h = 0.3\ m$

$\rho_{Hg} = \rho_{man} = 13\ 600\ kg/m^3$

$\rho_{H20} = \rho = 1000\ kg\ /m^3$

$Cd = 0.97$

$$\begin{aligned} Q_{actual} &= 0.97 \times 0.049 \times 0.005 \sqrt{\frac{2 \times 9.81 \times 0.3(13.6-1)}{0.049^2 - 0.005^2}} \\ &= 0.0002376 \sqrt{\frac{74.16}{0.002376}} \\ &= 0.042\ m^3/s \end{aligned}$$

Back To June 1997 Questions Page

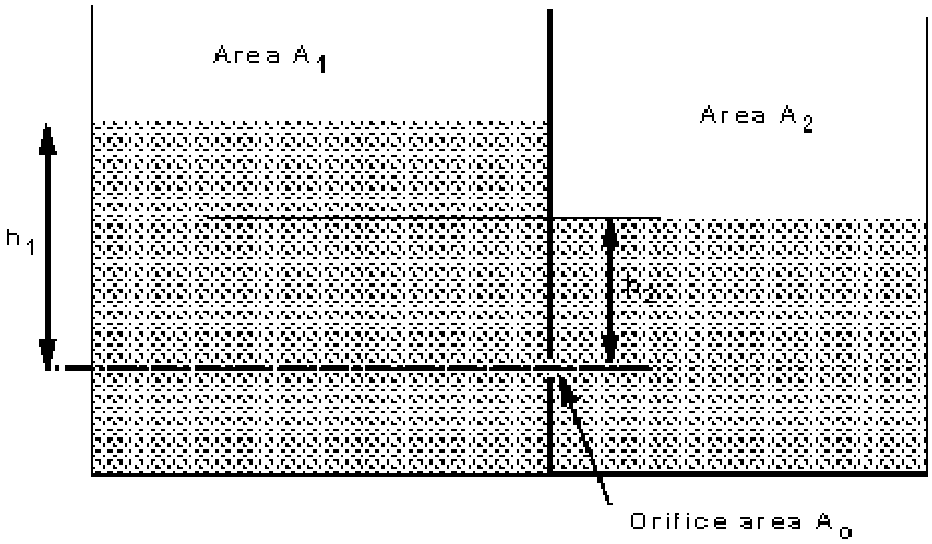
Back To June 1997 Questions Page

CIVE1400 Examination 1997. Answers

Question 4

4
Two vertical cylindrical tanks of 3m and 2m diameter containing water are joined at their bases by a pipe of diameter 0.05m. This pipe is short enough to be treated as an orifice with a coefficient of discharge of 0.58.
The $3m$ tank initially has a level $2m$ higher than the other, calculate how long it will take for the level difference to half.

(20 marks)



Two tanks of initially different levels joined by an orifice

Applying the continuity equation

$$Q = -A_1 \frac{\delta h_1}{\delta t} = A_2 \frac{\delta h_2}{\delta t}$$
$$Q\,\delta t = -A_1\,\delta h_1 = A_2\,\delta h_2$$

Also we can write $-\delta h_1 + \delta h_2 = \delta h$

So

$$-A_1\,\delta h_1 = A_2\,\delta h_1 - A_2\,\delta h$$
$$\delta h_1 = \frac{A_2\,\delta h}{A_1 + A_2}$$

Then we get

$$Q\,\delta t = -A_1\,\delta h_1$$
$$C_d A_o \sqrt{2g(h_1 - h_2)}\ \ \delta t = \frac{A_1 A_2}{A_1 + A_2}\ \delta h$$

Re arranging and integrating between the two levels we get

$$dt = \frac{A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \frac{dh}{\sqrt{h}}$$

$$t = \frac{A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \int_{h_{initial}}^{h_{final}} \frac{dh}{\sqrt{h}}$$

$$= \frac{2 A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \left[\sqrt{h} \right]_{h_{initial}}^{h_{final}}$$

$$= \frac{2 A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \left[\sqrt{h_{initial}} - \sqrt{h_{final}} \right]$$

h in this expression is the *difference* in height between the two levels ($h_2 - h_1$).

To get the time for the levels to equal use $h_{initial} = h_1$ and $h_{final} = 0$.

The question says $h_{initial} = 2$m and we want the time for this to half so, $h_{final} = 1$m

$A_1 = \pi\, 3^2 /4 = 7.069$ m^2
$A_2 = \pi\, 2^2 /4 = 3.142$ m^2
$A_o = \pi\, 0.05^2 /4 = 0.0019634$ m^2

$$t = \frac{2 A_1 A_2}{(A_1 + A_2) C_d A_o \sqrt{2g}} \left[\sqrt{h_{initial}} - \sqrt{h_{final}} \right]$$

$$= \frac{2 \times 7.069 \times 3.142}{(7.069 + 3.142) \times 0.58 \times 0.0019634 \sqrt{2g}} \left[\sqrt{2} - \sqrt{1} \right]$$

$$= \frac{44.42}{0.0514} 0.4142$$

$$= 357.95 \text{sec}$$

Back To June 1997 Questions Page
Back To June 1997 Questions Page

CIVE1400 Examination 1997. Answers

Question 5

5

In an experiment water is flowing over an 80° V-notch - Figure 2 - with a constant head of $0.3m$ into a vertical cylindrical tank of diameter $0.5m$.

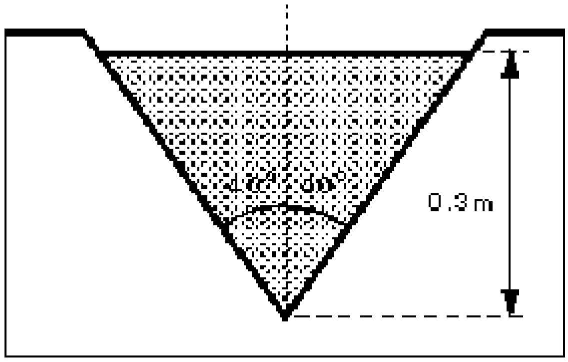


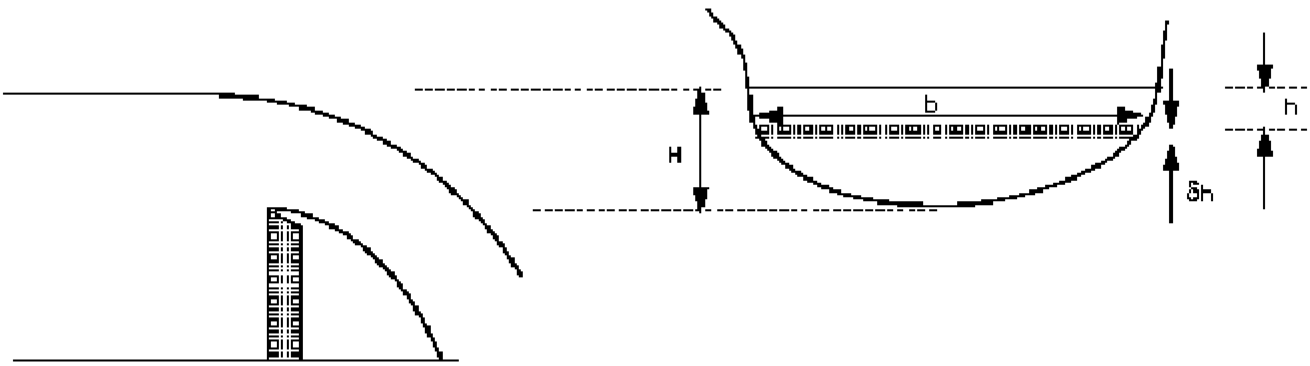
Figure 2

If the level in the tank rises $0.8m$ in 20 seconds, deriving all formulae, determine the coefficient of discharge of the notch.

(20 marks)

A General Weir Equation

Consider a horizontal strip of width *b* and depth *h* below the free surface, as shown in the figure below.



Elemental strip of flow through a notch

Assuming the velocity is only due to the head.

$$\text{velocity through the strip, } u = \sqrt{2gh}$$

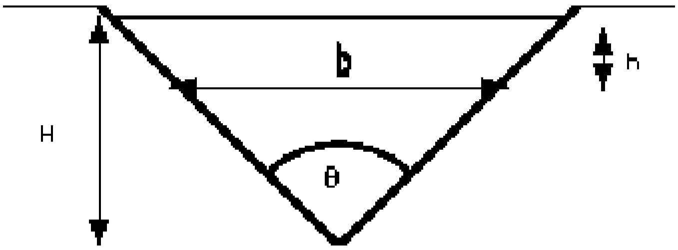
$$\text{discharge through the strip, } \delta Q = Au = b\,\delta h\sqrt{2gh}$$

Integrating from the free surface, $h = 0$, to the weir crest, $h = H$ gives the expression for the total theoretical discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H bh^{1/2}\,dh$$

This will be different for every differently shaped weir or notch. To make further use of this equation we need an expression relating the width of flow across the weir to the depth below the free surface.

For the "V" notch weir the relationship between width and depth is dependent on the angle of the "V".



"V" notch, or triangular, weir geometry.

If the angle of the "V" is θ then the width, *b*, a depth *h* from the free surface is

$$b = 2(H - h)\tan\left(\frac{\theta}{2}\right)$$

So the discharge is

$$\begin{aligned} Q_{\text{theoretical}} &= 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\int_0^H (H-h)h^{1/2}\,dh \\ &= 2\sqrt{2g}\tan\left(\frac{\theta}{2}\right)\left[\frac{2}{5}Hh^{3/2} - \frac{2}{5}h^{5/2}\right]_0^H \\ &= \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2} \end{aligned}$$

The actual discharge is obtained by introducing a coefficient of discharge

$$Q_{\text{actual}} = C_d \frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}$$

From the question:

$Q = A_{tank}\, h\,/\text{time} = (\pi\, 0.5^2 / 4) \times 0.8/20 = 0.00785\ m^3/s$

$\theta = 80°$

Head = H = 0.3

Re arranging the weir equation, and substituting in these values gives

$$\begin{aligned} C_d &= \frac{Q}{\frac{8}{15}\sqrt{2g}\tan\left(\frac{\theta}{2}\right)H^{5/2}} \\ &= \frac{0.00785}{0.533 \times 4.429 \times \tan 40 \times 0.3^{5/2}} \\ &= 0.08 \end{aligned}$$

Back To June 1997 Questions Page

Back To June 1997 Questions Page

CIVE1400 Examination 1997. Answers
Question 6

6
An emergency relief outlet from an ornamental pond in a public garden consists of a 2m wide sluice leading into a dry channel. This outlet is controlled by a sector gate of 1.5m radius as shown in Figure 3.

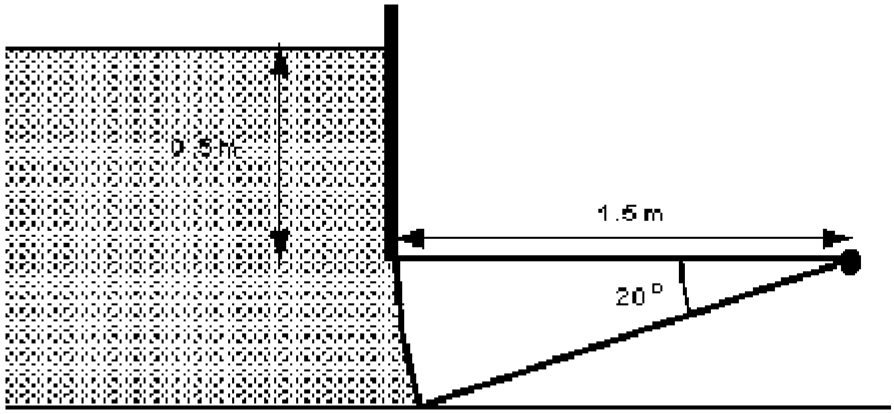


Figure 3.

Determine the resultant force on the gate and the angle that this makes to the horizontal.

