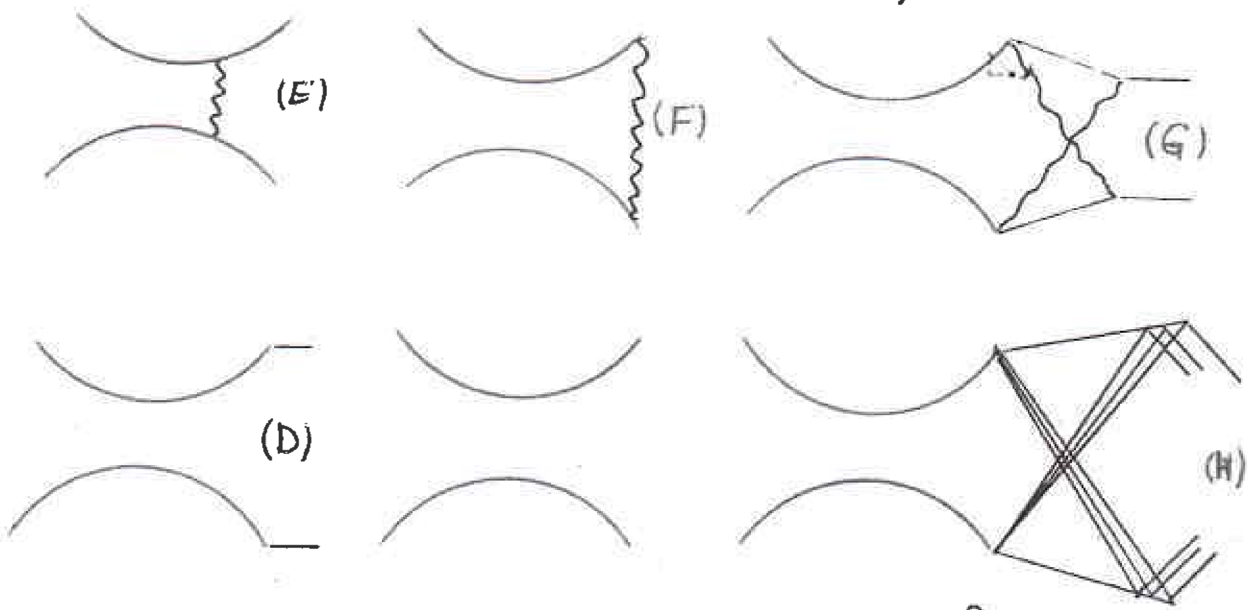
Curve E	Normal shock in the divergent part
Curve F	Normal shock at the exit plane.
Curve D	Perfectly expandend (design)
Curve G	overexpanded (oblique shock waves)
Curve H	underexpanded (expansion waves).



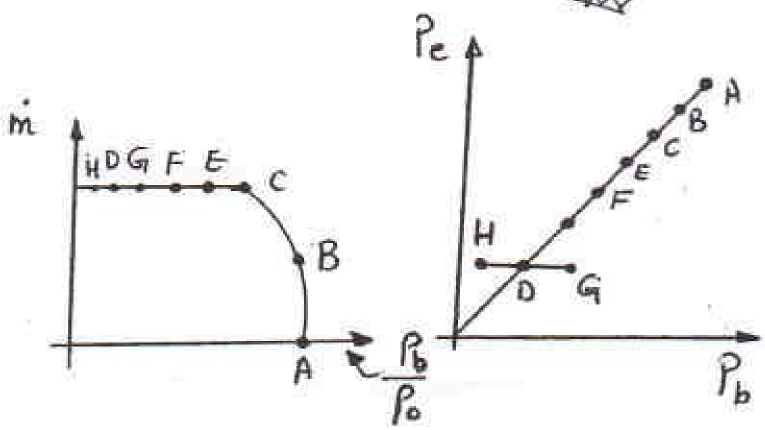


Figure (4. 7)

4.4 Normal Shock Wave Table ($\gamma=1.4$)

M_1	M_2	P_2/P_1	T_2/T_1	ρ_2/ρ_1	P_{o_2}/P_{o_1}

4.5. Moving Normal Shock Wave

In most cases the shock wave moves such as the shock that stands on a moving body or generated from an explosion or the one in a shock tube. Take a normal shock wave that moves with constant velocity (V_s) towards a stationary gas a shown in Figure (4.9a). If a control volume is fixed on the shock as shown in Figure(4.9.b), the relative motion of the gas will be considered.

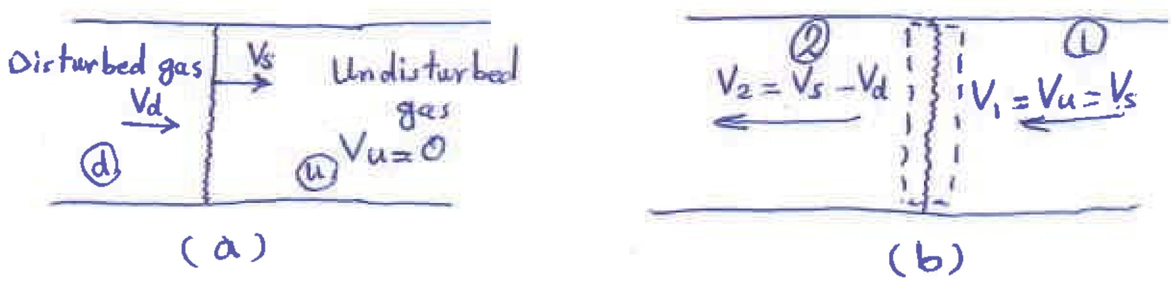


Figure 4.9: Moving shock wave. (a) original case
(b) Transformed case.

