The effect of oxygen demanding waste on rivers:

The amount of DO in water is one of the most commonly used indicators of river's health. In the extreme case, when anaerobic conditions exist, most higher forms of life are killed or driven off. Noxious conditions, including floating sludges, bubbling, odorous gases, and slimy fungal growths, then prevail.

A number of factors affect the amount of DO. available in a river:

1- Oxygen demanding waste.
2- Respiration of organisms living in the water and sediment's.
3- Respiration of plants in night.
4- Rising temperature in the summer reduce the solubility of oxygen.
5- Lower flows reduce the rate at which oxygen enters the water from the atmosphere.
6- Ice in the winter may form, blocking access to new atmospheric oxygen.

The oxygen sag curve:

The simplest model of the oxygen resources in a river focuses on two key processes: the removal of oxygen by microorganisms during biodegradation, and the replenishment of oxygen through re-aeration at the interface between the river and the atmosphere.

A- **De-oxygenation:** the rate of de-oxygenation at any point in the river is assumed to be proportional to the BOD remaining at the point. That is,

$$\text{Rate of de-oxygenation} = k_d L_t$$

Where:
k_d= the de-oxygenation rate constant (d^{-1})
L_t= the BOD remaining t (days) after the wastes enter the river, (mg/L).

$$\text{Rate of de-oxygenation} = k_d L_o e^{-k_d t}$$

where L_o is the BOD of the mixture of stream water and wastewater at the point of discharge.

B- Re-aeration: the rate at which oxygen is replenished is assumed to be proportional to the difference between the actual DO in the river at any given location and the saturated value of dissolved oxygen. This difference is called the oxygen deficit, D:

$$\text{Rate of re-aeration} = k_r D$$

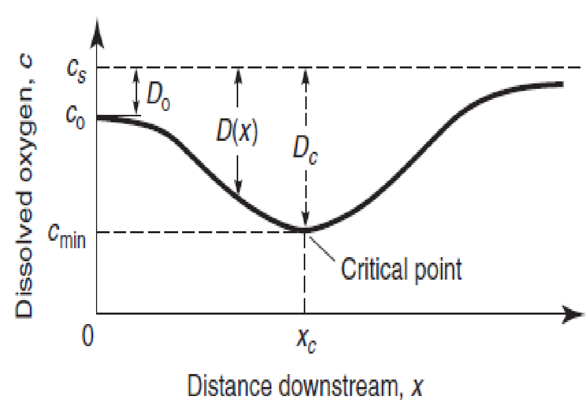
Where:
k_r= re-aeration constant (d^{-1}).
D= dissolved oxygen deficit = (DO_s – DO)
DO_s= saturated value of dissolved oxygen.
DO= actual dissolved oxygen at a given location downstream.

$$k_r = \frac{3.9\, u^{0.5}}{H^{1.5}}$$

Where:
k_r= re-aeration coefficient at 20 °C (d^{-1})
u = average stream velocity (m/s)
H = average stream depth (m)

$$k_{r\text{ at any temperature}} = k_{r\text{ at 20 °C}} \times 1.024^{T-20}$$

$$D_0 = DO_s - \frac{Q_w\, DO_w + Q_R\, DO_r}{Q_w + Q_r}$$



Where :
D_0 = initial oxygen deficit of the mixture of the river and WW
DO_s = saturated value of DO in water at the temperature of the river

The combined the two equations, Rate of deoxygenation= $k_d L_0 e^{-k_d t}$ and Rate of reaeration = $k_r D$, yields the following expression for the rate of increase of the oxygen deficit:

Rate of increase of the oxygen deficit = R_d - R_r

$$\frac{dD}{dt} = k_d L_0 e^{-k_d t} - k_r D$$

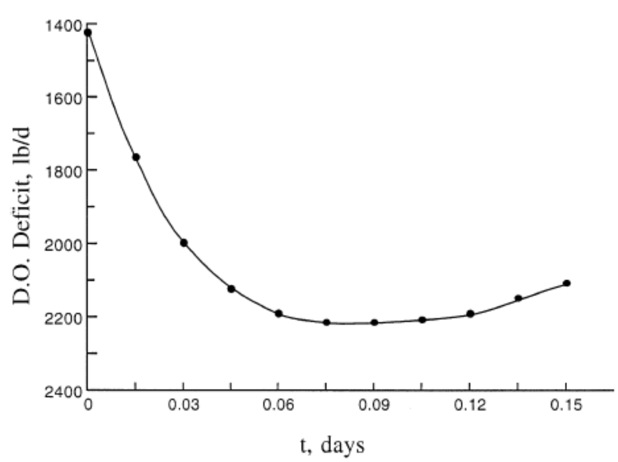
which has the solution:

$$D = \frac{k_d L_0}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_0\, e^{-k_r t}$$

Since the deficit D is the difference between the saturation value of dissolved oxygen DO_s and the actual value DO, we write the equation for the DO as:

$$DO = DO_s - [\frac{k_d L_0}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_0\, e^{-k_r t}]$$

This equation is called the " classic streeter-phelps oxygen sag curve"



$$T_c = \frac{1}{k_r - k_d} \ln[\frac{k_r}{k_d}\left(1 - \frac{D_0(k_r - k_d)}{k_d L_0}\right)]$$

T_c is the time required to reach the critical point.

$$D_C = \frac{k_r}{k_d}\left(L_0 \mathrm{e}^{-\mathrm{k_d} \mathrm{t}_c}\right)$$