(20 marks)

h = 1.5 sin 20 = 0.513 m
b = 1.5 cos 20 = 1.4095 m
a = 1.5 - b = 0.09046 m

Horizontal force

$$R_h = \text{pressure on a projection of curved surface on to a vertical wall}$$
$$= \text{pressure at centroid of vertical projection} \times \text{Area}$$
$$= \rho\, g\, h' \times A$$

$$h' = 0.5 + h/2 = 0.7565 \text{ m}$$

$$R_h = 1000 \times 9.81 \times 0.7565 \times 0.513 \times 2$$
$$= 7614.2 \text{ N}$$

Vertical force

$$R_v = \text{weight of imaginary fluid in volume ABC}$$
$$\text{Area of sector} = \pi \times 1.5^2 \times 20/360 = 0.3927 \text{ m}^2$$

Area of triangle = b h /2 = 0.513 × 1.4095 × 0.5 = 0.3615 m^2
Area of rectangle = 0.5 × a = 0.5 × 0.09064 = 0.04523 m^2
$R_v = \rho g (0.3927 - 0.3615 + 0.04523) \times 2$
$= 1500$ N

And the resultant force on the fluid is given by

$$R = \sqrt{R_h^2 - R_v^2} = 7760\ N$$

And the direction of application is

$$\phi = \tan^{-1}\left(\frac{R_v}{R_h}\right) = 11.14^\circ$$

Back To June 1997 Questions Page

Back To June 1997 Questions Page

CIVE1400 Examination 1997. Answers

Question 7

7
Describe the following phenomenon and explain why they occur:

Read any fluid mechanics book to find the text for these answers.

a. The boundary layer;
(5 marks)
b. Boundary layer separation;
(5 marks)
c. Boundary layer separation at a T-junction;
(5 marks)
d. The laminar sub-layer.
(5 marks)

Back To June 1997 Questions Page

Back To June 1998 Questions Page

1. A rectangular sluice gate is fitted at the base of a reservoir wall with a pivot in the arrangement shown in Figure 1. The gate is designed to regulate the level of water in the reservoir by opening when the water level to the right, h, reaches a certain depth. The gate has a width of 1.2m and its centre of gravity is 0.3m from the wall.

   Determine the weight, W, of the gate, if a water level of h = 2.779m will just cause the gate to open.

[The second moment of area of a rectangle about an axis through its centre is $bd^3/12$]

(20 marks)

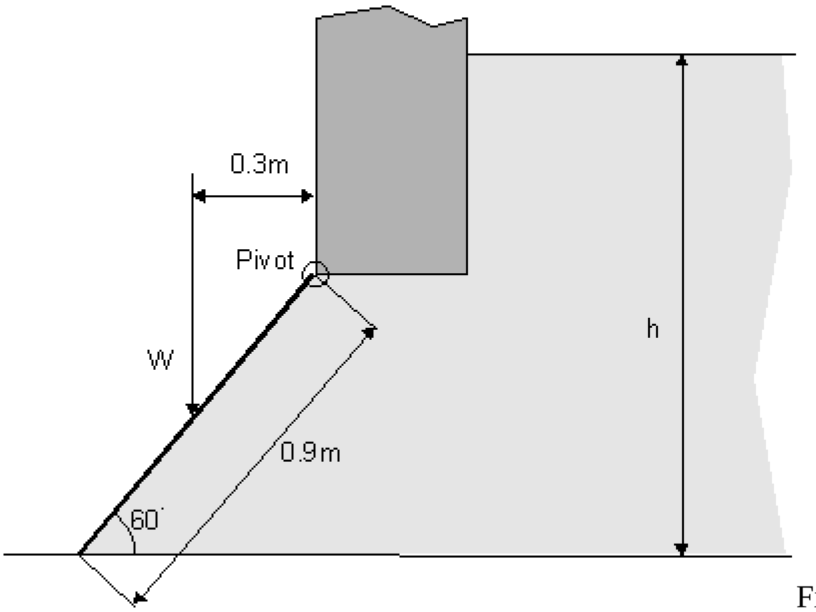


Figure 1

$$L = 0.9m$$
$$x = 0.9\cos 60 = 0.45m$$
$$y = 0.9\sin 60 = 0.779m$$

The gate opens when the moment at the pivot is clockwise.

That is when the moment due to the water > 0.3w.

<u>Method 1</u>

Force on plane = Area × Pressure at centroid

$$\begin{aligned}\text{Force} &= (1.2\times 0.9)\times\left(2.779-\frac{y}{2}\right)\rho g \\ &= 1.08\times(2.779-0.779/2)1000\times 9.81 \\ &= 25316N\end{aligned}$$

<u>Method 2</u>

Horizontal force = Hf = Area of projection on vertical plane × pressure at centroid

$$H_f = 1.2y\left(2.779 - \frac{y}{2}\right)\rho g$$
$$= 21912.6\,N$$

Vertical force = weight of water above gate

$$V_f = \left(2.779x - \frac{x \times y}{2}\right)\rho g 1.2$$
$$= (12268 - 1719)1.2$$
$$= 12658$$

Resultant force

$$R = \sqrt{{H_f}^2 + {V_f}^2}$$
$$= 25306\,N$$

Method 3

$$\text{Force} = \text{Shaded area}$$
$$= \left(2.779 \times 0.9 - \frac{0.779 \times 0.9}{2}\right)1.2\rho g$$
$$= 2.15055 \times 1.2\rho g$$
$$= 25316\,N$$

Point of action of the force = centre of pressure

$$x_2 = \frac{2}{\tan 60} = 3.444m$$
$$L_2 = \frac{2}{\sin 60} = 2.309m$$
$$\bar{x} = L_2 + \frac{0.9}{2} = 2.759m$$

$$Sc = \frac{\text{2nd moment of area about O}}{\text{1st moment of area about O}} = \frac{I_{oo}}{A\bar{x}}$$
$$I_{oo} = I_{GG} + A\bar{x}^2$$
$$Sc = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

For a rectangle, $I_{GG} = \frac{bd^3}{12}$

$$
\begin{aligned}
Sc &= \frac{d^2}{12\bar{x}} + \bar{x} \\
&= \frac{0.9^2}{12 \times 2.759} + 2.759 \\
&= 2.783m
\end{aligned}
$$

Need to find the lever arm, i.e. the distance from the pivot to the centre of pressure specified by Sc.

First find the position of the pivot, x_2, from the surface (along the inclined plane)

$$x_2 \cos 30 = 2$$

$$x_2 = 2.309\text{m}$$

Lever are, $x_1 = Sc - x_2 = 2.783 - 2.309 = 0.474$ m

Take moments to fine the weight of the gate, w

$$
\begin{aligned}
Rx_1 &= 0.3w \\
w &= \frac{25316 \times 0.474}{0.3} = 40000\,N
\end{aligned}
$$

Back To June 1998 Questions Page

Back To June 1998 Questions Page

CIVE1400 Examination 1998. Answers

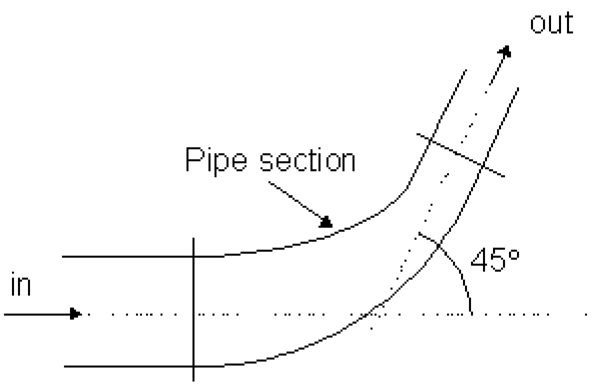
Question 2

2. A pipeline carries water around a horizontal 45° bend. The entrance diameter of the bend is 500mm and the velocity of flow is 1 m/s. The bend tapers gradually to 200 mm diam0eter at its exit. If the pressure just upstream of the entrance is measured at 200kN/m^2 , what is the force, and its line of action, exerted by the water on the bend?

(17 marks)

Comment on the reason why frictional losses may be neglected in this analysis.

(3 marks)



$A_1 = \pi d_1^2 / 4 = 0.1963\ m^2$

$A_2 = \pi d_2^2 / 4 = 0.0314\ m^2$

$u_1 = Q/A_1$;

$Q = u_1 A_1 = 1.0 \times 0.1963 = 0.1963\ m^3/s$

$u_2 = Q/A_2 = 0.1963/0.0314 = 6.248\ m/s$

$p_1 = 200\ kN/m^2 = 200\ 000\ N/m^2$

<u>Calculate the total force</u>

In the x-direction:

$$\begin{aligned}
F_{Tx} &= \rho Q(u_{2x} - u_{1x}) \\
u_{1x} &= u_1 \\
u_{2x} &= u_2 \cos\theta \\
F_{Tx} &= \rho Q(u_2 \cos\theta - u_1) \\
&= 1000 \times 0.1963(6.248 \cos 45 - 1.0) \\
&= 670.95N
\end{aligned}$$

In the y-direction:

$$\begin{aligned}
F_{Ty} &= \rho Q(u_{2y} - u_{1y}) \\
u_{1y} &= u_1 \sin 0 = 0 \\
u_{2y} &= u_2 \sin\theta \\
F_{Ty} &= \rho Q u_2 \sin\theta \\
&= 1000 \times 0.1963 \times 6.248 \sin 45 \\
&= 867.25N
\end{aligned}$$

<u>Calculate the pressure force</u>

Use Bernoulli to calculate force at exit, p_2

$$\frac{p_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{u_2^2}{2g}+z_2+h_f$$

the friction loss h_f can be ignored, $h_f=0$

As the pipe is in the horizontal plane, $z_1=z_2$

By continuity, $Q= u_1A_1 = u_2A_2$

$$\begin{aligned}p_2 &= p_1-\frac{\rho}{2}\left(u_2^2-u_1^2\right)\\ &= 200000-\frac{1000}{2}\left(6.248^2-1.0^2\right)\\ &= 200000-19018\\ &= 180980N\end{aligned}$$

$$F_p = \text{pressure force at 1 - pressure force at 2}$$

$$\begin{aligned}F_{p_x} &= p_1A_1\cos 0-p_2A_2\cos\theta = p_1A_1-p_2A_2\cos\theta\\ &= 200000\times 0.1963-180980\times 0.0314\cos 45\\ &= 39260-4018\\ &= 352412N\\ F_{p_y} &= p_1A_1\sin 0-p_2A_2\sin\theta = -p_2A_2\sin\theta\\ &= -180980\times 0.0314\sin 45\\ &= -4018N\end{aligned}$$

Calculate the body force

There are no body forces as the pipe is in the horizontal plane.

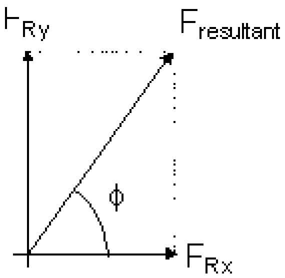
Calculate the resultant force

$$\begin{aligned}F_{T_x} &= F_{R_x}+F_{P_x}+F_{B_x}\\ F_{T_y} &= F_{R_y}+F_{P_y}+F_{B_y}\end{aligned}$$

$$
\begin{aligned}
F_{Rx} &= F_{Tx} - F_{Px} - 0 \\
&= \rho Q(u_2 \cos\theta - u_1) - p_1 A_1 + p_2 A_2 \cos\theta \\
&= 1000 \times 0.1963(6.248\cos 45 - 1.0) - 200000 \times 0.1963 + 180980 \times 0.0314 \times \cos 45 \\
&= 670 - 35242 \\
&= -34572 N
\end{aligned}
$$

$$
\begin{aligned}
F_{Ry} &= F_{Ty} - F_{Py} - 0 \\
&= \rho Q u_2 \sin\theta + p_2 A_2 \sin\theta \\
&= 1000 \times 0.1963 \times 6.248 \times \sin 45 + 180980 \times 0.0314 \sin 45 \\
&= 867 + 4018 \\
&= 4885 N
\end{aligned}
$$

And the resultant force on the fluid is given by



$$
F_R = \sqrt{F_{Rx}^2 - F_{Ry}^2} = 34815 N
$$

And the direction of application is

$$
\begin{aligned}
\phi &= \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{4885}{-34572}\right) \\
&= \tan^{-1}(-0.1413) \\
\phi &= -8.04^{\circ}
\end{aligned}
$$

the force on the bend is the same magnitude but in the opposite direction

$$
R = -F_R
$$

Friction can be ignored in the analysis because the pipe length is very short frictional effects it will be tiny in comparison to the other forces. Other losses are minimised as the bed is converging.