4.6. Shock Wave Strength(β)

It's a measure of the pressure increase across the shock.

$$\beta = \frac{P_2}{P_1} - 1 \qquad \text{———} \qquad (4.13)$$

Substituting Eq. (4.7) into Eq. (4.13)

$$\beta = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} - 1$$

$$\therefore \boxed{\beta = \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)} \qquad \text{———} \qquad (4.14)$$

4.7. Oblique Shock Wave (O.S.W.)

Oblique Shock wave appears when the surface changes its direction such as at airplane nose for super-sonic flow. However, a supersonic airplane does not necessarily generate an oblique shock that is attached to its nose. Instead, it may have a detached N.S.W. ahead of the airplane. As the airplane accelerates to its supersonic cruising speed the flow will develop from subsonic, through supersonic with a detached N.S.W. to attached Oblique Shock waves as shown in Figure

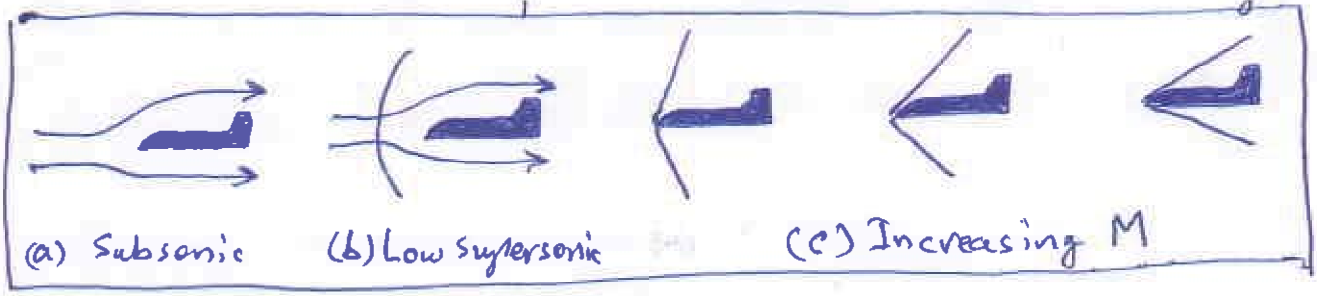


Figure 4.10. Development of shock wave for airplane with increasing speed.

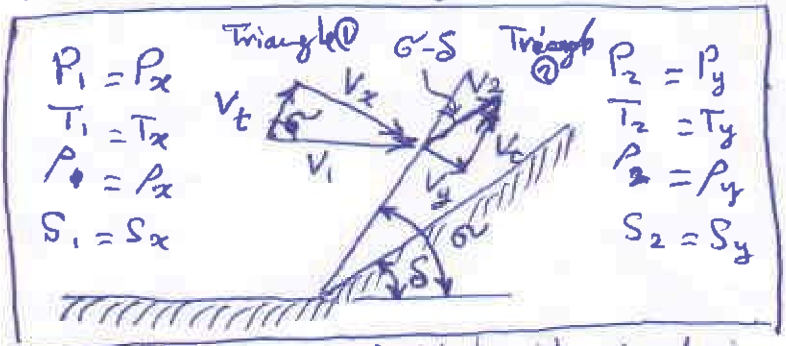


Figure (4.11) Velocity Analysis in Oblique Shock Wave

From triangle ①

$V_x = V_1 \sin\sigma$

$\therefore M_x = M_1 \sin\sigma$ —— (4.15)

From triangle ②

$V_y = V_2 \sin(\sigma-\delta)$

$\therefore M_y = M_2 \sin(\sigma-\delta)$ —— (4.16)

$$\Delta S = \cancel{C_p \ln \frac{T_{02}}{T_{01}}}^{\,0} - R \ln \frac{P_{02}}{P_{01}} \quad \text{(adiabatic)} \Rightarrow S_2 - S_1 = -R \ln \frac{P_{02}}{P_{01}}$$

$$\Delta S = \cancel{C_p \ln \frac{T_{0y}}{T_{0x}}}^{\,0} - R \ln \frac{P_{0y}}{P_{0x}} \quad \text{(adiabatic)} \Rightarrow S_y - S_x = -R \ln \frac{P_{0y}}{P_{0x}}$$

but $S_y - S_x = S_2 - S_1$

$$\therefore \frac{P_{0y}}{P_{0x}} = \frac{P_{02}}{P_{01}} \quad \text{———— (4.17)}$$ But $P_{0y} \neq P_{02}$ & $P_{0x} \neq P_{01}$

at $M_x \xrightarrow{\text{N.S.W. table}} \frac{P_y}{P_x} = \frac{P_2}{P_1}, \; \frac{T_y}{T_x} = \frac{T_2}{T_1}, \; \frac{\rho_y}{\rho_x} = \frac{\rho_2}{\rho_1}$ ---- etc. —(4.18)

To find the relation between the angles σ & δ, the velocity component triangles upstream and down stream can be used as follows:

$$\tan\delta = \frac{V_x}{V_t}$$

$$\tan(\sigma - \delta) = \frac{V_y}{V_t}$$

By dividing the second above equation to the first equation above, the following equation is obtained

$$\tan\delta = 2\cot\sigma \; \frac{M_1^2 \sin^2\sigma - 1}{M_1^2(\gamma + \cos 2\sigma) + 2} \quad \text{——— (4.19)}$$

This relation is represented by the following figure (4.12)

$\therefore M_1 \geq 1$ for N.S.W.

$\therefore M_1 \sin\sigma \geq 1$ for O.S.W.

$$\sin\sigma \geq \frac{1}{M_1}$$

$$\sigma \geq \sin^{-1}\frac{1}{M_1} \quad \text{——— (4.20)}$$

For a certain value of M_1, the minimum value of σ is $\left(\sigma_{min} = \sin^{-1}\frac{1}{M_1}\right)$ the Mach angle and the shock wave is Mach wave. On the other hand, the maximum value of σ is 90° (the wave is N.S.W)

