1.1 TRANSPORT PROCESSES

In most cases, contaminants are introduced into rivers over a particular subarea of the river cross section, For example, pipe discharges are typically over a small area along the side of the river, whereas submerged multiport diffuser discharges are over a larger portion of the river cross section. In these cases, two distinct zones are identified: the initial zone where the contaminant mixes across the cross section of the channel, and the well-mixed zone, where the contaminant is well-mixed across the cross section and further mixing is associated with longitudinal dispersion in the flow direction.

1.1.1 Initial Mixing

The turbulent velocity fluctuations in the vertical and transverse directions in rivers are on the same order of magnitude as the *shear velocity*, $u*$, which is defined by :

$$u* = \sqrt{\frac{f}{8}}\, V \quad \text{------------- Eq.(1)}$$

Where :

V = the mean (longitudinal) flow velocity, m/s

f = Darcy–Weisbach friction factor

The vertical turbulent diffusion coefficient, $\boldsymbol{\varepsilon_v}$, in a wide open channel can be estimated by the relation :

$$\boldsymbol{\varepsilon_v} = 0.067 du^* \quad \text{----------- Eq.(2)}$$

Where :

d = the depth of flow in the channel, m

Experimental results in straight rectangular channels indicate that the transverse turbulent diffusion coefficient, $\boldsymbol{\varepsilon_t}$, can be estimated by the relation:

$$\boldsymbol{\varepsilon_t} = 0.6 du^* \text{ (natural streams) ------------ Eq.(3)}$$

Considering a stream of characteristic depth d and width w, the time scale, T_d, for mixing over the depth of the channel can be estimated by :

$$T_d = \frac{d^2}{\boldsymbol{\varepsilon_v}} \quad \text{---------- Eq.(4)}$$

and the distance, L_d, downstream from the discharge point to where complete mixing over the depth occurs is given by :

$$L_d = VT_d = V\frac{d^2}{\varepsilon_v} \quad \text{---------- Eq.(5)}$$

Similarly, the time scale, T_w, for mixing over the width, w, can be estimated by:

$$T_w = \frac{w^2}{\varepsilon_t} \quad \text{---------- Eq.(6)}$$

and the corresponding downstream length scale, Lw, to where the tracer is well mixed over the width can be estimated by:

$$L_w = VT_w = V\frac{w^2}{\varepsilon_t} \quad \text{---------- Eq.(7)}$$

Fischer (1979) used field measurements to estimate the actual distance, L_w , for a single-port discharge located on the side of a channel to mix completely across a stream as :

$$L_w{}' = \frac{0.4\, Vw^2}{\varepsilon_t} \quad \text{at any discharge location in a stream --------- Eq.(8.a)}$$

if a multiport outfall of length L is placed in the center of a stream of width W, full cross-sectional mixing occurs when the contaminant mixes over a width, w=(W-L)/2, and the downstream distance, L_w, to complete cross-sectional mixing is given by:

$$L_w{}' = \frac{0.1\, V(W-L)^2}{\varepsilon_t} \quad \text{---------------- Eq.(8.b)}$$

Clearly, cross-sectional mixing can be accelerated by using multiport diffusers rather than single-port outlets.

Example 4.1 A municipality discharges wastewater from the side of a stream that is 10 m wide and 2 m deep. The average flow velocity in the stream is 1.5 m/s, and the friction factor is estimated to be 0.03 (calculated using the Colebrook equation). (a) Estimate the time for the wastewater to become well mixed over the channel cross section. (b) How far downstream from the discharge location can the effluent be considered well mixed across the stream?

SOLUTION (a) From the data given, $f = 0.03$ and $V = 1.5$ m/s. Therefore, the shear velocity, u_*, is given by Equation 4.3 as

$$u_* = \sqrt{\frac{f}{8}}V = \sqrt{\frac{0.03}{8}}(1.5) = 0.092 \text{ m/s}$$

Since $d = 2$ m, the vertical and transverse diffusion coefficients are

$$\varepsilon_v = 0.067du_* = 0.067(2)(0.092) = 0.012 \text{ m}^2/\text{s}$$
$$\varepsilon_t = 0.6du_* = 0.6(2)(0.092) = 0.11 \text{ m}^2/\text{s}$$

The time scale for vertical mixing, T_d, is given by

$$T_d = \frac{d^2}{\varepsilon_v} = \frac{2^2}{0.012} = 333 \text{ s} = 5.6 \text{ min}$$

and the time scale for transverse mixing, T_w, is given by

$$T_w = \frac{w^2}{\varepsilon_t} = \frac{10^2}{0.11} = 909 \text{ s} = 15 \text{ min}$$

The discharge is well mixed over the channel cross section when it is well mixed over both the depth and the width, which in this case occurs after about 15 min.