Back To June 1998 Questions Page

Back To June 1998 Questions Page

CIVE1400 Examination 1998. Answers

Question 3

3.a The expression below calculates the discharge in a pipeline from measurement of the pressure at the tapping points of a Venturimeter.

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}}$$

Show that if the pressure is measured using a manometer, as shown in Figure 2, then the inclination of the metre is not relevant.

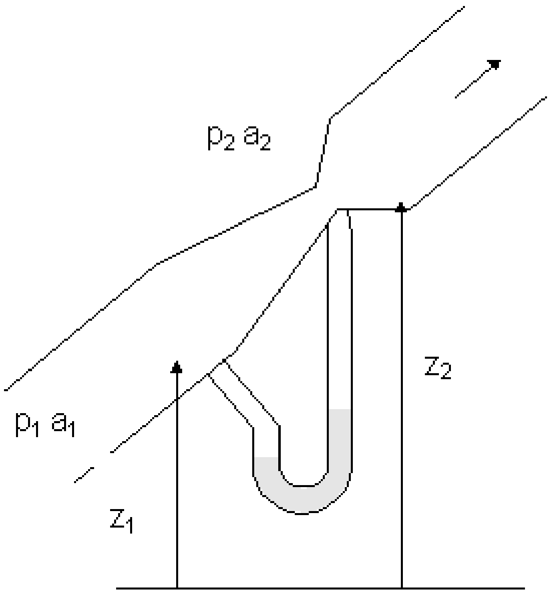


Figure 2

(15 marks)

3.b A Venturimeter with a throat diameter of 40mm is connected to a 100mm pipeline. When the flow in the pipeline is 12 litres/s the difference in pressure on a mercury manometer is 375mm. What is the coefficient of discharge at this flow?

(The specific gravity of mercury is 13.6).

(5 marks)

3.a

This can be expressed in terms of the manometer readings (see figure below)

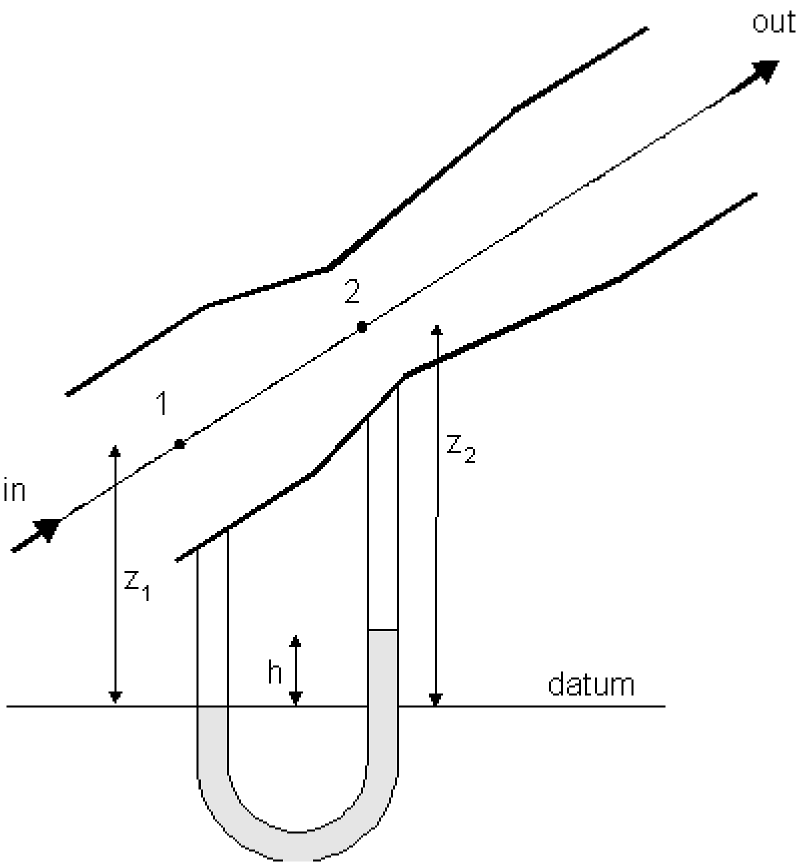
$$p_1 + \rho g z_1 = p_2 + \rho_{man} g h + \rho g (z_2 - h)$$

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

Thus the discharge can be expressed in terms of the manometer reading::

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}{A_1^2 - A_2^2}}$$

There are no terms that require the angle or height on the Venturi meter in this expression.



3.b

d_1 = 100mm = 0.1 m

$A_1 = \pi\ 0.1^2 /4 = 0.00785\ m^2$

d_2 = 40mm = 0.04 m

$A_2 = \pi\ 0.04^2 /4 = 0.001257\ m^2$

h = 375mm = 0.375 m

$\rho_{Hg} = \rho_{man}$ = 13 600 kg/m^3

$\rho_{H20} = \rho$ = 1000 kg /m^3

Q = 10 litres/s = 0.01 m^3/s

Cd = ?

$$0.01 = Cd \times 0.00785 \times 0.001257 \sqrt{\frac{2 \times 9.81 \times 0.375(13.6 - 1)}{0.00785^2 - 0.01257^2}}$$

$$0.01 = Cd\, 0.01195$$

$$Cd = 0.84$$

4.a Starting from the Bernoulli equation develop a relationship between the flow over a sharp edged rectangular weir and the depth of water above the base of the weir. State any assumptions made.

(15 marks)

4.b A sharp edged rectangular weir has a width of 400mm and a coefficient of discharge of 0.65. What is the flow of water when the upstream depth above the base of the weir is 250mm.

(5 marks)

Back To June 1998 Questions Page

Back To June 1998 Questions Page

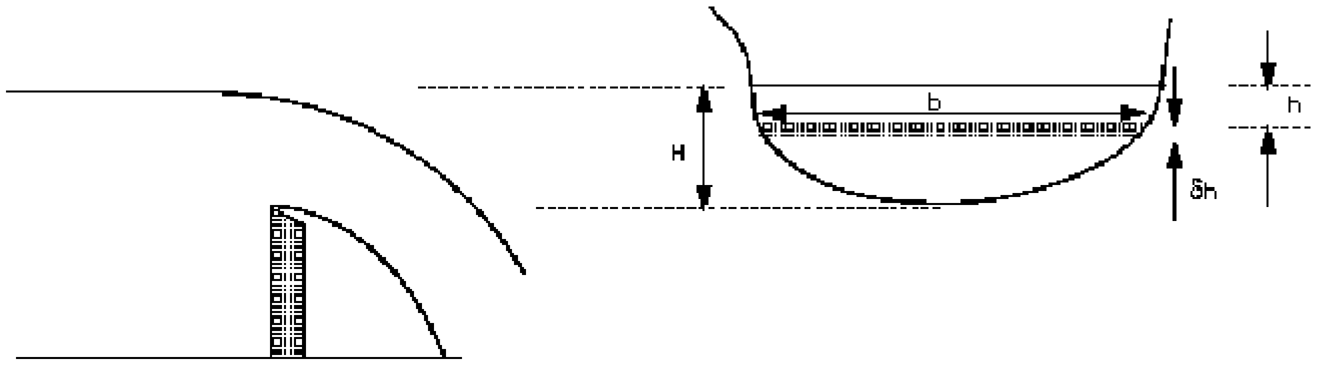
CIVE1400 Examination 1998. Answers

Question 4

4.a

A General Weir Equation

Consider a horizontal strip of width *b* and depth *h* below the free surface, as shown in the figure below.



Elemental strip of flow through a notch

Assuming the velocity is only due to the head.

$$\text{velocity through the strip, } u = \sqrt{2gh}$$
$$\text{discharge through the strip, } \delta Q = Au = b\,\delta h\sqrt{2gh}$$

Integrating from the free surface, $h = 0$, to the weir crest, $h = H$ gives the expression for the total theoretical discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H bh^{1/2}\,dh$$

This will be different for every differently shaped weir or notch. To make further use of this equation we need an expression relating the width of flow across the weir to the depth below the free surface.

For a rectangular weir b = B = constant

The actual discharge is obtained by introducing a coefficient of discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H Bh^{1/2}\,dh$$
$$Q_{actual} = Cd\,\frac{2}{3}\sqrt{2g}\,H^{3/2}$$

$$Q = 0.65\times 0.4\times\frac{2}{3}\times\sqrt{19.62}\times 0.25^{3/2}$$
$$= 0.096\ m/s$$

Back To June 1998 Questions Page

Back To June 1998 Questions Page

CIVE1400 Examination 1998. Answers

Question 5

5. Explain, with a complete description of the mechanisms at work, what is meant by the following phrases.

a. Laminar flow

(5 marks)

b. Turbulent flow

(5 marks)

c. Boundary layer

(5 marks)

d. Boundary layer separation

(5 marks)

These require book work.

A minimum of two pages of descriptions will be required.

Answers with full explanations of the processes and how they interact (with diagrams) will get good marks.

Back To June 1998 Questions Page

Back To June 1998 Questions Page

CIVE1400 Examination 1998. Answers

Question 6

6.a Describe some uses for dimensional analysis. Your explanation should include the meanings and relevance of the terms *geometric similarity, dynamic similarity* and *kinematic similarity*.

(8 marks)

6.b The drag force, F, exerted on a body in a moving fluid can be said to be a function of the following parameters

fluid density ρ

fluid viscosity μ

diameter d

velocity u

Show that an expression for the drag force is

$$F = d^2 u^2 \rho \, \phi(\mathrm{Re})$$

where ϕ is some unknown function and Re is the Reynolds number.

(12 marks)

6.a

Book work - requires at least 5 lines per description.

One line answers are NOT sufficient.

6.b

$$F = f(\rho, \mu, d, u)$$
$$0 = \phi(F, \rho, \mu, d, u)$$

Assume the governing variables ρ , *u*, *d*

According to Buckingham's π theorem there are n-m groups

where

n = number of variables (5) and

m = number of dimensions (i.e. MLT, giving 3)

n-m = 5-3 = 2 groups

$$0 = \phi(\pi_1, \pi_2)$$
$$\pi_1 = \rho^{a_1} u^{b_1} d^{c_1} \mu$$
$$\pi_2 = \rho^{a_2} u^{b_2} d^{c_2} F$$

Dimensions of the variables are:

ρ = density (kg/m^3) = ML^{-3}

μ = viscosity (kg/m/s) = ML^{-1} T^{-1}

u = velocity (m/s) = ML^{-1}

d = length (m) = L

F = newtons (kg m /s^2) = MLT^{-2}

For π_1

$$0 = \left(ML^{-3}\right)^{a_1}\left(LT^{-1}\right)^{b_1}\left(L\right)^{c_1} ML^{-1}T^{-1}$$
$$0 = a_1 + 1$$
$$0 = -3a_1 + b_1 + c_1 - 1$$
$$0 = -b_1 + 1$$
$$a_1 = -1$$
$$b_1 = -1$$
$$c_1 = -1$$
$$\pi_1 = \frac{\mu}{\rho u d}$$

For π_2

$$0 = \left(ML^{-3}\right)^{a_2}\left(LT^{-1}\right)^{b_2}\left(L\right)^{c_2} MLT^{-2}$$
$$0 = a_2 + 1$$
$$0 = -3a_2 + b_2 + c_2 + 1$$
$$0 = -b_2 - 2$$
$$a_2 = -1$$
$$b_2 = -2$$
$$c_2 = -2$$
$$\pi_2 = \frac{F}{\rho u^2 d^2}$$

$$0 = \phi(\pi_1, \pi_2)$$
$$0 = \phi\left(\frac{u}{\rho u d}, \frac{F}{\rho u^2 d^2}\right)$$

Inverting π_1 gives $\mathrm{Re} = \dfrac{\rho u d}{\mu}$

$$0 = \phi\left(\mathrm{Re}, \frac{F}{\rho u^2 d^2}\right)$$

Rearranging this gives $F = \rho u^2 d^2 \phi(\mathrm{Re})$

BacBack To June 1998 Questions Page

CIVE1400 Examination 1998. Answers

Question 7

7.a Define the following terms in connection with the flow of a liquid

i. Uniform flow
ii. Steady flow
iii. Unsteady flow
iv. Mean velocity
v. Discharge
vi. Mass flow rate
vii. Continuity

(14 marks)

7.b Oil flows in a pipe which contracts from 450mm diameter at point A to 300mm at point B then splits into to branches of diameters 150mm and 225mm discharging at C and D respectively.

