Gas Dynamics

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References :

1- "Compressible Fluid Flow", Patrick H. Oosthuizen & William E. Carscallen, 1997.

2- "Gas Dynamics", James E. A. John & Theo G. keith, 2006.

3. "ديناميك الغازات"، د. منذر الدروبي، 1980

CH.1. Introduction

Fluid Mechanics

- Static
- Dynamics
  - Gas Dynamics
  - Aerodynamics
  - Hydrodynamics

1.1. Definitions

Gas Dynamics: It is the subject which deals with the flow of compessible gases (often internal flow) and is a one branch of the fluid dynamics.

Compressible fluid: It is the fluid that have the ability of compression, i.e., preceptible variation in density of fluid ($\Delta\rho \neq 0$).

Compressible flow: It is the flow with $\Delta\rho \neq 0$ when the velocity changes.

Incompressible fluid: It is the fluid that withstand the external effects with $\Delta\rho = 0$

Incompressible flow: It is the flow in which no change in density of the fluid is happened ($\Delta\rho = 0$).

Steady flow: It means that the fluid properties, in any point in the space, do not depend on the time.

$$\frac{\partial(\text{Property})}{\partial t} = 0$$

Unsteady flow: It means that the fluid properties, in any point in the space, depend on the time.

$$\frac{\partial(\text{Property})}{\partial t} \neq 0$$

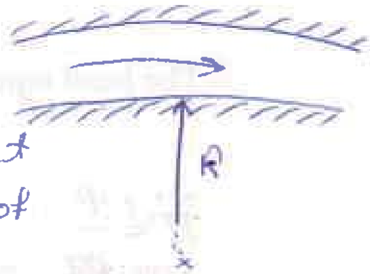
1.2. One - Dimensional Flow

It can be said that the flow is one-dimensional flow when the fluid properties are uniform in any cross-section of a duct and variable along the flow direction only.

$$\frac{\partial(\text{Property})}{\partial x} \neq 0 , \quad \frac{\partial(\text{Property})}{\partial y} = 0 , \quad \frac{\partial(\text{Property})}{\partial z} = 0$$

- The flow in slight chang cross-section area can be considered one-dimensional flow

- In larg radius of duct curvature, the flow can be considered one-dimensional flow.



- In the airofoil case can not be considered one-dimensional flow, where it is important to determine the drag & lift forces inspite of larg radius of flow path curvature.

1.3. Method of Analysis

To analyse the flow problem mathmatically, there are two types of laws, the first type is called Conservation Laws, and the second type is called Constitutive Laws.

The first type consists of the following laws:

1. Conservation of Mass law (Continuity Equation).
2. Conservation of Momentum Law (Newton Second law).
3. Conservation of Energy law (First Law of Thermodynamics).
4. Second Law of Thermodynamics.

The second type deals with the nature of the used gas and relates the physical properties of the gas for example the equation of state.

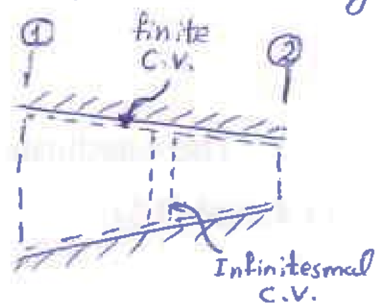
1.4. System and Control Volume

The system is a constant quntity of the matter seperated by boundaries from the surrounding. The system can be moved and exchang energy but not matter. The system is not applicable for fluid dynamics.

The control volume is an arbitrary fixed volume in space bounded by control surface. The matter and energy can be transmitted through the control surface. The control volume can be applied in fluid dynanics.

The choice of control volume must be appropriate to the nature of problem for simplification of mathmatical analysis.

The control volume can be a finite for a certain cross-section, and an infinitesmal for the whole domaine from ① to ②.