(b) In a time interval of 15 min (= 909 s), the discharged effluent travels a distance, VT_w, given by

$$VT_w = 1.5(909) = 1364 \text{ m}$$

The Fischer et al. (1979) relation given by Equation 4.15 indicates that the actual downstream distance required for complete cross-sectional mixing is $0.4VT_w = 0.4(1364) = 546$ m.

Example 4.2 Estimate the distance downstream to where the wastewater described in Example 4.1 is well mixed across the stream if (a) the wastewater is discharged from the center of the stream, and (b) the wastewater is discharged through a 5-m-long multiport diffuser placed in the middle of the stream.

SOLUTION From the previous analysis: $\varepsilon_v = 0.012\,\text{m}^2/\text{s}$, $\varepsilon_t = 0.11\,\text{m}^2/\text{s}$, and the time scale, T_s, for transverse mixing over a distance s is given by

$$T_s = \frac{s^2}{\varepsilon_t}$$

In Example 4.1 the wastewater was discharged from the side of the channel, so the mixing width, s, was the width of the channel, w.

(a) If the wastewater is discharged from the center of the stream, the mixing width, s, for the wastewater to become well mixed over the channel cross section is given by

$$s = \frac{w}{2} = \frac{10}{2} = 5\,\text{m}$$

and the corresponding time scale, T_s, is given by

$$T_s = \frac{5^2}{0.11} = 227\text{ s} = 3.8\,\text{min}$$

Since the flow velocity, V, is 1.5 m/s, the downstream distance, L, for the wastewater to become well mixed is

$$L = 0.4VT_s = 0.4(1.5)(227) = 136\,\text{m}$$

(b) If the wastewater is discharged from a 5-m-long diffuser centered in the stream, the mixing width, s, for the wastewater to become well mixed over the channel cross section is given by

$$s = \frac{w-5}{2} = \frac{10-5}{2} = 2.5\text{ m}$$

and the corresponding time scale, T_s, is given by

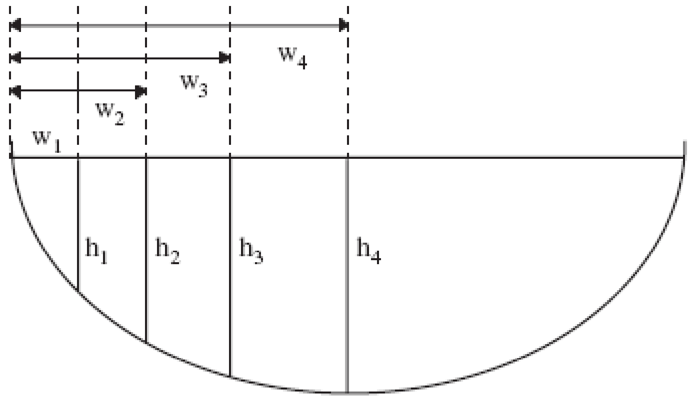
$$T_s = \frac{2.5^2}{0.11} = 56.8\,\text{s} = 0.95\text{ min}$$

Since the flow velocity, V, is 1.5 m/s, the downstream distance, L, for the wastewater to become well mixed is

$$L = 0.4VT_s = 0.4(1.5)(56.8) = 34\,\text{m}$$

DISCHARGE MEASUREMENT

Discharge (flow rate) measurement is very important to provide the basic data required for river or stream water quality. The discharge in a stream cross-section can be measured from a sub-section by the following formula:



$$Q = \text{Sum (mean depth} \times \text{width} \times \text{mean velocity)}$$

$$Q = \sum_{n=1}^{n} \frac{1}{2}(h_n + h_{n-1})(w_n - w_{n-1}) \times \frac{1}{2}(v_n + v_{n-1}) \quad (2.2)$$

If equal width w

$$Q = \sum_{n=1}^{n} \frac{w}{4}(h_n + h_{n-1})(v_n + v_{n-1}) \quad (2.2a)$$

where Q = discharge, cfs
w_n = nth distance from initial point 0, ft
h_n = nth water depth, ft
v_n = nth velocity, ft/s

EXAMPLE: Data obtained from the velocity measurement are listed in the first three columns of Table 2.1. Determine the flow rate at this cross-section.

Solution: Summarized field data and complete computations are shown in Table 2.1.