If the velocity at A is 1.8m/s and the velocity at D is 3.6m/s, what is the discharge at C and D and the velocity at B and C?

(3 marks)

7.c If point A is 10m higher than point B and the pressure at A is $10kN/m^2$, what is the pressure at point B?

(3 marks)

7.a

A minimum of 3 lines per description is required - preferably more with simple examples. Four or five words is NOT sufficient and you will get no marks.

7.b

$v_A = 1.8$ m/s

$A_A = \pi\, 0.45^2 / 4 = 0.159$ m^2

$v_D = 3.6$ m/s

$A_D = \pi\, 0.225^2 / 4 = 0.0398$ m^2

$Q_A = v_A\, A_A = 0.286$ m^3/s

$A_B = \pi\, 0.3^2 / 4 = 0.0707$ m^2

$A_C = \pi\, 0.15^2 / 4 = 0.0177$ m^2

$Q_A = Q_B = 0.286 = A_B\, v_B$

$v_B = 0.286 / 0.0707 = 4.0545$ m/s

$Q_D = A_D\, v_D = 3.6 \times 0.398 = 0.1439$ m^3/s

$Q_C = Q_A - Q_D = 0.1427$ m^3/s

$v_C = Q_C / A_C = 0.14272 / 0.0177 = 8.063$ m/s

7.c

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$

$$\frac{p_A}{\rho g} + \frac{v_A^2 - v_B^2}{2g} + z_A - z_B = \frac{p_B}{\rho g}$$

$$\left(\frac{10000}{9810} + \frac{1.8^2 - 4.045^2}{19.62} + 10\right) 9810 = p_B$$

$$p_B = 101539\, N/m^2$$

Back To June 1998 Questions Page

k To June 1998 Questions Page

Back To June 1999 Questions Page

1.a A concrete dam has the cross-sectional profile shown in Figure 1. Calculate the magnitude, direction and position of action of the resultant force exerted by the water per unit width of dam?

(15 marks)

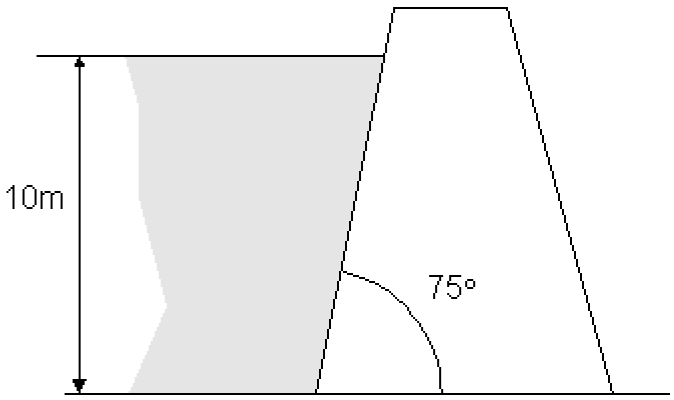


Figure 1

1.b A second design for the same dam has the cross-sectional profile composed of a vertical face with a circular curved section at the base as shown in Figure 2. Calculate the resultant force and its direction of application per unit width of this dam.

(10 marks)

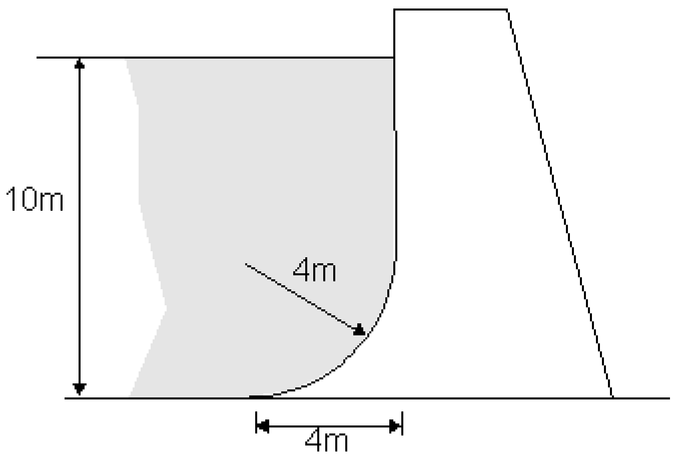


Figure 2

1.a.

Method 1

Vertical force = weight of water = ρ g A b

Horizontal force = force on a projection of the vertical plane

$$= \rho g h \frac{h}{2} b$$

$$L = h \tan \theta = 10 \tan 15 = 2.679m$$
$$A = 0.5hL = 0.5 \times 10 \times 2.679 = 13.397m^2$$

$$Rv = 1000 \times 9.81 \times 13.397 \times 1 = 131425N$$
$$Rh = 1000 \times 9.81 \times 10 \times (10/2) \times 1 = 490500N$$

$$R = \sqrt{Rv^2 + Rh^2} = 507802N$$

Acting at right angle to the wall 15° to the horizontal. Also $\tan^{-1}\left(Rv / Rh\right) = \phi = 15°$

<u>Method 2</u>

Force on wall = pressure at centroid × area of wall

= ρ g × depth to centroid × area of wall

Sloping wall length, $v = h / \cos 15 = 10.35m$

$$F = 1000 \times 9.81 \times (10.35 \times 1) \times 5$$
$$= 507668N$$

Position of this force is through the centre of pressure, Sc.

Using the parallel axis theorem,

$$Sc = \frac{I_{oo}}{A\bar{x}} = \frac{\text{2nd momnt of area}}{\text{1st moment of area}}$$
$$I_{oo} = I_{GG} - A\bar{x}^2$$
$$Sc = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

$\bar{x}$ is the distance along the face to the centroid = v/2 = 5.175m

$$I_{GG} = \frac{bd^3}{12} = \frac{1 \times 10.35^2}{12} = 92.39$$

$$Sc = \frac{92.39}{10.35 \times 1 \times (10.35 \times 0.5)} + (10.35 \times 0.5)$$
$$= 6.9m$$

This is the distance to the centre of pressure from O.

1.b.

$$b = 1m$$
$$a_1 = 4 \times 6 = 24m^2$$
$$a_2 = \frac{\pi 4^2}{2} = 12.566m^2$$

Vertical force

$$Rv = \text{weight of water}$$
$$= \rho g(a_1 + a_2)b$$
$$= 1000 \times 9.81 \times (24 + 12.566) \times 1$$
$$= 358712N$$

Horizontal force = force on the projection of vertical plan.

This is the same as in part a of this question.

$$Rh = 490500N$$

Resultant force

$$R = \sqrt{Rv^2 + Rh^2} = 607671N$$
$$\tan \phi = \frac{Rv}{Rh}$$
$$\phi = \tan^{-1}\left(\frac{358712}{490500}\right) = 36.178°$$

Back To June 1999 Questions Page

Back To June 1999 Questions Page

CIVE1400 Examination 1999. Answers

Question 2

2.a A differential "U"-tube manometer containing mercury of density $13000 kg/m^3$ is used to measure the pressure drop along a horizontal pipe. If the fluid in the pipe is water and the manometer reading is 0.65*m*, what is the pressure difference between the two tapping points?

(8 marks)

2.b A square tank of side length $1.5m$ is filled with oil to a depth of 1.8*m*. If the density of the oil is $800 kg/m^3$, find the resultant force and its point of action on one wall.

(6 marks)

2.c A jet of water of diameter 0.2*m* is being fired horizontally at a vertical wall. If the velocity of the jet is 20.0*m/s* estimate the force exerted on the wall.

(6 marks)

2.d Water is flowing from a horizontal nozzle into the atmosphere at 20*m/s*. The diameter of the pipe feeding the nozzle is 80*mm* and the nozle has a diameter of 20*mm* at its exit. Find the pressure just upstream of the nozzle.

(5 marks)

2.a

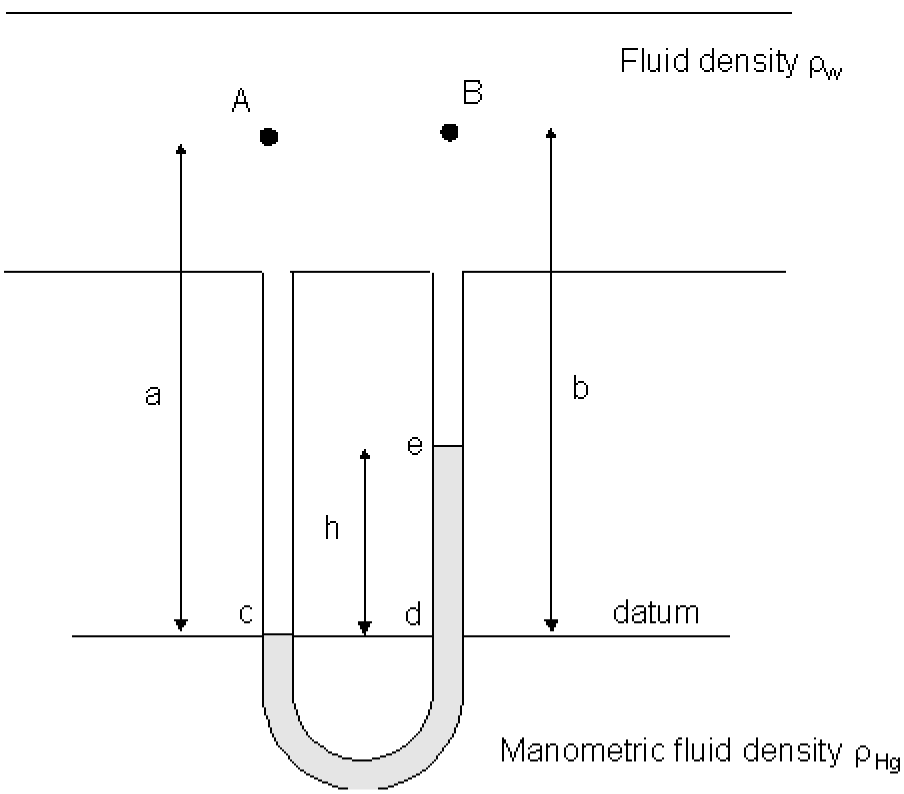


Figure of manometer setup

density of mercury $\rho = 13000\ kg/m^3$

pressure at C and D is equal:

$p_C = p_D$

$p_A + \rho_w\, g\, a = p_B + \rho_w\, g\, (b - h) + \rho_{Hg}\, g\, h$

$p_A - p_B = \rho_w\, g\, b - \rho_w\, g\, h - \rho_w\, g\, a + \rho_{Hg}\, g\, h$

$= \rho_w\, g\, (b - a) + hg\, (\rho_{Hg} - \rho_w)$

As horizontal a = b

$p_A - p_B = hg\, (\rho_{Hg} - \rho_w)$

$= 0.65 \times 9.81 \times (13000 - 1000)$

$= 76\ 518\ N/m^2$

$= 76.5\ kN/m^2$

2.b

Force per unit width R,

$R = \rho\, g\, H \times H \times 0.5$

$= 800 \times 9.81 \times 1.8^2\ 0.5$

$= 12\ 713\ N/m$

Total force on wall = 12713 × 1.5 = 19070 N

Point of action , normal to wall through centroid of pressure diagram. 2/3 from surface.