$$\sin^{-1}\frac{1}{M_1} \leq \sigma \leq 90^\circ \quad \text{——— (4.21)}$$

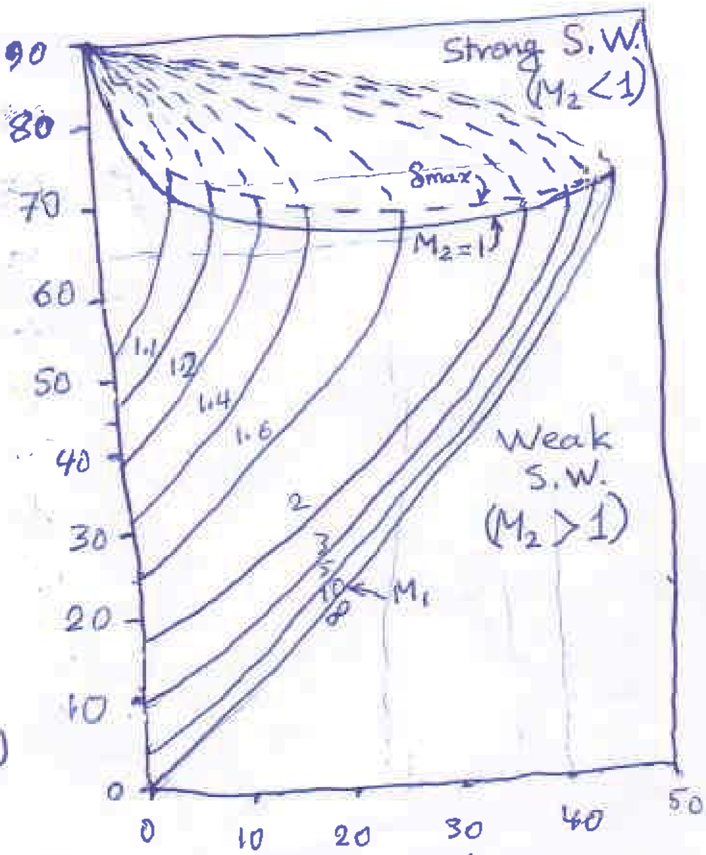


Figure (4.12) Oblique shock Chart

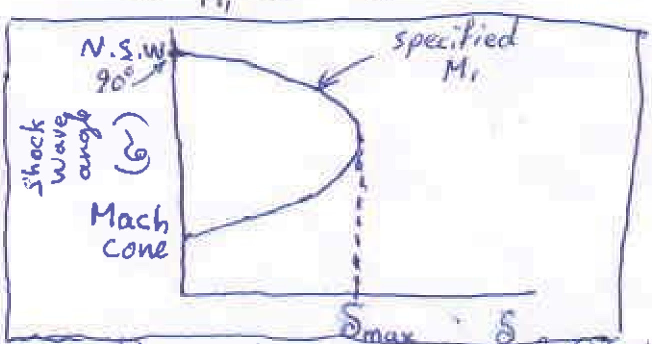


Figure (4.13) Limits of O.S.W. angle

There are two cases :

(1) If $\delta > \delta_{max}$, there is no solution as shown in Figure (4.12), while a detached bow O.S.W. is appeared a head as shown in Figure (4.14).

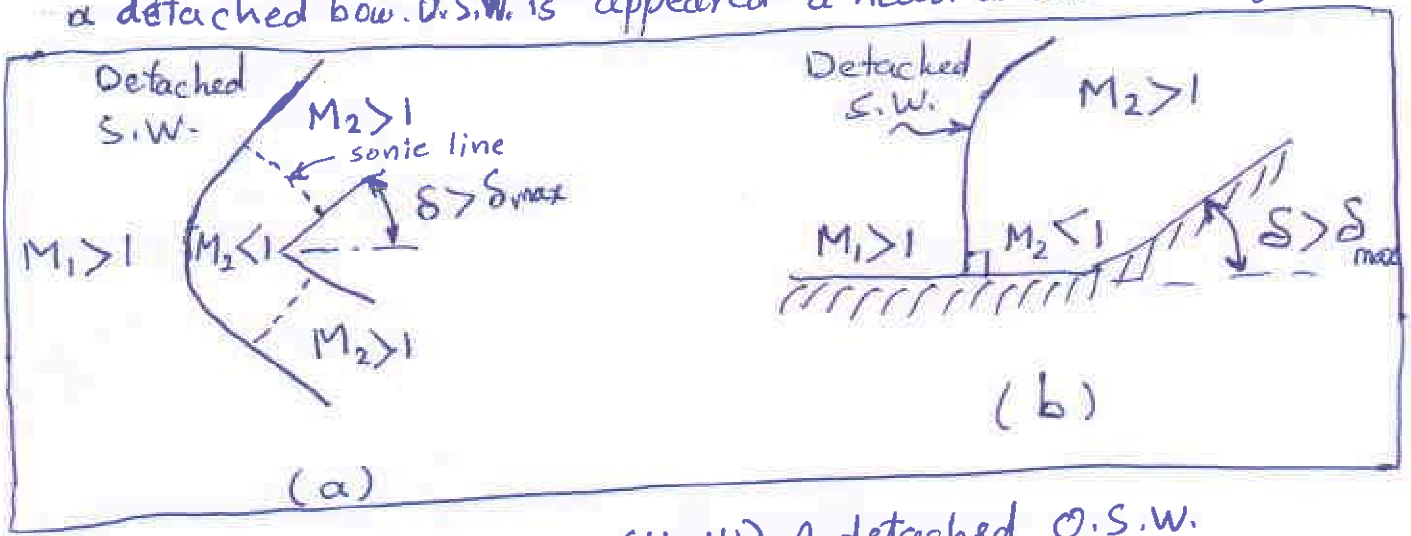


Figure (4.14) A detached O.S.W. ahead (a) a wedge & (b) 2D Passage with concave sudden turn

Same thing happens when $M_1 < M_{1\,min}$ where $M_{1\,min}$ is the value of M_1 at which O.S.W. is attached the leading edge of the wedge or the deflection in a 2-D passage. The region down stream the detached O.S.W. is very complicated where both subsonic & supersonic field are existed. The detached bow O.S.W. is strong ($M_2<1$) as σ is large while it is weak ($M_2>1$) as σ has a small value.

(2) If $\delta < \delta_{max}$ ($M_1 > M_{1\,min}$) there are two solutions : Strong shock solution when σ has a high value & weak shock solution when σ is at its low value as descriped in Figure (4.15).

Experimentally, it is found that for a given M_1 & δ in external flows the shock wave is weak.