1.5. Thermodynamics Concepts

- Property : It is the property of the system. The property may be internal for the system matter or external for the system position. There are two types of properties:
  Extensive ~ depends on the mass of the system, for example the energy (E).
  Intensive ~ don't depend on the mass of the system, for example the energy per unit mass (e).
- State : It is the situation of the system described by its properties values.
- Process : It is the event that happened when the state of system has been changed.
- Cycle : It is series of processes that the state in the end should be identical to the state in the begining.

- Potential Energy (PE) : $PE = mgz$

- Kinetic Energy (KE) : $KE = \frac{1}{2} m V^2$

- Internal Energy (U) : It is caused by the motion and construction of matter particles.

- Enthalpy (h) : $h = u + p\upsilon = u + \frac{P}{\rho}$ (KJ/Kg)

  υ : specific volume

- Specific Heat at Constant Pressure (C_p) : $C_p = \left(\frac{\partial h}{\partial T}\right)_p$

- Specific Heat at Constant Volume (C_v) : $C_v = \left(\frac{\partial u}{\partial T}\right)_v$

- Specific Heat Ratio (k) : $\cancel{\gamma} = \frac{C_p}{C_v}$

- Adiabatic Process : It is a process in which no heat transfer between the system and the surrounding media.

$$dq = 0$$

- Reversible Process : It is the process that can be inversed with coming back the system and surrounding media to the starting state. The reversible process is an ideal process.

- Irreversible Process : It is the process that can not be inversed to the starting state.

- Entropy (s) : $\Delta s = \int_1^2 \left(\frac{dq}{T}\right)_{rev}$

$$ds = \frac{dq}{T} \qquad (J/kg \cdot k)$$

1.6. Conservation of Mass Law (Continuity Equation)

The conservation of mass for a system is given as:

$$\frac{dm_{system}}{dt} = 0$$

* The mass of the system doesn't change neither with place nor with time.

For control volume

$$\frac{\partial m_{cv}}{\partial t} + \dot{m}_{out} - \dot{m}_{in} = 0 \quad \text{—— (1)} \quad \text{(Continuity Equation)}$$

For One-dimensional steady flow

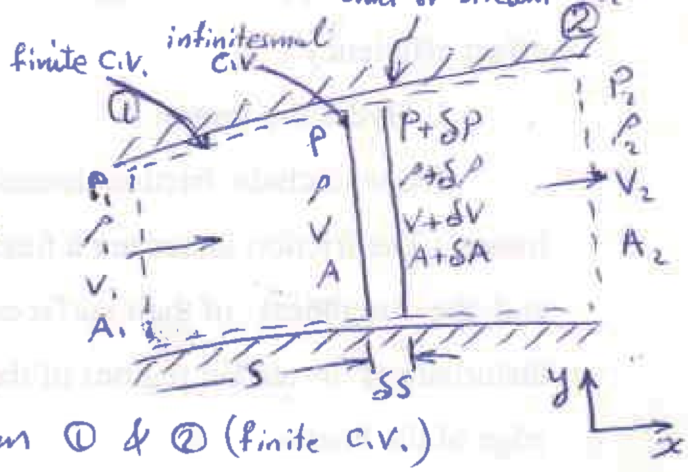
$$\frac{\partial m_{cv}}{\partial t} = 0 \quad \text{—— (2)}$$

$$\dot{m}_{out} = \sum_{\substack{all \\ outlets}} \rho V A \quad , \text{where } V \perp A \quad \text{—— (3)}$$

$$\dot{m}_{in} = \sum_{\substack{all \\ inlets}} \rho V A \quad \text{—— (4)}$$

Substituting Eq's (2), (3) & (4) in Eq. (1) yeilds:

$$\sum_{\substack{all \\ inlets}} \rho V A = \sum_{\substack{all \\ outlet}} \rho V A$$



For C.V. between (1) & (2) there are one inlet & one outlet.

$$\dot{m} = \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For all cross-sections between (1) & (2) (finite C.V.)