Distance from surface to point of action of resultant = 2H/3 = 1.2 m

2.c

Area of the jet

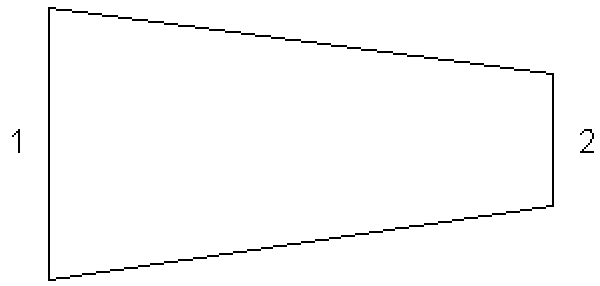
$$A = \frac{\pi d^2}{4} = \frac{3.14159 \times 0.2^2}{4} = 0.0314 m^2$$

Force on the water:

$$
\begin{aligned}
F &= \dot{m}(u_2 - u_1) \\
&= \rho Q(u_2 - u_1) \\
&= \rho A u_1(u_2 - u_1) \\
&= 1000 \times 0.0314 \times 20.0(0 - 20.0) \\
&= -12560N (\text{in the opposite direction to the jet})
\end{aligned}
$$

Force on the wall = R = -F = 12560 N (in the direction of the jet)

2.d



u_1 = unknown

d_1 = 0.08m

u_2 = 20 m/s

d_2 = 0.02m

Use continuity to calculate the unknow velocity

$$
\begin{aligned}
A_1 u_1 &= A_2 u_2 \\
\frac{\pi d_1^2}{4} u_1 &= \frac{\pi d_2^2}{4} u_2 \\
u_2 &= u_1 \left(\frac{d_1}{d_2} \right)^2 \\
&= 20 \left(\frac{0.02}{0.08} \right)^2 = 1.25 m/s
\end{aligned}
$$

Use Bernoulli to calculate the unknown pressure

$$
\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2
$$

As horizontal $z_1 = z_2$

p_2 = atmospheric = 0.0

$$p_1 + \rho\frac{u_1^2}{2} = p_2 + \rho\frac{u_2^2}{2}$$

$$p_1 = \frac{\rho}{2}\left(u_2^2 - u_1^2\right)$$

$$= \frac{1000}{2}\left(20^2 - 1.25^2\right)$$

$$= 199218\ N/m^2$$

$$= 199.2\ kN/m^2$$

Back To June 1999 Questions Page

Back To June 1999 Questions Page

CIVE1400 Examination 1999. Answers

Question 3

3 A 5*m* wide tank with an L-shaped cross section, as shown in Figure 3, has a gate which is hinged at the top at it right hand end. If the tank is filled with water to a level of 8*m* determine the torque required at the hinge to just keep the gate closed.

(20 marks)

Determine also the force on the base of the tank and comment on why this is not the same as the weight of the water.

(5 marks)

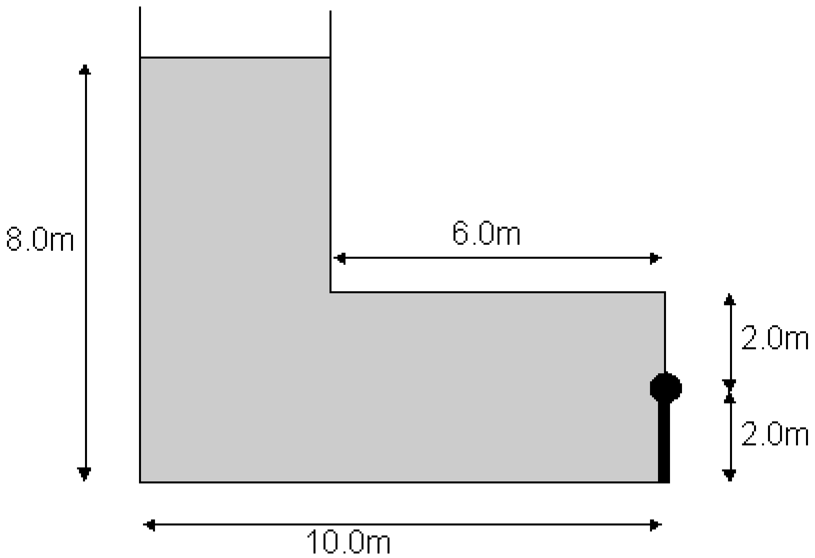
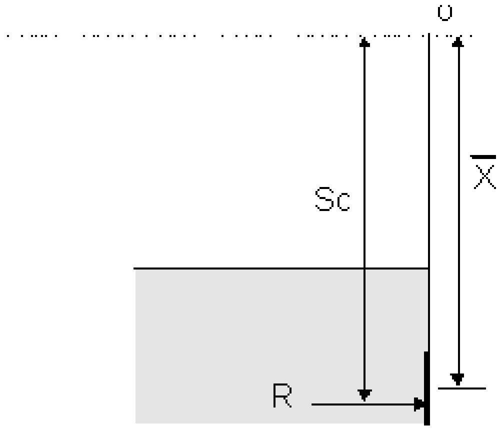


Figure 3

$$
\begin{aligned}
\text{Force on gate} &= \text{pressure at centroid} \times \text{area of gate} \\
&= \rho g(6+1)\times(2\times 5) \\
&= 9810\times 7\times 10 \\
&= 686700\ N
\end{aligned}
$$

Position of action of force = Sc from the point O

The point O is the point where a line from the gate crosses the top water surface level



Position of this force is through the centre of pressure, Sc.

Using the parallel axis theorem,

$$
\begin{aligned}
Sc &= \frac{I_{oo}}{A\bar{x}} = \frac{\text{2nd momnt of area}}{\text{1st moment of area}} \\
I_{oo} &= I_{GG} - A\bar{x}^2 \\
Sc &= \frac{I_{GG}}{A\bar{x}} + \bar{x}
\end{aligned}
$$

$\bar{x}$ is the distance along the face to the centroid

$$
I_{GG} = \frac{bd^3}{12} = \frac{5\times 2^2}{12} = 3.333
$$

$$
Sc = \frac{bd^3}{12}\frac{1}{bd}\frac{1}{7} + 7 = \frac{4}{84} + 7 = 7.0476m
$$

Torque required = $(7.0476-6)\times 686700 = 719400$ Nm clockwise

Force on base = PA = pghA = 3924 kN

Weight of water = (10×8×5 - 4×6×5)×9810 = 2746800 kN

Back To June 1999 Questions Page

Back To June 1999 Questions Page

CIVE1400 Examination 1999. Answers

Question 4

4 Water flows at a rate of 1.0m^3/s round a 45° contracting pipe bend which lies in a horizontal plane. The diameter at the bend entrance is 800*mm* and at the exit 400*mm* - as shown in Figure 4.

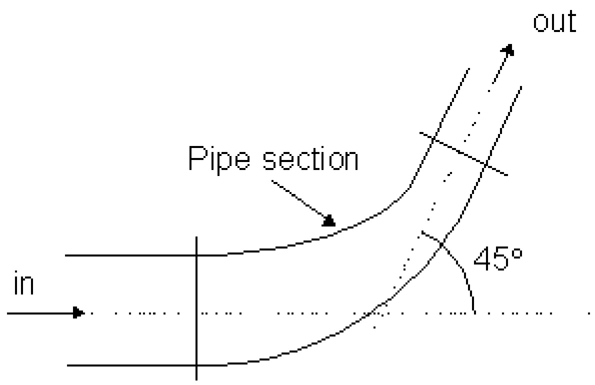


Figure 4.

If the pressure at the entrance to the bend is100 kN/m^2, what is the magnitude and direction of the force exerted by the fluid on the bend?

(22 marks)

Comment on the reason why frictional losses may be neglected in this analysis.

(3 marks)

$A_1 = \pi d_1^2 / 4 = 0.5027$ m^2

$A_2 = \pi d_2^2 / 4 = 0.1257$ m^2

$u_1 = Q/A_1 = 1.0/0.5027 = 1.989$ m/s

$u_2 = Q/A_2 = 1.0/0.1257 = 7.955$ m/s

$p_1 = 100$ kN/m^2 = 100 000 N/m^2

<u>Calculate the total force</u>

In the x-direction:

$$
\begin{aligned}
F_{T_x} &= \rho Q(u_{2_x} - u_{1_x}) \\
u_{1_x} &= u_1 \\
u_{2_x} &= u_2 \cos\theta \\
F_{T_x} &= \rho Q(u_2 \cos\theta - u_1) \\
&= 1000 \times 1.0(7.955\cos 45 - 1.989) \\
&= 3636 N
\end{aligned}
$$

In the y-direction:

$$
\begin{aligned}
F_{T_y} &= \rho Q(u_{2_y} - u_{1_y}) \\
u_{1_y} &= u_1 \sin 0 = 0 \\
u_{2_y} &= u_2 \sin\theta \\
F_{T_y} &= \rho Q u_2 \sin\theta \\
&= 1000 \times 1.0 \times 7.955 \sin 45 \\
&= 5625 N
\end{aligned}
$$

<u>Calculate the pressure force</u>

Use Bernoulli to calculate force at exit, p_2

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2 + h_f$$

the friction loss h_f can be ignored, $h_f=0$

As the pipe is in the horizontal plane, $z_1=z_2$

By continuity, $Q= u_1A_1 = u_2A_2$

$$
\begin{aligned}
p_2 &= p_1 - \frac{\rho}{2}(u_2^2 - u_1^2) \\
&= 100000 - \frac{1000}{2}(7.955^2 - 1.989^2) \\
&= 100000 - 29663 \\
&= 70337 N
\end{aligned}
$$

$$F_P = \text{pressure force at 1} - \text{pressure force at 2}$$

$$
\begin{aligned}
F_{P_x} &= p_1 A_1 \cos 0 - p_2 A_2 \cos \theta = p_1 A_1 - p_2 A_2 \cos \theta \\
&= 100000 \times 0.5027 - 70337 \times 0.1257 \cos 45 \\
&= 50270 - 6252 \\
&= 44018 N \\
F_{P_y} &= p_1 A_1 \sin 0 - p_2 A_2 \sin \theta = -p_2 A_2 \sin \theta \\
&= -70337 \times 0.1257 \sin 45 \\
&= -6252 N
\end{aligned}
$$

<u>Calculate the body force</u>

There are no body forces as the pipe is in the horizontal plane.

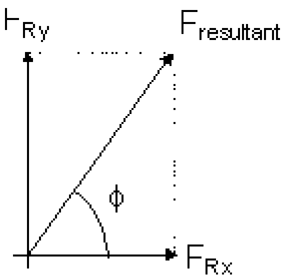
<u>Calculate the resultant force</u>

$$
\begin{aligned}
F_{T_x} &= F_{R_x} + F_{P_x} + F_{B_x} \\
F_{T_y} &= F_{R_y} + F_{P_y} + F_{B_y}
\end{aligned}
$$

$$
\begin{aligned}
F_{R_x} &= F_{T_x} - F_{P_x} - 0 \\
&= \rho Q(u_2 \cos \theta - u_1) - p_1 A_1 + p_2 A_2 \cos \theta \\
&= 1000 \times 1.0(7.955 \cos 45 - 1.989) - 100000 \times 0.5027 + 70337 \times 0.1257 \times \cos 45 \\
&= 3636 - 44018 \\
&= -40382 N
\end{aligned}
$$

$$
\begin{aligned}
F_{R_y} &= F_{T_y} - F_{P_y} - 0 \\
&= \rho Q u_2 \sin \theta + p_2 A_2 \sin \theta \\
&= 1000 \times 1.0 \times 7.955 \times \sin 45 + 70337 \times 0.1257 \sin 45 \\
&= 5625 - 6252 \\
&= 11877 N
\end{aligned}
$$

And the resultant force on the fluid is given by



$$F_R = \sqrt{F_{R_x}^2 - F_{R_y}^2} = 42092 N$$

And the direction of application is

$$\phi = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{11877}{-40382}\right)$$
$$= \tan^{-1}(-0.294)$$
$$\phi = -16.4^{\circ}$$

the force on the bend is the same magnitude but in the opposite direction

$$R = -F_R$$

Back To June 1999 Questions Page

Back To June 1999 Questions Page

CIVE1400 Examination 1999. Answers

Question 5

5.a Using continuity and the Bernoulli equation derive the following expression that can be used to measure flow rate with a Venturi meter.