TIME OF TRAVEL

The river time of travel and stream geometry characteristics can be computed using a volume displacement model. The time of travel is determined at any specific reach as the channel volume of the reach divided by the flow as follows:

$$t = \frac{V}{Q} \times \frac{1}{86{,}400} \quad (2.3)$$

where t = time of travel at a stream reach, days
V = stream reach volume, ft^3 or m^3
Q = average stream flow in the reach, ft^3/sec (cfs) or m^3/s

EXAMPLE: The cross-section areas at river miles 62.5, 63.0, 63.5, 64.0, 64.5, and 64.8 are, respectively, 271, 265, 263, 259, 258, and 260 ft^2 at a surface water elevation. The average flow is 34.8 cfs. Find the time of travel for a reach between river miles 62.5 and 64.8.

TABLE 2.1 Velocity and Discharge Measurements

1 Distance from 0, ft	2 Depth, ft	3 Velocity, ft/s	4 Width, ft	5 Mean depth, ft	6 Mean velocity, ft/s	7 = 4 × 5 × 6 Discharge, cfs
0	0	0				
2	1.1	0.52	2	0.55	0.26	0.3
4	1.9	0.84	2	1.50	0.68	2.0
7	2.7	1.46	3	2.30	1.15	7.9
10	3.6	2.64	3	3.15	2.05	19.4
14	4.5	4.28	4	4.05	3.46	56.1
18	5.5	6.16	4	5.00	5.22	104.4
23	6.6	8.30	5	6.05	7.23	349.9
29	6.9	8.88	6	6.75	8.59	302.3
35	6.5	8.15	6	6.70	7.52	302.3
40	6.2	7.08	5	6.35	6.62	210.2
44	5.5	5.96	4	5.85	6.52	152.2
48	4.3	4.20	4	4.90	5.08	99.6
50	3.2	2.22	2	3.75	3.21	24.1
52	2.2	1.54	2	2.70	1.88	10.2
54	1.2	0.75	2	1.45	1.15	3.3
55	0	0	1	0.35	0.38	0.1
						1559.0*

*The discharge is 1559 cfs.

Solution:

Step 1. Find average area in the reach

$$\text{Average area} = \frac{1}{6}(271 + 265 + 263 + 259 + 258 + 260)\ \text{ft}^2$$
$$= 262.7\ \text{ft}^2$$

Step 2. Find volume

$$\text{Distance of the reach} = (64.8 - 62.5)\ \text{miles}$$
$$= 2.3\ \text{miles} \times 5280\ \frac{\text{ft}}{\text{mile}}$$
$$= 12{,}144\ \text{ft}$$
$$V = 262.7\ \text{ft}^2 \times 12{,}144\ \text{ft}$$
$$= 3{,}190{,}000\ \text{ft}^3$$

Step 3. Find t

$$t = \frac{V}{Q} \times \frac{1}{86{,}400}$$
$$= \frac{3{,}190{,}000\ \text{ft}^3}{34.8\ \text{ft}^3/\text{s} \times 86{,}400\ \text{s/d}}$$
$$= 1.06\ \text{days}$$

Dissolved Oxygen Saturation

DO saturation (D.O. $_{sat}$) values for various water temperatures can be computed using the American Society of Civil Engineers′ formula (ASCE, 1960)

$$DO_{sat} = 14.652 - 0.41022T + 0.0079910T^2 - 0.000077774T^3 \qquad (2.4)$$

where DO_{sat} = dissolved oxygen saturation concentration, mg/L
T = water temperature, °C

This formula represents saturation values for distilled water at sea level pressure. Water impurities can increase the saturation level or decrease the saturation level, depending on the surfactant characteristics of the contaminant. The D.O. $_{sat}$ values calculated from the above formula are listed in Table 2.2 .

EXAMPLE 1: Calculate DO saturation concentration for a water temperature at 0, 10, 20 and 30°C, assuming $\beta = 1.0$.