$$\boxed{\dot{m} = \rho V A = \text{Constant}}$$

For infinitesmal C.V.

$$\rho V A = (\rho + \delta\rho)(V + \delta V)(A + \delta A)$$

$$\Longrightarrow \boxed{\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0} \qquad \text{(Derive)}$$

1.7. Conservation of Momentum

Newton 2^{nd} Law of motion for a system can be given as:

$$\sum \vec{F} = \frac{d(m\vec{V})}{dt}$$

For the Control Volume,

$$\sum \vec{F} = \frac{\partial (mV)_{cv}}{\partial t} + \int_{out} \vec{V}\, d\dot{m} - \int_{in} \vec{V}\, d\dot{m} \quad \text{(Momentum Equation)}$$

For Three-dimensional flow there are three equations of (x, y, z) directions, for example in x-direction the momentum equation is written as:

$$\sum \vec{F}_x = \frac{\partial (mV_x)_{cv}}{\partial t} + \int_{out} V_x\, d\dot{m} - \int_{in} V_x\, d\dot{m}$$

For one-dimensional steady flow,

$$\sum F_x = \sum_{\text{all outlets}} (\dot{m} V_x) - \sum_{\text{all inlets}} (\dot{m} V_x)$$

For one inlet and one outlet flow,

$$\sum F_x = \dot{m}\,(V_{2x} - V_{1x})$$

There are two types of forces included in the term $(\sum \vec{F})$. The 1^{st} type is the surface forces applied on the fluid on the control surface (Pressure force, viscose force or friction force). The 2^{nd} type is the body forces (weight) and it is neglected in gas flow. For frictionless steady flow the last equation becomes:

$$\boxed{\frac{dP}{\rho} + V dV + g dz = 0} \quad \text{Euler Equation}$$

1.8. Conservation of Energy

First law of Thermodynamics for a system can be given as:

$$\dot{Q} - \dot{W} = \frac{dE}{dt}$$

For the Control volume

$$\dot{Q} - \dot{W} = \frac{\partial E_{cv}}{\partial t} + \int_{out} e\, d\dot{m} - \int_{in} e\, d\dot{m}$$

Where

$$e = u + \frac{P}{\rho} + \frac{V^2}{2} + gz$$

1.9. Universal Law of Gases (Equation of State)

Perfect Gas is an ideal compressible fluid which obeys the Universal law of Gases that is given as :

$$P v = R T$$

where :

P: Absolute pressure ($P_{abs} = P_{gauge} + P_{atm}$) $P_{atm} = 1.013 \times 10^5$ Pa

v : Specific volume ($v = \frac{1}{\rho}$) & ($v = \frac{V}{m}$)

R : Gas constant

T : Absolute temperature ($^\circ K$)

The equation of state for a perfect gas or a mole basis, can be written as ,

$$P V = m R T = n \bar{R} T$$

Where: (V) is the volume of the mass (m) or the (n) moles of the gas.

$\bar{R} = \bar{M} R$: Universal gas constant

$= 8314.3$ J/kg-mole.$^\circ$K

$\bar{M}$: Molal mass of gas الوزن الجزيئي للغاز

Example : For atmospheric air between 0 to 100 km altitude the molal mass $\bar{M} = 28.966$ kg/kg-mol. Find the gas constant.

$$R = \frac{\bar{R}}{\bar{M}} = \frac{8314.3}{28.966} = 287 \text{ J/kg.}^\circ\text{K}$$

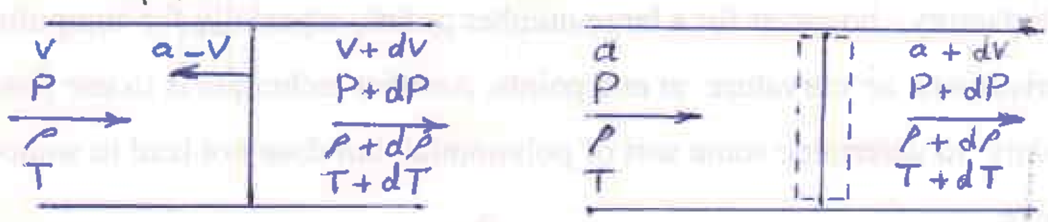
CH.2. Basic Concepts of Compressible Fluid Flow

2.1. Velocity of Sound

The acoustic waves are small pressure waves that transfers in a plane, cylindrical or spherical shape, these waves are formed due to disturbances in the flow.