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}}$$

(12 marks)

5.b A manomter may be attached to the Venturi meter to measure the pressure difference between point 1 and 2 in figure 5 below. Show that in this situation it is not necessary to know the inclination of the Venturi meter to measure flow rate.

(8 marks)

5.c A Venturi meter is being used to measure flow in a pipeline of diameter $250\,mm$ which carries water. When the pressure difference between the throat and the entrance of the Venturi meter is $300\,mm$ on a mercury manometer, determine the flow in the pipeline.

The Venturi meter has a throat diameter of $80\,mm$ and a coefficient of discharge of 0.97. The relative density of mercury is 13.6.

(5 marks)

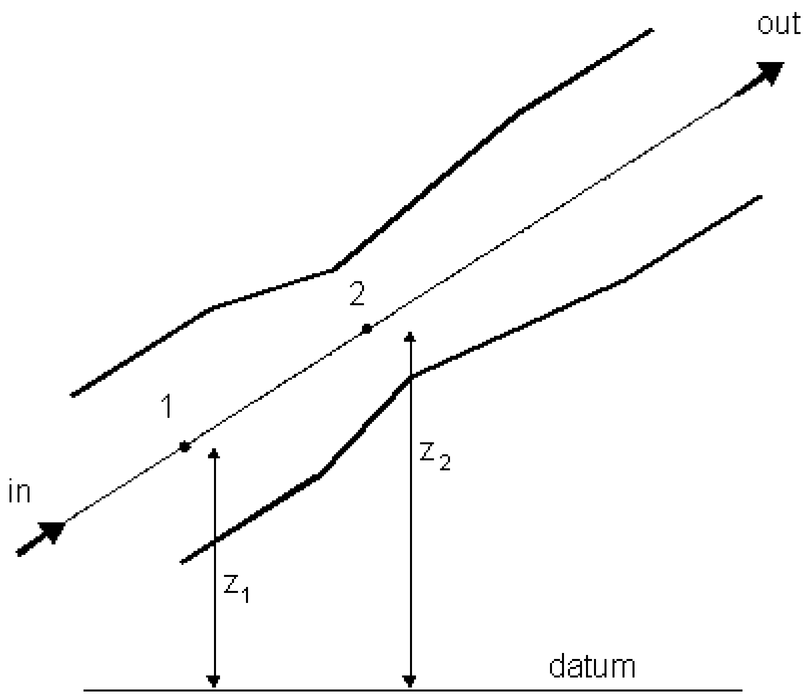


Figure 5

5.a

Applying Bernoulli along the streamline from point 1 to point 2 in the narrow *throat* of the Venturi meter we have

$$\frac{p_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

By the using the continuity equation we can eliminate the velocity u_2,

$$Q = u_1 A_1 = u_2 A_2$$
$$u_2 = \frac{u_1 A_1}{A_2}$$

Substituting this into and rearranging the Bernoulli equation we get

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = \frac{u_1^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$u_1 = \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

$$= A_2\sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 + A_2^2}}$$

To get the theoretical discharge this is multiplied by the area. To get the actual discharge taking in to account the losses due to friction, we include a coefficient of discharge

$$Q_{ideal} = u_1 A_1$$

$$Q_{actual} = C_d Q_{ideal} = C_d u_1 A_1$$

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2g\left[\frac{p_1 - p_2}{\rho g} + z_1 - z_2\right]}{A_1^2 - A_2^2}}$$

5.b

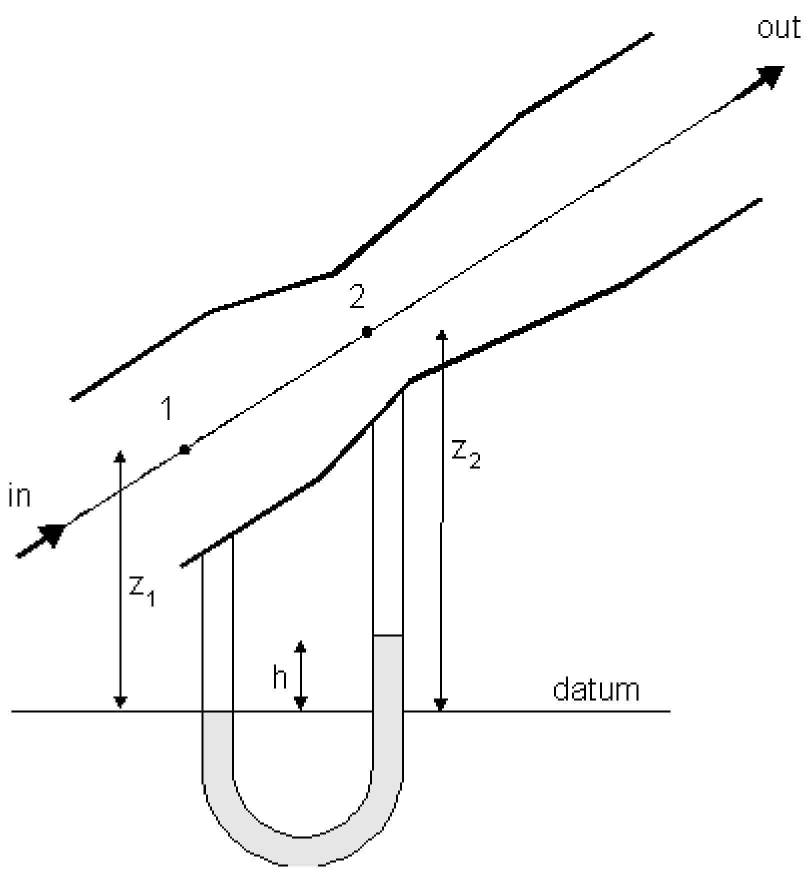
This can also be expressed in terms of the manometer readings

$$p_1 + \rho g z_1 = p_2 + \rho_{man} g h + \rho g(z_2 - h)$$

$$\frac{p_1 - p_2}{\rho g} + z_1 - z_2 = h\left(\frac{\rho_{man}}{\rho} - 1\right)$$

Thus the discharge can be expressed in terms of the manometer reading::

$$Q_{actual} = C_d A_1 A_2 \sqrt{\frac{2gh\left(\frac{\rho_{man}}{\rho} - 1\right)}{A_1^2 - A_2^2}}$$



5.c

$A_1 = \pi\ 0.25^2 /4 = 0.049\ m^2$

$A_2 = \pi\ 0.08^2 /4 = 0.005\ m^2$

h = 0.3 m

$\rho_{Hg} = \rho_{man} = 13\ 600\ kg/m^3$

$\rho_{H20} = \rho = 1000\ kg\ /m^3$

Cd = 0.97

$$Q_{actual} = 0.97 \times 0.049 \times 0.005\sqrt{\frac{2 \times 9.81 \times 0.3(13.6-1)}{0.049^2 - 0.005^2}}$$
$$= 0.0002376\sqrt{\frac{74.16}{0.002376}}$$
$$= 0.042\ m^3/s$$

Back To June 1999 Questions Page

Back To June 1999 Questions Page

CIVE1400 Examination 1999. Answers

Question 6

6.a Assuming the drag force, F, exerted on a body is a function of the following

fluid density ρ

fluid viscosity μ

diameter d

velocity u

Show that the drag force can be expressed as

$$F = d^2 u^2 \rho \phi(\mathrm{Re})$$

where ϕ is some unknown function and Re is the Reynolds number.

(10 marks)

6.b The diameter of a ball is 0.1m and it is required to predict the force it will experience when exposed to a flow of water travelling at 5 m/s. A sphere of 1.0 m diameter is available and can be exposed to a flow of air. To obtain the dynamically similar conditions at what velocity should this flow of air opperate?

(8 marks)

6.c Under these conditions the 1m diameter sphere exeperiences a force of 14 200 *N*. What force would you expect on the 0.1m diameter ball when in water flow?

(7 Marks)

$\mu_{water} = 1.0 \times 10^{-6}$ kg/ms

$\mu_{air} = 1.7 \times 10^{-5}$ kg/ms

$\rho_{water} = 1000$ kg/m^3

$\rho_{air} = 12.5$ kg/m^3

6.a.

$$F = f(\rho, \mu, d, u)$$
$$0 = \phi(F, \rho, \mu, d, u)$$

Assume the governing variables ρ , *u, d*

According to Buckingham's π theorem there are n-m groups

where

n = number of variables (5) and

m = number of dimensions (i.e. MLT, giving 3)

n-m = 5-3 = 2 groups

$$0 = \phi(\pi_1, \pi_2)$$
$$\pi_1 = \rho^{a_1} u^{b_1} d^{c_1} \mu$$
$$\pi_2 = \rho^{a_2} u^{b_2} d^{c_2} F$$

Dimensions of the variables are:

ρ = density (kg/m^3) = ML^{-3}

μ = viscosity (kg/m/s) = ML^{-1} T^{-1}

u = velocity (m/s) = ML^{-1}

d = length (m) = L

F = newtons (kg m /s^2) = MLT^{-2}

For π_1

$$0 = \left(ML^{-3}\right)^{a_1} \left(LT^{-1}\right)^{b_1} \left(L\right)^{c_1} ML^{-1}T^{-1}$$
$$0 = a_1 + 1$$
$$0 = -3a_1 + b_1 + c_1 - 1$$
$$0 = -b_1 + 1$$
$$a_1 = -1$$
$$b_1 = -1$$
$$c_1 = -1$$
$$\pi_1 = \frac{\mu}{\rho u d}$$

For π_2

$$
\begin{aligned}
0 &= \left(ML^{-3}\right)^{a_2}\left(LT^{-1}\right)^{b_2}(L)^{c_2} MLT^{-2} \\
0 &= a_2 + 1 \\
0 &= -3a_2 + b_2 + c_2 + 1 \\
0 &= -b_2 - 2 \\
a_2 &= -1 \\
b_2 &= -2 \\
c_2 &= -2 \\
\pi_2 &= \frac{F}{\rho u^2 d^2}
\end{aligned}
$$

$$
\begin{aligned}
0 &= \phi(\pi_1, \pi_2) \\
0 &= \phi\left(\frac{u}{\rho u d}, \frac{F}{\rho u^2 d^2}\right)
\end{aligned}
$$

Inverting π_1 gives $\mathrm{Re} = \dfrac{\rho u d}{\mu}$

$$0 = \phi\left(\mathrm{Re}, \frac{F}{\rho u^2 d^2}\right)$$

Rearranging this gives $F = \rho u^2 d^2 \phi(\mathrm{Re})$

6.b

For dynamic similarity the Reynolds numbers are equal for both the water and air situation

$$\mathrm{Re}_{\text{air}} = \mathrm{Re}_{\text{water}}$$

$$\left(\frac{\rho u d}{\mu}\right)_{\text{air}} = \left(\frac{\rho u d}{\mu}\right)_{\text{water}}$$

$$\frac{12.5 \times u_{\text{air}} \times 1.0}{1.7 \times 10^{-5}} = \frac{1000 \times 5.0 \times 0.1}{1.0 \times 10^{-6}}$$

$$u_{\text{air}} = 680 m/s$$

6.c.