Solution:
(a) at $T = 0$°C

$$DO_{sat} = 14.652 - 0 + 0 - 0$$
$$= 14.652 \text{ mg/L}$$

TABLE 2.2 Dissolved Oxygen Saturation Values in mg/L

Temp., °C	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	14.65	14.61	14.57	14.53	14.49	14.45	14.41	14.37	14.33	14.29
1	14.25	14.21	14.17	14.13	14.09	14.05	14.02	13.98	13.94	13.90
2	13.86	13.82	13.79	13.75	13.71	13.68	13.64	13.60	13.56	13.53
3	13.49	13.46	13.42	13.38	13.35	13.31	13.28	13.24	13.20	13.17
4	13.13	13.10	13.06	13.03	13.00	12.96	12.93	12.89	12.86	12.82
5	12.79	12.76	12.72	12.69	12.66	12.62	12.59	12.56	12.53	12.49
6	12.46	12.43	12.40	12.36	12.33	12.30	12.27	12.24	12.21	12.18
7	12.14	12.11	12.08	12.05	12.02	11.99	11.96	11.93	11.90	11.87
8	11.84	11.81	11.78	11.75	11.72	11.70	11.67	11.64	11.61	11.58
9	11.55	11.52	11.49	11.47	11.44	11.41	11.38	11.35	11.33	11.30
10	11.27	11.24	11.22	11.19	11.16	11.14	11.11	11.08	11.06	11.03
11	11.00	10.98	10.95	10.93	10.90	10.87	10.85	10.82	10.80	10.77
12	10.75	10.72	10.70	10.67	10.65	10.62	10.60	10.57	10.55	10.52
13	10.50	10.48	10.45	10.43	10.40	10.38	10.36	10.33	10.31	10.28
14	10.26	10.24	10.22	10.19	10.17	10.15	10.12	10.10	10.08	10.06
15	10.03	10.01	9.99	9.97	9.95	9.92	9.90	9.88	9.86	9.84
16	9.82	9.79	9.77	9.75	9.73	9.71	9.69	9.67	9.65	9.63
17	9.61	9.58	9.56	9.54	9.52	9.50	9.48	9.46	9.44	9.42
18	9.40	9.38	9.36	9.34	9.32	9.30	9.29	9.27	9.25	9.23
19	9.21	9.19	9.17	9.15	9.13	9.12	9.10	9.08	9.06	9.04
20	9.02	9.00	8.98	8.97	8.95	8.93	8.91	8.90	8.88	8.86
21	8.84	8.82	8.81	8.79	8.77	8.75	8.74	8.72	8.70	8.68
22	8.67	8.65	8.63	8.62	8.60	8.58	8.56	8.55	8.53	8.52
23	8.50	8.48	8.46	8.45	8.43	8.42	8.40	8.38	8.37	8.35
24	8.33	8.32	8.30	8.29	8.27	8.25	8.24	8.22	8.21	8.19
25	8.18	8.16	8.14	8.13	8.11	8.10	8.08	8.07	8.05	8.04
26	8.02	8.01	7.99	7.98	7.96	7.95	7.93	7.92	7.90	7.89
27	7.87	7.86	7.84	7.83	7.81	7.80	7.78	7.77	7.75	7.74
28	7.72	7.71	7.69	7.68	7.66	7.65	7.64	7.62	7.61	7.59
29	7.58	7.56	7.55	7.54	7.52	7.51	7.49	7.48	7.47	7.45
30	7.44	7.42	7.41	7.40	7.38	7.37	7.35	7.34	7.32	7.31

Source: American Society of Civil Engineering Committee on Sanitary Engineering Research, 1960.

(b) at $T = 10°C$

$$\begin{aligned} DO_{sat} &= 14.652 - 0.41022 \times 10 + 0.0079910 \times 10^2 - 0.000077774 \times 10^3 \\ &= 11.27 \text{ (mg/L)} \end{aligned}$$

(c) at $T = 20°C$

$$\begin{aligned} DO_{sat} &= 14.652 - 0.41022 \times 20 + 0.0079910 \times 20^2 - 0.000077774 \times 20^3 \\ &= 9.02 \text{ (mg/L)} \end{aligned}$$

(d) at $T = 30°C$

$$\begin{aligned} DO_{sat} &= 14.652 - 0.41022 \times 30 + 0.0079910 \times 30^2 - 0.000077774 \times 30^3 \\ &= 7.44 \text{ (mg/L)} \end{aligned}$$

The DO saturation concentrations generated by the formula must be corrected for differences in air pressure caused by air temperature changes and for elevation above the mean sea level (MSL). The correction factor can be calculated as follows:

$$f = \frac{2116.8 - (0.08 - 0.000115A)E}{2116.8} \tag{2.5}$$

where f = correction factor for above MSL
A = air temperature, °C
E = elevation of the site, feet above MSL

EXAMPLE 2: Find the correction factor of DO_{sat} value for water at 620 ft above the MSL and air temperature of 25°C? What is DO_{sat} at a water temperature of 20°C?

Solution:

Step 1. Using Eq. (2.5)

$$\begin{aligned} f &= \frac{2116.8 - (0.08 - 0.000115A)E}{2116.8} \\ &= \frac{2116.8 - (0.08 - 0.000115 \times 25)620}{2116.8} \\ &= \frac{2116.8 - 47.8}{2116.8} \\ &= 0.977 \end{aligned}$$

Step 2. Compute DO_{sat}
From Example 1, at $T = 20°C$

$$DO_{sat} = 9.02 \text{ mg/L}$$

With an elevation correction factor of 0.977

$$DO_{sat} = 9.02 \text{ mg/L} \times 0.977 = 8.81 \text{ mg/L}$$