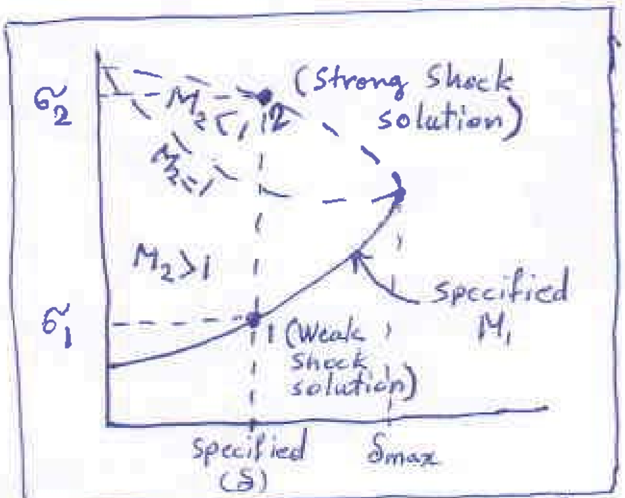


Figure 4.15 Strong & Weak O.S.W. solutions

For external flow such the flow over a wedge shown in Figure (4.16), either both solutions are considered, weak wave solution is considered if the experimental experience is adopted (mentioned in the problem), strong wave solution is considered if it is required in the problem or the solution can be concluded:

① if $M_2 < 1 \Rightarrow$ strong wave solution

② if there are series of waves (shock system) $\Rightarrow$ weak wave solution.

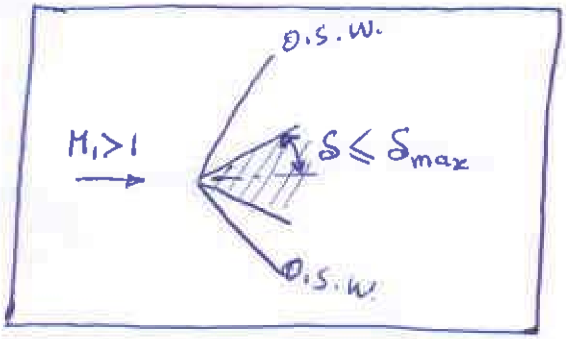


Figure 4.16. Attached O.S.W. over a wedge

For a supersonic flow through a 2-D passage with sudden turn with $\delta \leq \delta_{max}$ (or $M_1 \geq M_{1,min}$), the two solutions are described as follows:

① Strong wave solution

If the downstream pressure is sufficiently high, an O.S.W. will appear attached to the lower wall and N.S.W. to the upper wall ($\delta = 0$) and the flow is non uniform downstream the shock wave as shown in Figure (4.17)

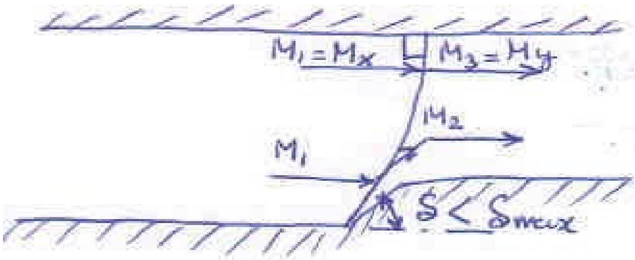
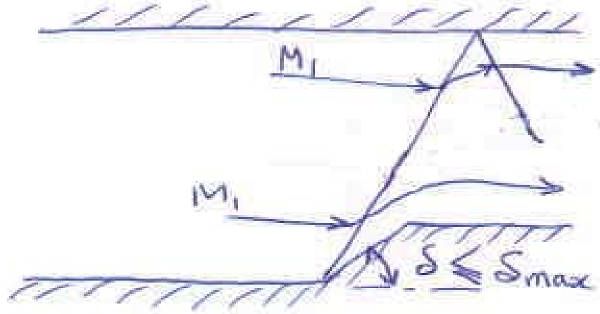


Figure (4.17) Strong O.S.W. in a 2-D passage with sudden turn.

② Weak Wave Solution

The wave stay's straight and it is reflected from the upper wall at different σ. The flow down stream the weak O.S.W. remains uniform as shown in Figure 4.18



supersonic

CH. 5 : Constant Area Duct with Friction or with Heat Transfer

5.1. Governing Equations of Flow in Constant Area Duct with Friction

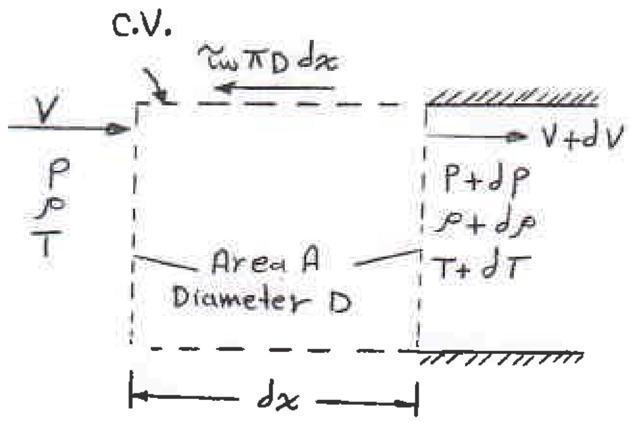


Figure (5.1) C.V. for flow in a constant area duct with friction.