With dynamic similarity $\phi(\mathrm{Re}_{water}) = \phi(\mathrm{Re}_{air})$

So

$$\frac{F_{water}}{F_{air}} = \frac{d^2_{water} u^2_{water} \rho_{water} \phi(\mathrm{Re}_{water})}{d^2_{air} u^2_{air} \rho_{air} \phi(\mathrm{Re}_{air})}$$

$$\frac{F_{water}}{14200} = \frac{0.1^2}{1.0^2} \frac{5^2}{680^2} \frac{1000}{12.5}$$

$$F_{water} = 0.614\,N$$

Back To June 1999 Questions Page

Back To June 2000 Questions Page

CIVE1400 Examination 2000. Answers

Question 1

1 A 5*m* wide tank with an L-shaped cross section, as shown in Figure 1, has a gate which is hinged at the top at it right hand end. If the tank is filled with water to a level of 8*m* determine the torque required at the hinge to just keep the gate closed.

(20 marks)

Determine also the force on the base of the tank and comment on why this is not the same as the weight of the water.

(5 marks)

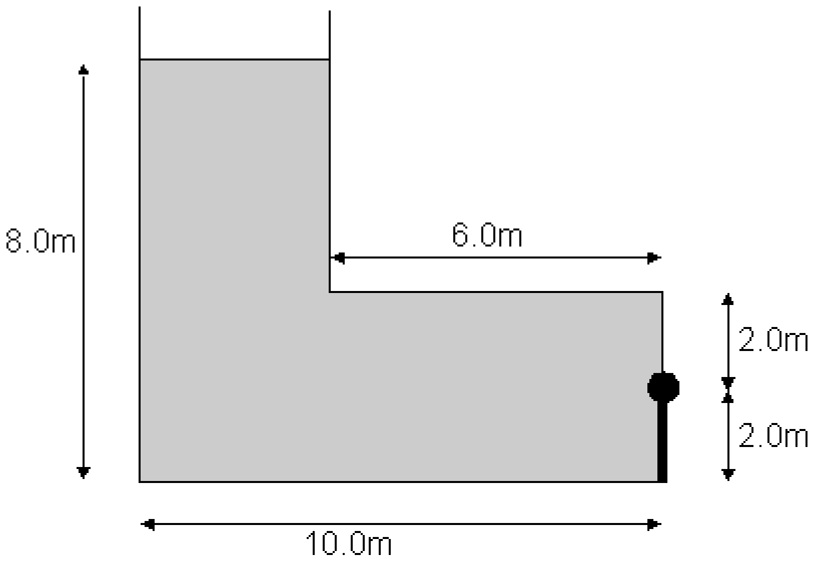
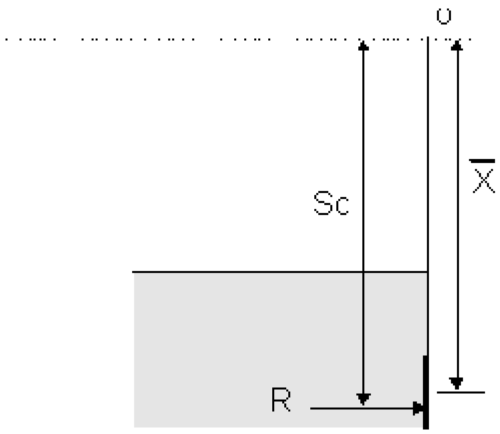


Figure 1

$$
\begin{aligned}
\text{Force on gate} &= \text{pressure at centroid} \times \text{area of gate} \\
&= \rho g(6+1)\times(2\times 5) \\
&= 9810\times 7\times 10 \\
&= 686\,700\ N
\end{aligned}
$$

Position of action of force = Sc from the point O

The point O is the point where a line from the gate crosses the top water surface level



Position of this force is through the centre of pressure, Sc.

Using the parallel axis theorem,

$$Sc = \frac{I_{oo}}{A\bar{x}} = \frac{\text{2nd momnt of area}}{\text{1st moment of area}}$$

$$I_{oo} = I_{GG} - A\bar{x}^2$$

$$Sc = \frac{I_{GG}}{A\bar{x}} + \bar{x}$$

$\bar{x}$ is the distance along the face to the centroid

$$I_{GG} = \frac{bd^3}{12} = \frac{5 \times 2^2}{12} = 3.333$$

$$Sc = \frac{bd^3}{12}\frac{1}{bd}\frac{1}{7} + 7 = \frac{4}{84} + 7 = 7.0476m$$

Torque required = $(7.0476 - 6) \times 686700 = 719400$ Nm clockwise

Force on base = PA = pghA = 3924 kN

Weight of water = (10×8×5 - 4×6×5)×9810 = 2746800 N

Back To June 2000 Questions Page

Back To June 2000 Questions Page

CIVE1400 Examination 2000. Answers

Question 2

2 Water flows along a circular pipe and is turned vertically through 180° by a reducing bend as shown in figure 2. The rate of flow in the pipe is 20 litres/s, the pressure measured at the entrance to the bend is 120 kN/m^3 and the volume of fluid in the bend is 0.1m^3. What is the magnitude and direction of the force exerted by the fluid on the bend? Ignore any friction losses.

(25 marks)

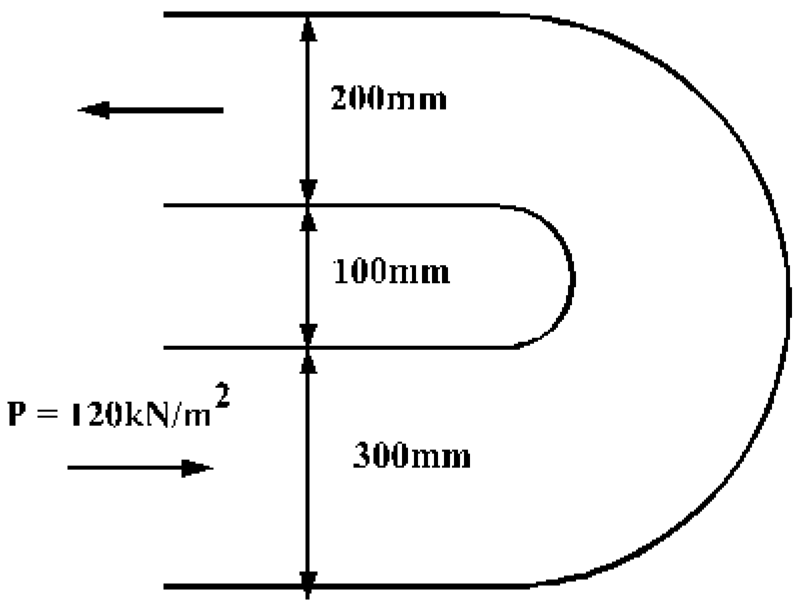


Figure 2

Take the inlet as point 1 and the outlet as point 2.

$$Q = 2 \text{ l/s} = 0.02 \text{ m}^3/\text{s}$$

$$p_1 = 120000 \text{ N/m}^2$$

Height difference between the two pipe centres,

$$h = 0.35\text{m}$$

$$p_1 = 120000 \text{ N/m}^2 \quad p_2 = ?$$

$$d_1 = 0.3\text{m} \quad d_2 = 0.2 \text{ m}$$

$$a_1 = \frac{\pi d_1^2}{4} = 0.0707\, m^2 \quad a_2 = \frac{\pi d_2^2}{4} = 0.0314\, m^2$$

$$Q = a_1 u_1 = a_2 u_2$$
$$u_1 = \frac{0.02}{0.0707} = 0.2829\, m/s$$
$$u_2 = \frac{0.02}{0.0314} = 0.6369\, m/s$$

<u>Calculate the total force on the bend.</u>

In the x- direction

$$\begin{aligned} F_{Tx} &= \rho Q(u_{2x} - u_{1x}) \\ &= \rho Q(u_2 \cos\theta - u_1) \\ &= 1000 \times 0.02(-0.6369 - 0.2829) \\ &= -18.40\,N \end{aligned}$$

In the y-direction there is no component i.e.

$$F_{Ty} = \rho Q(u_{2y} - u_{1y}) = 0$$

<u>Calculate the pressure force</u>

F_p = pressure force at 1 - pressure force at 2

$$\begin{aligned} F_{px} &= p_1 A_1 \cos 0 - p_2 A_2 \cos 180 \\ &= p_1 A_1 + p_2 A_2 \end{aligned}$$

$$F_{py} = 0$$

We know p_1 but need to find p_2 using Bernoulli from point 1 to point 2.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

$$\frac{12000}{9810} + \frac{0.2829^2}{19.62} + 0 = \frac{p_2}{9810} + \frac{0.6369^2}{19.62} + 0.35$$

$$p_2 = 161404\,N/m^2$$

$$\begin{aligned} F_{px} &= p_1 A_1 + p_2 A_2 \\ &= 120000 \times 0.0707 + 161404 \times 0.0314 \\ &= 121369 N \end{aligned}$$

<u>Calculate the body forces</u>

The only body force is that due to gravity i.e the weight of the water.

We are given the volume = 0.1 m^3

$$F_{Ry} = \rho g \times volume$$
$$= 981 N$$

$$F_{Bx} = 0$$

Calculate the resultant force acting on the fluid

$$F_{Tx} = F_{Rx} + F_{Px} + F_{Bx}$$
$$-18.40 = F_{Rx} + 12139$$
$$F_{Rx} = 12157 N$$

$$F_{Ty} = F_{Ry} + F_{Py} + F_{By}$$
$$0 = F_{Ry} + 0 - 981$$
$$F_{Ry} = 981 N$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = 12197 N$$

Acting at the angle θ

$$\tan \theta = \frac{F_{Ry}}{F_{Rx}}$$
$$\theta = 4.61^\circ$$

The force acting on the bend is -12197 N i.e. equal in magnitude to the force on the fluid, but in the opposite direction.

Back To June 2000 Questions Page

Back To June 2000 Questions Page

CIVE1400 Examination 2000. Answers

Question 3

3 Describe with the aid of diagrams the following phenomena explaining why and when they occur.

(Each part requires at least a half page description of the phenomenon plus diagrams.)

i. The laminar boundary layer

(5 marks)

ii. The turbulent boundary layer

(5 marks)

iii. The laminar sublayer

(5 marks)

iv. Boundary layer separation

(5 marks)

v. Methods to prevent boundary layer separation

(5 marks)

Back To June 2000 Questions Page

Back To June 2000 Questions Page

CIVE1400 Examination 2000. Answers

Question 4

4.a Starting from the Bernoulli equation, develop the equation shown below for discharge over a sharp edged rectangular weir. State **all** assumptions made.

$$Q_{actual} = C_d \frac{2}{3} B \sqrt{2g} H^{3/2}$$

(15 marks)

4.b At the end of a channel is a sharp edged rectangular weir with a width of 400mm and a coefficient of discharge of 0.65. The water is flowing at a depth 0.16m above the base of the weir. If this weir is replaced by a 90° V-notch weir with the same coefficient of discharge, what will be the necessary upstream depth of water to achieve the same discharge as the rectangular weir.

The equation for discharge over a v-notch weir is:

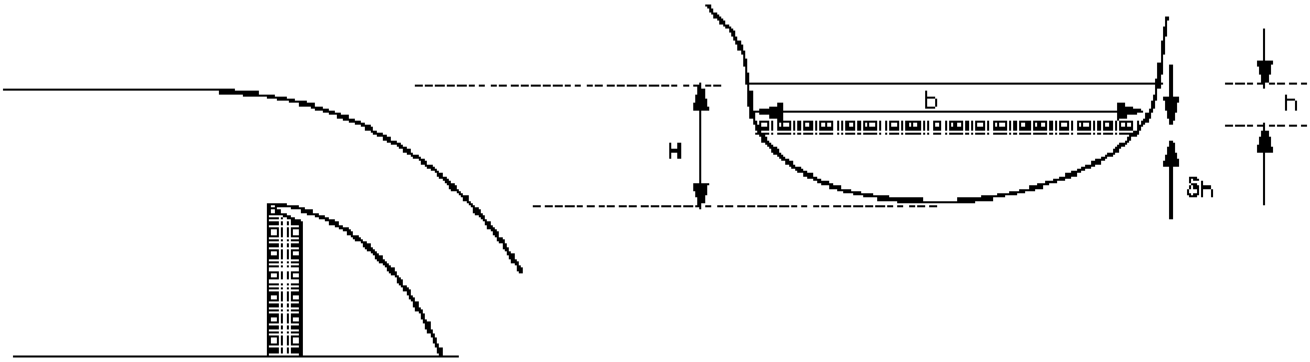
$$Q_{actual} = C_d \frac{8}{15} \sqrt{2g} \tan\left(\frac{\theta}{2}\right) H^{5/2}$$

(10 marks)

4.a

A General Weir Equation

Consider a horizontal strip of width *b* and depth *h* below the free surface, as shown in the figure below.