Consider a plane wave as shown in figure

relative to the fluid, the velocity of the wave is (a).



(a) Moving wave at a velocity inside a flow

(b) The motion is relative to a C.V. fixed on a wave

Changes in flow properties are happened due to the movement of sound wave. To analyse this problem mathmatically a C.V. is considered fixed on the sound wave.

Applying Continuity Eq.

$$\rho a \cancel{A} = (\rho + d\rho)(a + dV)\cancel{A} \quad \div \rho a$$

$$\Rightarrow \frac{d\rho}{\rho} + \frac{dV}{a} = 0 \quad \text{———} \textcircled{1} \qquad (\text{Derive})$$

To apply the momentum equation, it must be noted that the thickness of the wave is very small, therefore the viscosity force, which is proportional to the area of C.V. wall, can be neglected, The momentum eq. becomes,

$$PA - (P + dP)A = \dot{m}\left[(a + dV) - a\right]$$

but $\dot{m} = \rho a A$

$$\therefore \; dP + \rho a \, dV = 0 \quad \text{————} \textcircled{2}$$

This equation is similar to Euler's equation for non-viscous fluid through infinitesmal C.V.

By substituting (dV) from eq. ① into eq. ② we get,

$$a^2 = \left(\frac{dP}{d\rho}\right)$$

The gas that a sound wave passes through has experienced an isentropic process since the process is adiabatic reversible.

$$\therefore \quad a = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s}$$

For isentropic flow $\quad \frac{P}{\rho^\gamma} = \text{constant} = C$

$$P = C\rho^\gamma \quad \Rightarrow \left(\frac{\partial P}{\partial \rho}\right)_s = C\,\gamma\,\rho^{\gamma-1} \quad \Rightarrow \left(\frac{\partial P}{\partial \rho}\right)_s = \frac{P}{\rho^\gamma}\cdot \gamma\rho^{\gamma-1}$$

$$\Rightarrow \left(\frac{\partial P}{\partial \rho}\right)_s = \frac{kP}{\rho}$$, but for perfect gas $\frac{P}{\rho} = RT$,

$$\therefore \left(\frac{\partial P}{\partial \rho}\right)_s = \gamma RT \quad \Rightarrow \quad a^2 = \gamma RT \quad \Rightarrow a = \sqrt{\gamma RT}$$

For air $\gamma = 1.4$ & $R = 287$ J/kg.k ,

$$\therefore \quad \boxed{a = 20.04\sqrt{T}} \quad m/s \text{ , } T \text{ in } K \text{ units}$$