Continuity

$$\rho V A = \dot m \longrightarrow \rho V = \frac{\dot m}{A} = G = \text{constant} \longrightarrow \frac{d\rho}{\rho} + \frac{dV}{V} = 0 \quad \text{------- (5.1)}$$

x – Momentum

$$PA - (P+dP)\underbrace{A}_{\frac{\pi}{4}D^2} - \tau_w \pi D\, dx = \dot m(\cancel{V} + dV - \cancel{V})$$

$$\text{or} \quad dP + \frac{4\tau_w\, dx}{D} + \rho V\, dV = 0 \quad \text{------------ (5.2)}$$

Energy Equation

$$h + \frac{1}{2}V^2 = h_o = C_p T_o = C_p T + \frac{V^2}{2} = \text{constant}$$

$$\text{or} \quad C_p\, dT + V\, dV = 0 \quad \text{------- (5.3)}$$

Perfect Gas Law

$$P = \rho R T \quad \text{or} \quad \frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \quad \text{----- (5.4)}$$

Fanning Friction Factor

$$\tau_w = \frac{1}{2} f \rho V^2 = \frac{1}{2} f \gamma P M^2 \quad \text{-------- (5.5)}$$

$$\therefore V^2 = \gamma R T M^2 \quad \text{or} \longrightarrow \underbrace{\frac{dV^2}{V^2}}_{\frac{2dV}{V}} = \frac{dM^2}{M^2} + \frac{dT}{T} \quad \text{------ (5.6)}$$

Equation (5.2) can be written as

$$dp = -\frac{2f\gamma p M^2 dx}{D} - \rho V dV \quad \div p$$

$$\frac{dp}{p} = -2\gamma M^2 \frac{f}{D} dx - \overset{\frac{1}{RT}}{\frac{\rho}{p}} V dV = -2\gamma M^2 \frac{f}{D} dx - \frac{\gamma V^2}{\gamma RT} \frac{dV}{V}$$

$$\frac{dp}{p} = -2 \frac{f}{D} \gamma M^2 dx - \gamma M^2 \frac{dV}{V} \quad \text{------} \quad (5.7)$$

rewrite equation (5.4) as

$$\frac{dp}{p} = \overset{d\rho/\rho}{\left(-\frac{dV}{V}\right)} + \frac{dT}{T}$$

From equation (5.3), we can write $dT + \frac{V}{\varphi} dV = 0 \quad \div T$

$$\frac{dT}{T} = -\frac{V}{\varphi T} dV = -\frac{V^2 \gamma R}{\varphi T \gamma R} \frac{dV}{V} = -(\gamma - 1) M^2 \frac{dV}{V}$$

$$\therefore \frac{dp}{p} = -\frac{dV}{V} - (\gamma - 1) M^2 \frac{dV}{V} \quad \text{------} \quad (5.8)$$

~~By equation~~ Equate eqs. (5.7) and (5.8), we get

$$-2 \frac{f}{D} \gamma M^2 dx - \cancel{\gamma M^2 \frac{dV}{V}} = -\frac{dV}{V} - \cancel{\gamma M^2 \frac{dV}{V}} + M^2 \frac{dV}{V}$$

$$\therefore \boxed{\frac{dV}{V} = 2f \frac{dx}{D} \frac{\gamma M^2}{(1 - M^2)}} \quad \text{------} \quad (5.9)$$

Substitution of equation (5.9) into (5.8), gives

$$\boxed{\frac{dp}{p} = -\frac{[1 + (\gamma - 1) M^2] \gamma M^2}{(1 - M^2)} 2f \frac{dx}{D}} \quad \text{------} \quad (5.10)$$

and

$$\boxed{\frac{dT}{T} = -(\gamma - 1) M^2 \frac{dV}{V} = -\frac{(\gamma - 1) \gamma M^4}{(1 - M^2)} 2f \frac{dx}{D}} \quad \text{------} \quad (5.11)$$

Substitution of equ. (5.9) into equ. (5.1), gives:-

$$\boxed{\frac{d\rho}{\rho} = -2f \frac{dx}{D} \frac{\gamma M^2}{(1 - M^2)}} \quad \text{------} \quad (5.12)$$

Substitution of eqs. (5.9) and (5.11) into equ. (5.6), gives:-

$$\boxed{\frac{dM^2}{M^2} = \frac{\gamma M^2\left[1+\frac{\gamma-1}{2}M^2\right]}{(1-M^2)}\,4f\,\frac{dx}{D}} \quad \text{------ (5.13)}$$

Property		Subsonic		Supersonic
V	⟶	increases	⟶	decreases
P	⟶	decreases	⟶	increases
T	⟶	decreases	⟶	increases
ρ	⟶	decreases	⟶	increases
M	⟶	increases	⟶	decreases

5.2. Fanno Line

From 1^{st} & 2^{nd} Laws of Thermodynamics, the following Eq. can be derived,

$$ds = c_p\frac{dT}{T} - \frac{R}{P}dP \qquad \div c_p \qquad \frac{R}{c_p} = \frac{\gamma-1}{\gamma}$$

$$\frac{ds}{c_p} = \frac{dT}{T} - \frac{\gamma-1}{\gamma}\,\frac{dP}{P} \quad \text{------ (5.14)}$$

from energy equation $c_p dT + V dV = 0 \longrightarrow \frac{dV}{V} = -\frac{c_p dT}{V^2}$ ---- (5.15)

From equation (5.4) and (5.1) and (5.15)

$$\frac{dP}{P} = \frac{dT}{T} + \frac{c_p}{V^2}dT \quad \text{------ (5.16)}$$

substitution of equ. (5.16) into (5.14) gives

$$\frac{ds}{c_p} = \frac{dT}{T} - \frac{\gamma-1}{\gamma}\left(\frac{dT}{T} + \frac{c_p}{V^2}dT\right) \quad \text{------ (5.17)}$$

$$\because\ T_o = T + \frac{V^2}{2c_p} \longrightarrow V^2 = 2c_p(T_o - T) \quad \text{---- (5.18)}$$

From equ. (5.17) and (5.18) we have:-

$$\frac{ds}{c_p} = \left(1 - \frac{\gamma-1}{\gamma}\right)\frac{dT}{T} - \frac{\gamma-1}{\gamma}\,\frac{\cancel{c_p}}{2\cancel{c_p}(T_o-T)}\,dT$$

or

$$\frac{ds}{c_p} = \frac{1}{\gamma}\,\frac{dT}{T} - \frac{\gamma-1}{2\gamma}\,\frac{dT}{T_o-T} \quad \text{------ (5.19)}$$

By Integrating equation (5.19) between arbitrary state (T_1 & S_1) and any Value of (T) and (S), we have :-

$$\frac{S-S_1}{C_p} = \ln\left[\left(\frac{T}{T_1}\right)^{\frac{1}{\gamma}}\left(\frac{T_o-T}{T_o-T_1}\right)^{\frac{\gamma-1}{2\gamma}}\right] \quad ------- (5.20)$$

Equation (5.20) is the equation of Fanno line for a perfect gas.