Elemental strip of flow through a notch

Assuming the velocity is only due to the head i.e. a very slow flow towards the weir. Then from the Bernoulli equation we get this expression for:

$$\text{velocity through the strip, } u = \sqrt{2gh}$$
$$\text{discharge through the strip, } \delta Q = Au = b\,\delta h\sqrt{2gh}$$

Integrating from the free surface, $h = 0$, to the weir crest, $h = H$ gives the expression for the total theoretical discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H b h^{1/2}\,dh$$

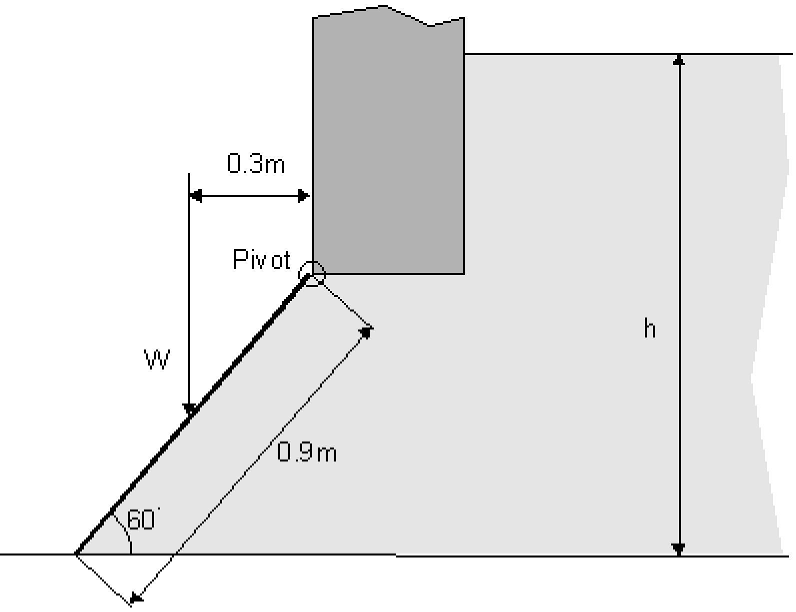
This will be different for every differently shaped weir or notch. To make further use of this equation we need an expression relating the width of flow across the weir to the depth below the free surface.

For a rectangular weir b = B = constant

The actual discharge is obtained by introducing a coefficient of discharge

$$Q_{\text{theoretical}} = \sqrt{2g}\int_0^H B h^{1/2}\,dh$$
$$Q_{actual} = Cd\,\frac{2}{3}\sqrt{2g}\,H^{3/2}$$

For the rectangular weir



$$Q = 0.65 \times 0.4 \times \frac{2}{3} \times \sqrt{19.62} \times 0.16^{3/2}$$
$$= 0.049\ m^3/s$$

For the v-notch weir

$$0.049 = 0.65 \times \frac{8}{15} \times \sqrt{19.62} \times \tan 45 \times H^{5/2}$$
$$0.049 = 1.5355 H^{5/2}$$
$$H^{5/2} = 0.0319$$
$$H = 0.252 m$$

(Note you have to use the inverse-root function of your calculator to get the value for *H* from $H^{5/2}$)

Back To June 2000 Questions Page

Back To June 2000 Questions Page

CIVE1400 Examination 2000. Answers

Question 5

5 A rectangular sluice gate is fitted at the base of a reservoir wall with a pivot in the arrangement shown in Figure 3. The gate is designed to regulate the level of water in the reservoir by opening when the water level to the right, h, reaches a certain depth. The gate has a width of 1.2m and its centre of gravity is 0.3m from the wall.

Determine the weight, W, of the gate, if a water level of h = 2.779m will just cause the gate to open.

(25 marks)

Figure 3

$$L = 0.9m$$
$$x = 0.9\cos 60 = 0.45m$$
$$y = 0.9\sin\ 60 = 0.779m$$

The gate opens when the moment at the pivot is clockwise.

That is when the moment due to the water > 0.3w.

<u>Method 1</u>

Force on plane = Area × Pressure at centroid

$$\begin{aligned}\text{Force} &= (1.2\times 0.9)\times\left(2.779-\frac{y}{2}\right)\rho g\\ &= 1.08\times(2.779-0.779/2)1000\times 9.81\\ &= 25316N\end{aligned}$$

<u>Method 2</u>

Horizontal force = Hf = Area of projection on vertical plane × pressure at centroid

$$\begin{aligned}H_f &= 1.2y\left(2.779-\frac{y}{2}\right)\rho g\\ &= 21912.6N\end{aligned}$$

Vertical force = weight of water above gate

$$\begin{aligned}V_f &= \left(2.779x-\frac{x\times y}{2}\right)\rho g 1.2\\ &= (12268-1719)1.2\\ &= 12658\end{aligned}$$

Resultant force

$$\begin{aligned}R &= \sqrt{H_f^{\ 2}+V_f^{\ 2}}\\ &= 25306N\end{aligned}$$

<u>Method 3</u>

$$
\begin{aligned}
\text{Force} &= \text{Shaded area} \\
&= \left(2.779\times 0.9 - \frac{0.779\times 0.9}{2}\right)1.2\rho g \\
&= 2.15055\times 1.2\rho g \\
&= 25316\,N
\end{aligned}
$$

Point of action of the force = centre of pressure

$$
\begin{aligned}
x_2 &= \frac{2}{\tan 60} = 3.444m \\
L_2 &= \frac{2}{\sin 60} = 2.309m \\
\bar{x} &= L_2 + \frac{0.9}{2} = 2.759m
\end{aligned}
$$

$$
\begin{aligned}
Sc &= \frac{\text{2nd moment of area about O}}{\text{1st moment of area about O}} = \frac{I_{oo}}{A\bar{x}} \\
I_{oo} &= I_{GG} + A\bar{x}^2 \\
Sc &= \frac{I_{GG}}{A\bar{x}} + \bar{x}
\end{aligned}
$$

For a rectangle, $I_{GG} = \dfrac{bd^3}{12}$

$$
\begin{aligned}
Sc &= \frac{d^2}{12\bar{x}} + \bar{x} \\
&= \frac{0.9^2}{12\times 2.759} + 2.759 \\
&= 2.783m
\end{aligned}
$$

Need to find the lever arm, i.e. the distance from the pivot to the centre of pressure specified by Sc.

First find the position of the pivot, x_2, from the surface (along the inclined plane)

$$x_2 \cos 30 = 2$$

$$x_2 = 2.309\text{m}$$

Lever are, $x_1 = Sc - x_2 = 2.783 - 2.309 = 0.474$ m

Take moments to fine the weight of the gate, w

$$Rx_1 = 0.3w$$

$$w = \frac{25316 \times 0.474}{0.3} = 40000\,N$$

Back To June 2000 Questions Page

Back To June 2000 Questions Page

CIVE1400 Examination 2000. Answers

Question 6

6.a Describe what is meant by the term dimensional analysis. Your explanation should include the meanings and relevance of the terms *geometric similarity, dynamic similarity* and *kinematic similarity* as well as identifying some uses form the technique.

(8 marks)

6.b Assuming the drag force, F, exerted on a body is a function of the following

fluid density ρ

fluid viscosity μ

diameter d

velocity u

Show that the drag force can be expressed as

$$F = d^2 u^2 \rho \phi(\mathrm{Re})$$

where ϕ is some unknown function and Re is the Reynolds number.

(10 marks)

6.c A prototype boat propeller has a diameter of 1.0m. It is necessary to determine the force it will experience when water flows past at 5 m/s. A model propeller is available of diameter 0.1m and can be placed in a wind tunnel. To obtain the dynamically similar conditions at what velocity would the air need to flow in the wind tunnel?

(7 marks)

$\mu_{water} = 1.0 \times 10^{6}$ kg/ms $\mu_{air} = 1.7 \times 10^{5}$ kg/ms

$\rho_{water} = 1000$ kg/m^3 $\rho_{air} = 12.5$ kg/m^3

6.a.

Dimensional analysis is used when constructing physical models of prototype structures. Physical models are used when the fluid flow is particularly complex and difficult to analyse by other means. It enables physical measurements - forces, velocities etc. - taken from the scale models to be converted to the equivalent measurement which would be found on a prototype.

The term similarity relates to physical a scale models.

Geometric similarity - all dimensions are in the in the same ratio.

Dynamic similarity - all velocities are in the same ratio - requires geometric similarity

Kinematic similarity - all forces are in the same ration - requires dynamic similarity.

6.b.

$$F = f(\rho, \mu, d, u)$$
$$0 = \phi(F, \rho, \mu, d, u)$$

Assume the governing variables ρ , *u*, *d*

According to Buckingham's π theorem there are n-m groups

where

n = number of variables (5) and

m = number of dimensions (i.e. MLT, giving 3)

n-m = 5-3 = 2 groups

$$0 = \phi(\pi_1, \pi_2)$$
$$\pi_1 = \rho^{a_1} u^{b_1} d^{c_1} \mu$$
$$\pi_2 = \rho^{a_2} u^{b_2} d^{c_2} F$$

Dimensions of the variables are:

ρ = density (kg/m^3) = ML^{-3}

μ = viscosity (kg/m/s) = $ML^{-1} T^{-1}$

u = velocity (m/s) = ML^{-1}

d = length (m) = L

F = newtons (kg m /s^2) = MLT^{-2}

For π_1

$$
\begin{aligned}
0 &= \left(ML^{-3}\right)^{a_1}\left(LT^{-1}\right)^{b_1}\left(L\right)^{c_1} ML^{-1}T^{-1} \\
0 &= a_1 + 1 \\
0 &= -3a_1 + b_1 + c_1 - 1 \\
0 &= -b_1 + 1 \\
a_1 &= -1 \\
b_1 &= -1 \\
c_1 &= -1 \\
\pi_1 &= \frac{\mu}{\rho u d}
\end{aligned}
$$

For π_2

$$
\begin{aligned}
0 &= \left(ML^{-3}\right)^{a_2}\left(LT^{-1}\right)^{b_2}\left(L\right)^{c_2} MLT^{-2} \\
0 &= a_2 + 1 \\
0 &= -3a_2 + b_2 + c_2 + 1 \\
0 &= -b_2 - 2 \\
a_2 &= -1 \\
b_2 &= -2 \\
c_2 &= -2 \\
\pi_2 &= \frac{F}{\rho u^2 d^2}
\end{aligned}
$$

$$
\begin{aligned}
0 &= \phi\left(\pi_1, \pi_2\right) \\
0 &= \phi\left(\frac{u}{\rho u d}, \frac{F}{\rho u^2 d^2}\right)
\end{aligned}
$$

Inverting π_1 gives $\mathrm{Re} = \frac{\rho u d}{\mu}$

$$
0 = \phi\left(\mathrm{Re}, \frac{F}{\rho u^2 d^2}\right)
$$

Rearranging this gives $F = \rho u^2 d^2 \phi(\mathrm{Re})$

6.c

For dynamic similarity the Reynolds numbers are equal for both the water and air situation

$$\mathrm{Re}_{air} = \mathrm{Re}_{water}$$

$$\left(\frac{\rho u d}{\mu}\right)_{air} = \left(\frac{\rho u d}{\mu}\right)_{water}$$

$$\frac{12.5 \times u_{air} \times 1.0}{1.7 \times 10^{-5}} = \frac{1000 \times 5.0 \times 0.1}{1.0 \times 10^{-6}}$$

$$u_{air} = 680 m/s$$

Back To June 2000 Questions Page