2.2. Mach Number

It is defined by the following equation,

$$M = \frac{V}{a}$$

where (V) is the fluid velocity

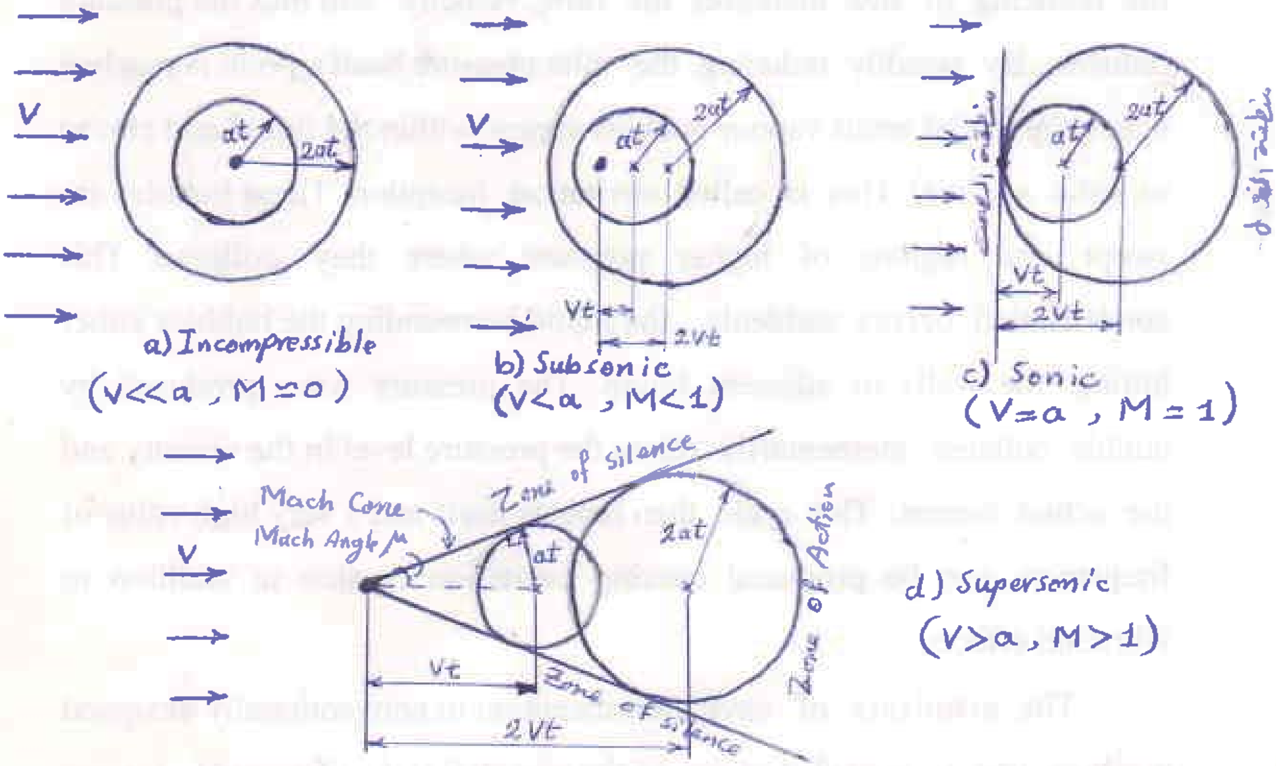
(a) is the sound velocity corresponding to the fluid condition that the fluid velocity (V) is measured.

According to the Mach number the flow is divided into ;

Incompressible flow	$M << 1 \approx 0$	
Subsonic flow	$M < 1$	
Sonic flow	$M = 1$	Transonic flow
Supersonic flow	$M > 1$	
Hypersonic flow	$M >> 1$	

* M له فائدتان : ① تقسيم الجريان الى انواع ② في التحليل البعدي للجريانات الانضغاطية

2.3. Physical Differences Between The Flow Types



$$\sin\mu = \frac{at}{Vt} = \frac{1}{M} \Longrightarrow \boxed{\mu = \sin^{-1}\frac{1}{M}}$$

2.4. The Adiabat. Steady Flow Ellipse

The adiabatic steady flow energy Eqn

$$h_o = h + \frac{V^2}{2} = \text{const.}$$

$$2C_p(T_o - T) = V^2$$

$$V = \sqrt{2C_p(T_o - T)}$$

or

$$V = \sqrt{\frac{2\gamma R\,(T_o - T)}{\gamma - 1}}$$

when a theoretically $= 0$, $T = 0$

hence

$$V_{max} = \sqrt{\frac{2\gamma R T_o}{\gamma - 1}} = \sqrt{\frac{2}{\gamma - 1}\,a_o^2} \quad \text{——— (1)}$$