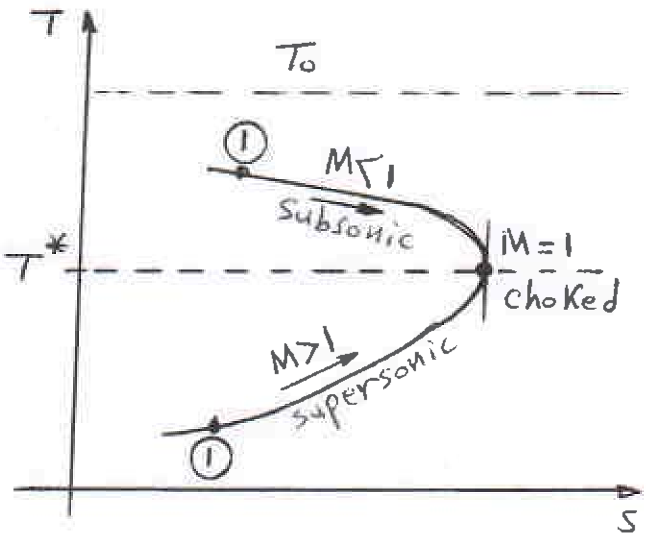


Figure (5.2) Fanno line

According to 2nd law, entropy must be increase, so that the path of states along any one of the Fanno lines must be towards right.

Case ① subsonic The effects of friction tend to increase Mach number and to decrease the temperature.
So that the upper part of Fanno curve is for subsonic flow.

Case ① Supersonic The effects of friction tend to decrease Mach number and to increase the temperature.
So that the lower part of Fanno curve is for supersonic flow.

To find the point of maximum entropy on Fanno line. From equation (5.19)

$$\frac{ds}{C_p} = \frac{1}{\gamma}\frac{dT}{T} - \frac{\gamma-1}{2\gamma}\frac{dT}{T_o-T} \qquad \div dT$$

$$\frac{ds}{dT} = \frac{1}{\gamma} C_p \frac{1}{T} - \frac{\gamma-1}{2\gamma} C_p \frac{1}{T_o-T} \qquad (\text{put } \frac{ds}{dT}=0)$$

$$\frac{1}{\gamma}\frac{1}{T} = \frac{\gamma-1}{2\gamma}\frac{1}{T_o-T} \qquad \because V^2 = 2C_p(T_o-T) \quad \text{and} \quad C_p = \frac{\gamma R}{\gamma-1}$$

$$\therefore \frac{1}{T} = \frac{\cancel{\gamma T}}{V^2}\,\frac{\gamma R}{\cancel{\gamma T}} \longrightarrow V^2 = \gamma R T \longrightarrow \boxed{M=1}$$

This means that $M=1$ at point of maximum entropy.
For Friction Flow and according to 2nd law A subsonic flow never becomes supersonic, and supersonic flow never becomes subsonic, unless a shock wave is present.

5.3. Relations For the Frictional Flow

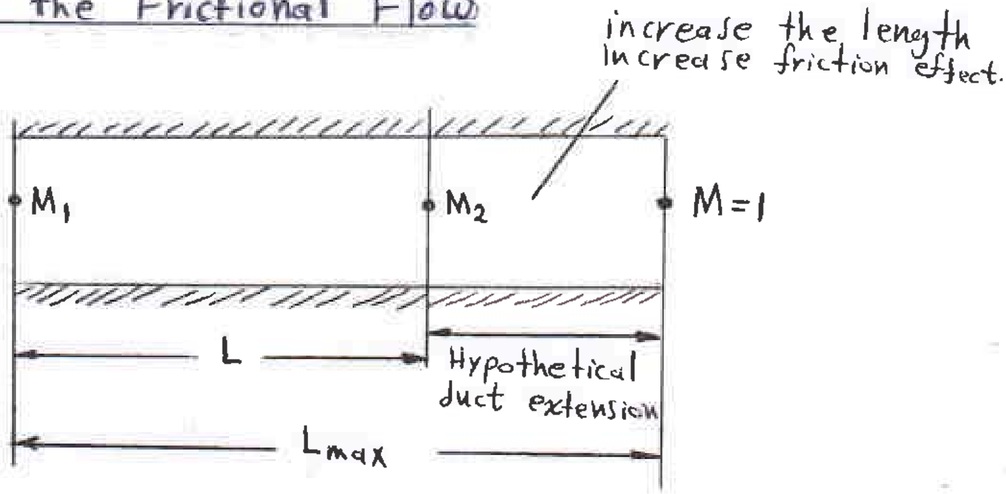


Figure (5.3)

By integrating equation (5.13) between a section where Mach number is (M) and the section where (M=1), we get:-

$$\int_{M^2}^{1} \frac{1-M^2}{\gamma M^4 \left(1+\frac{\gamma-1}{2}M^2\right)}\, dM^2 = \int_0^{L_{max}} 4f\,\frac{dx}{D} \quad \text{------} \quad (5.21)$$

Integration the left side by partial fraction.

$$\frac{4\bar{f}L_{max}}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M^2}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right) \quad \text{------} \quad (5.22)$$

where $\bar{f}$ is the mean friction factor, defined over the duct length as:-

$$\bar{f} = \frac{1}{L_{max}}\int_0^{L_{max}} f\,dx$$

The duct length (L) required for Mach number to change from M_1 to M_2 can be found as

$$\frac{4\bar{f}L}{D} = \left(\frac{4\bar{f}L_{max}}{D}\right)_{M_1} - \left(\frac{4\bar{f}L_{max}}{D}\right)_{M_2} \quad \text{------} \quad (5.23)$$

$$\because \frac{T}{T^*} = \frac{T}{T_o}\frac{T_o}{T^*} \qquad \frac{T_o}{T} = 1+\frac{\gamma-1}{2}M^2, \text{ and } \frac{T_o}{T^*} = 1+\frac{\gamma-1}{2} = \frac{\gamma+1}{2}$$

$$\boxed{\therefore \frac{T}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M^2}} \quad \text{------} (5.24)$$

$$\frac{V}{V^*} = \frac{M\sqrt{\gamma RT}}{\sqrt{\gamma RT^*}} = M\sqrt{\frac{T}{T^*}}$$

$$\boxed{\therefore \frac{V}{V^*} = M\sqrt{\frac{\gamma+1}{2+(\gamma-1)M^2}}} \quad \text{------} (5.25)$$

By continuity equation for steady flow in constant area duct

$$\rho V = \rho^* V^*$$

$$\boxed{\frac{\rho}{\rho^*} = \frac{V^*}{V} = \frac{1}{M}\sqrt{\frac{2+(\gamma-1)M^2}{\gamma+1}}} \quad \text{------} (5.26)$$

and by perfect gas law

$$\frac{P}{P^*} = \frac{\rho}{\rho^*}\frac{T}{T^*}$$

By using eqs. (5.24) and (5.26)

$$\boxed{\frac{P}{P^*} = \frac{1}{M}\sqrt{\frac{\gamma+1}{2+(\gamma-1)M^2}}} \quad \text{------} (5.27)$$

$$\frac{P_o}{P_o^*} = \frac{P_o}{P}\frac{P}{P^*}\frac{P^*}{P_o^*}$$

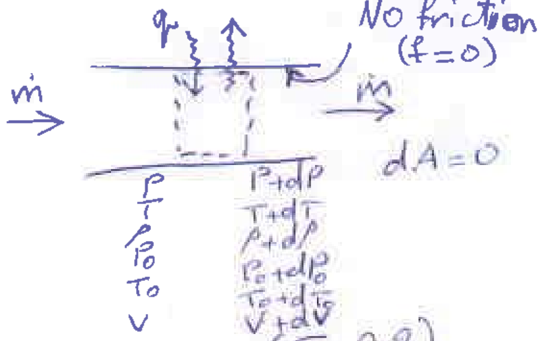
$$= \left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{\gamma}{\gamma-1}} \quad \xrightarrow{5.27} \frac{1}{M}\sqrt{\frac{\gamma+1}{2+(\gamma-1)M^2}} \quad \frac{1}{\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{1}{M}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{-\frac{1}{2}}$$

$$\boxed{\frac{P_o}{P_o^*} = \frac{1}{M}\left(\frac{2+(\gamma-1)M^2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}} \quad \text{------} (5.28)$$

5.4. Frictionless Flow with Heat Transfer in Constant Area Duct

It is the flow with heat added or cooling with no friction. This is unreal hypothesis since it is impossible to neglect the friction except in very small duct length. Combustion chamber is a good example of this case.



Basic Equations

$dA = 0$ & $f = 0$

$$\therefore \frac{dT_o}{T_o} = \frac{1-M^2}{(1+\gamma M^2)(1+\frac{\gamma-1}{2}M^2)} \frac{dM^2}{M^2} \qquad (5.29)$$

$$\frac{dV}{V} = \frac{1}{1+\gamma M^2} \frac{dM^2}{M^2} \qquad (5.30)$$

$$\frac{dP}{P} = -\frac{\gamma M^2}{1+\gamma M^2} \frac{dM^2}{M^2} \qquad (5.31)$$

$$\frac{d\rho}{\rho} = -\frac{1}{1+\gamma M^2} \frac{dM^2}{M^2} \qquad (5.32)$$

$$\frac{dT}{T} = \frac{(1-\gamma M^2)(1-M^2)}{(1+\gamma M^2)} \frac{dM^2}{M^2} \qquad (5.33)$$

$$\frac{dP_o}{P_o} = -\frac{\gamma M^2(1-M^2)}{2(1+\gamma M^2)(1+\frac{\gamma-1}{2}M^2)} \frac{dM^2}{M^2} \qquad (5.34)$$

$$\frac{dS}{C_p} = \frac{1-M^2}{1+\gamma M^2} \frac{dM^2}{M^2} \qquad (5.35)$$

$$dq = C_p \, dT_o \qquad (5.36)$$

From Eq. (5.29) & Eq. (5.36),

at $M=1 \Longrightarrow \frac{dq}{dM}=0$.

This means that the heat exchanged with the flow per unit of mass (q) reaches its maximum value. Therefor, there is no additional heat exchange for the same inlet conditions as the duct exit at the critical state. Hence, the critical state represents the state of the end point in the duct (exit).

By integrating Eq. (5.29) — Eq (5.35) between a certain point and critical state point, the following relations are obtained:

$$\frac{T_o}{T_o^*} = \frac{2(\gamma+1)M^2\left(1+\frac{\gamma-1}{2}M^2\right)}{1+\gamma M^2} \quad \text{———} \quad (5.37)$$

$$\frac{V}{V^*} = \frac{(\gamma+1)M^2}{1+\gamma M^2} \quad \text{———} \quad (5.38)$$

$$\frac{P}{P^*} = \frac{\gamma+1}{1+\gamma M^2} \quad \text{———} \quad (5.39)$$

$$\frac{\rho}{\rho^*} = \frac{1+\gamma M^2}{(\gamma+1)M^2} \quad \text{———} \quad (5.40)$$

$$\frac{T}{T^*} = \frac{(\gamma+1)^2 M^2}{(1+\gamma M^2)^2} \quad \text{———} \quad (5.41)$$

$$\frac{P_o}{P_o^*} = \frac{\gamma+1}{1+\gamma M^2}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}} \quad \text{———} \quad (5.42)$$

$$\frac{S-S^*}{C_p} = \ln\left[M^2\left(\frac{\gamma+1}{1+\gamma M^2}\right)^{\frac{\gamma}{\gamma-1}}\right] \quad \text{———} \quad (5.43)$$

Now, q between any two points in the duct can be calculated from

$$\boxed{q = C_p\,(T_{o2} - T_{o1})} \quad \text{———} \quad (5.44)$$

5.5 Rayleigh Line

It represents all points that can be reached from a starting point in a constant area duct that has a simple change in T_0. This line can be drawn in the T-S diagram using the following equation which is derived from Eq. (5.41) & Eq (5.43) as shown in Figure

$$\frac{S-S^*}{C_p} = \ln\left[\frac{T}{T^*}\left(\frac{1}{(\gamma+1)-\sqrt{(\gamma+1)^2-4\gamma\frac{T}{T^*}}}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad (5.45)$$

Heat addition moves Mach number towards 1, while heat removal moves it away from 1.