where $a_o^2 = \gamma R T_o$

Consider a stream tube in which the flow dosenot exchange heat with the fluid in neighboring stream tubes. The steady energy eqn is

$$h + \frac{V^2}{2} = \text{const.} \quad \text{——— (2)}$$

For perfect gas

$$h = C_p\,.T = \frac{\gamma R}{\gamma - 1}\,T \quad ; \quad RT = \frac{P}{\rho}$$

but

$$a^2 = \gamma R T$$

Therefore ,

$$h = \frac{a^2}{\gamma - 1} = \frac{\gamma P}{(\gamma - 1)\rho} \quad \text{——— (3)}$$

From eqn's (2) & (3)

$$\frac{a^2}{\gamma-1} + \frac{V^2}{2} = \text{const.} \qquad \text{——— (4)}$$

For stagnation conditions ($V=0$), eqn (4) will be

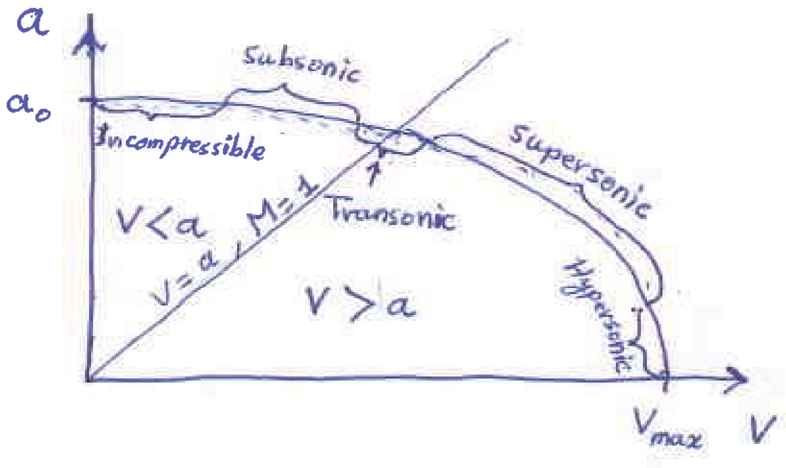
$$\frac{a_o^2}{\gamma-1} = \text{const.} \qquad \text{——— (5)}$$

where a_o is the speed of sound at stagnation conditions. Substitute eqn (1) & (5) in eqn (4) to get,

$$\boxed{\frac{a^2}{a_o^2} + \frac{V^2}{V_{max}^2} = 1} \qquad \text{——— (6)}$$

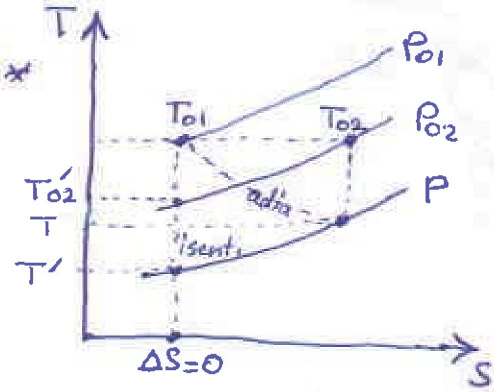
Eqn (6) is called adiabatic Steady flow ellipse which is a useful device to examine the relationship between the flow velocity and speed of sound. The ellipse plotted for $\gamma = 1.4$ is shown below

Incompressible flow ($M < 0.3$)
Subsonic flow ($0.3 < M < 1$)
Transonic flow ($0.8 < M < 1.2$)
Supersonic flow ($1.2 < M < 5$)
Hypersonic flow ($M > 5$)



2.5. Stagnation State

* هي الحالة التي تنتج اذا أوقفنا الجريان تماماً في نفس النقطة بصورة ايسنتروبية.