Comparing with Fanno line, Rayleigh line has same behavior in heating but opposit result in cooling.

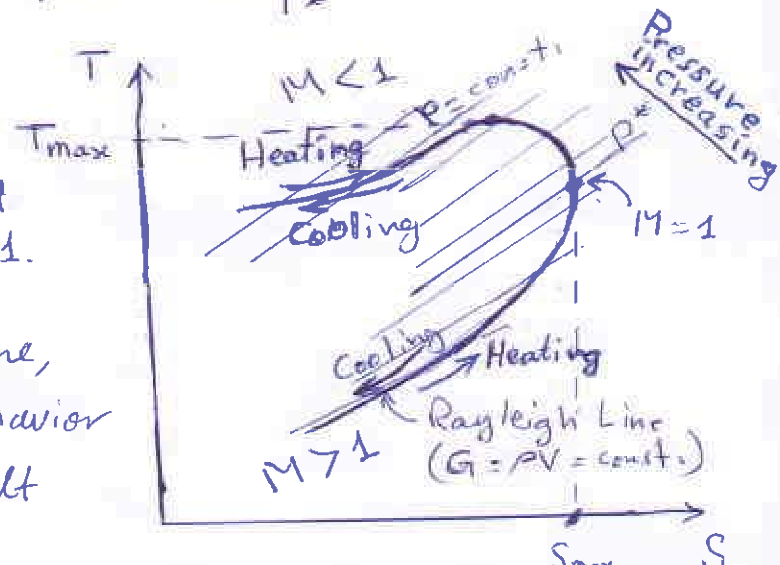


Figure 5.4. Rayleigh Line

For ease of calculations, Rayleigh flow characteristics of gas with $\gamma = 1.4$ (such air) are listed in Rayleigh Flow table

Example (1)

Air flows in frictionless constant area duct from poin(1) at $V_1 = 100$ m/s , $P_1 = 50$ kPa and $T_1 = 50\,^\circ C$. If heat is added between point (1) and point (2) by a rate of 100 kca/Kg,

a) compute M_2 , P_2 , T_2 , V_2 & P_{o2}

b) What is the maximum heat per unit of mass that can be added to the duct at same inlet conditions ?

Solution

a)

① $$M_1 = \frac{V_1}{a_1} = \frac{V_1}{\sqrt{\gamma R T_1}} = \frac{100}{\sqrt{1.4 * 287 * 323}} = 0.278$$

② From isentropic flow table at $M_1 = 0.278$,

$$\frac{P_1}{P_{o1}} = 0.94772 \Longrightarrow P_{o1} = \frac{P_1}{0.94772} = \frac{50}{0.94772} = 52.8 \text{ kPa}$$

$$\frac{T_1}{T_{o1}} = 0.98477 \Longrightarrow T_{o1} = \frac{T_1}{0.98477} = \frac{323}{0.98477} = 328\,^\circ K$$

③ Also, from Rayleigh flow table at $M_1 = 0.278$,

$$\frac{T_{o1}}{T_o^*} = 0.30674 \Longrightarrow T_o^* = \frac{T_{o1}}{0.30674} = \frac{328}{0.30674} = 1069.3\,^\circ K$$

$$\frac{T_1}{T^*} = 0.36248 \Longrightarrow T^* = \frac{T_1}{0.36248} = \frac{323}{0.36248} = 891\,^\circ K$$

$$\frac{P_{o1}}{P_o^*} = 1.2072 \Longrightarrow P_o^* = \frac{P_{o1}}{1.2072} = \frac{52.8}{1.2072} = 43.7 \text{ kPa}$$

$$\frac{P_1}{P^*} = 2.1656 \Longrightarrow P^* = \frac{P_1}{2.1656} = \frac{50}{2.1656} = 23 \text{ kPa}$$

$$\frac{V_1}{V^*} = 0.16739 \Longrightarrow V^* = \frac{V_1}{0.16739} = \frac{100}{0.16739} = 597.4 \text{ m/s}$$

Now, T_{02} can be calculated from Eq (5.44),

④ $$T_{02} = T_{01} + \frac{q}{C_p} = 328 + \frac{100}{0.24} = 744\ ^\circ K$$

$$\therefore \frac{T_{02}}{T_0^*} = \frac{744}{1069.3} = 0.6957$$

From Rayleigh table at $\frac{T_{02}}{T_0^*} = 0.695$

$M_2 = 0.503$ (subsonic) or $M_2 = 2.61$ (supersonic)

We select $\boxed{M_2 = 0.503}$ because $M_1 = 0.278$ (subsonic)

⑤ $$\frac{T_2}{T^*} = 0.79389 \Rightarrow T_2 = T^* * 0.79389 = 891 * 0.79389 = \boxed{707\ ^\circ K}$$

$$\frac{P_2}{P^*} = 1.7732 \Rightarrow P_2 = P^* * 1.7732 = 23 * 1.7732 = \boxed{40.8\ kPa}$$

$$\frac{P_{02}}{P_0^*} = 1.113 \Rightarrow P_{02} = P_0^* * 1.113 = 43.7 * 1.113 = \boxed{48.6\ kPa}$$

$$\frac{V_2}{V^*} = 0.44776 \Rightarrow V_2 = V^* * 0.44776 = 597.4 * 0.44776 = \boxed{267.5\ m/s}$$

b) $q = q_{max}$ at $T_{02} = T_0^*$

$$\therefore q_{max} = C_p (T_0^* - T_{01})$$

$$= 0.24 (1069.3 - 328) = \boxed{178\ Kcal/kg}$$