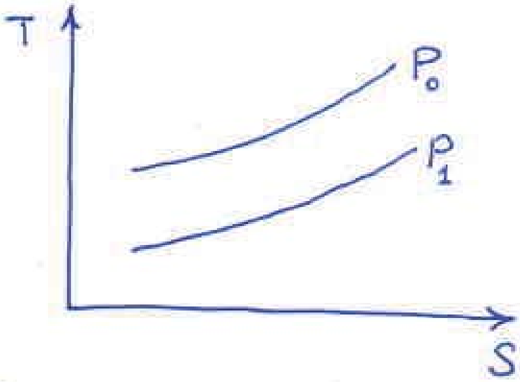
From 1^{st} law of Thermodynamics

$$\Delta E = \cancel{\Delta W}^{\,0} + \cancel{\Delta Q}^{\,0}$$

$$\Delta E = 0 \Longrightarrow E = \text{const.}$$

$$h_0 + \cancel{\frac{V_0^2}{2}}^{\,0} = h + \frac{V^2}{2} = \text{const.} \quad \text{Adiabatic E.E.}$$

For ideal gas



$$C_p T_0 = C_p T + \frac{V^2}{2}$$

For reversible

$$h_{01} = h_{02} \Longrightarrow C_p T_{01} = C_p T_{02} \Longrightarrow T_{01} = T_{02}$$

Increase of entropy

$$\Delta S = C_p \ln \frac{T_{02}}{T_{01}} - R \ln \frac{P_{02}}{P_{01}}$$

$$\Delta S = -R \ln \frac{P_{02}}{P_{01}}$$

For isentropic flow $\Delta S = 0 \Rightarrow \therefore P_{01} = P_{02}$

$0 \longrightarrow 1$

$$C_p T_0 + \frac{V_0^2}{2} = C_p T_1 + \frac{V_1^2}{2}$$

$$C_p T_0 = C_p T_1 + \frac{V_1^2}{2}$$

$$\boxed{T_0 = T_1 \left(1 + \frac{V_1^2}{2 C_p T_1}\right)}$$

$$T_0 = T_1 \left(1 + \frac{V_1^2 (\gamma - 1)}{2 \gamma R T_1}\right)$$

$$\boxed{T_0 = T_1 \left[1 + \frac{\gamma - 1}{2} M_1^2\right]}$$

Very IMPORTANT

$$\frac{T_o}{T_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)$$

$$\left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_o}{T_1} \implies \frac{P_o}{P_1} = \left(\frac{T_o}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \quad \frac{P_o}{P_1} = \left[1 + \frac{\gamma-1}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

Derive all relations $\frac{P}{P_o}$, $\frac{\rho}{\rho_o}$, $\frac{T}{T_o}$

2.6. Critical State

Sonic flow $(M=1)$ is called also critical state. Critical state is considered an important reference state. The superscript (*) refers to that the property is at critical state.

It is clear that

$$V^* = a^* = \sqrt{\gamma R T^*}$$

because $M = M^* = 1$

Also, it is easy to derive $\frac{P^*}{P_o}$, $\frac{\rho^*}{\rho_o}$ & $\frac{T^*}{T_o}$ by substituting $M=1$ in $\frac{P}{P_o}$, $\frac{\rho}{\rho_o}$ & $\frac{T}{T_o}$.

Note: P^*, ρ^* & T^* can be found when stagnation properties are known. (How?)

Ch.3. Isentropic Flow in Variable Area Duct

Baisically, there are three effcts that may cause changes in the properties of compressible flow as follows:

1. Variation of duct's area
2. Friction
3. Heat exchange with surrounding.

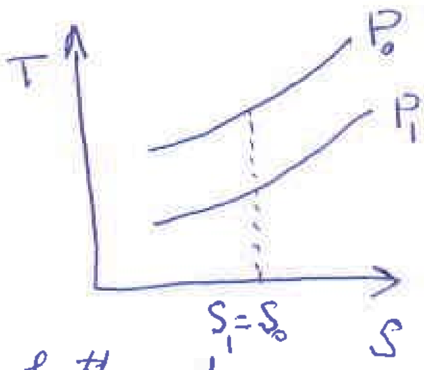
In this chapter, the effect of area variation of duct will be considered only with neglecting the other two effects.

3.1. General Features

The isentropic flow is an adiabatic reversible flow and it is called also variable area duct simple flow. In other word, the only effect on the flow is the varation of area.

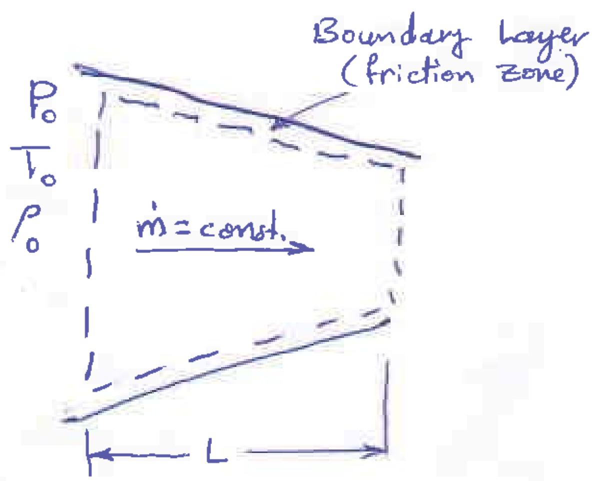
Why do we study this isentropic flow?

Although the isentropic flow is an ideal flow, it is considered an efficient tool by which real flows are to be compared and studied such as flow inside nozzles and diffusers. Also, any relatively short adiabatic flow can be considered an isentropic flow due to negligible friction effect.



2^{nd} Law of thermodynamics

$S_1 = S_o$



Continuity Equ.

$$\dot{m} = \rho V A \implies \frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad \text{——— (3-1)}$$

3.2. Dependance of Mach Number on Area Variation

From eqn. (3-1)

$$\frac{dV}{V} = -\frac{d\rho}{\rho} - \frac{dA}{A} \quad \text{—— ⓐ}$$

and Euler's Eqn.

$$V\,dV + \frac{dP}{\rho} = 0 \quad \xRightarrow{\div V^2} \quad \frac{dV}{V} = -\frac{dP}{\rho V^2} \quad \text{—— ⓑ}$$

Substitute for $\frac{dV}{V}$ from eqns. ⓐ & ⓑ, we get,

$$dP = \rho V^2 \left(\frac{d\rho}{\rho} + \frac{dA}{A} \right) \quad \text{—— (3-2)}$$

Notes:

1. At incompressible flow $d\rho = 0$, $dP > 0$ when $dA > 0$.
2. At low speed $\frac{d\rho}{\rho}$ is very small and can be ignored.
3. At high speed $\frac{d\rho}{\rho}$ with ρV^2 cannot be ignored.

We know that

$$a^2 = \frac{dP}{d\rho} \implies d\rho = \frac{dP}{a^2} \quad \text{—— ⓒ}$$

therefor, by substituting eqn ⓒ into eqn. (3-2), we get,

$$dP = \rho V^2 \frac{dP}{\rho a^2} + \rho V^2 \frac{dA}{A}$$

$$dP - M^2 dP = \rho V^2 \frac{dA}{A} \implies (1 - M^2)\,dP = \rho V^2 \frac{dA}{A} \quad \text{—— (3-3)}$$

Now, substitute eqn ⓒ into eqn (3-3) for dP to get,

$$\frac{d\rho}{\rho} = \frac{M^2}{1 - M^2} \frac{dA}{A} \quad \text{—— (3-4)}$$

By using eqn ⓐ, eqn (3-4) becomes,

$$\frac{dV}{V} = -\frac{1}{1 - M^2} \frac{dA}{A} \quad \text{—— (3-5)}$$

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