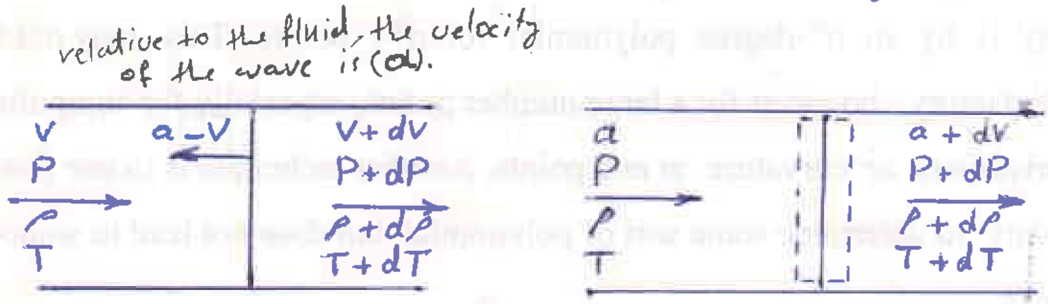
CH. 2. Basic Concepts of Compressible Fluid Flow

2.1. Velocity of Sound

The acoustic waves are small pressure waves that transfers in a plane, cylindrical or spherical shape, these waves are formed due to disturbances in the flow.

Consider a plane wave as shown in figure



(a) Moving wave at a velocity inside a flow

(b) The motion is relative to a C.V. fixed on a wave

Changes in flow properties are happened due to the movement of sound wave. To analyse this problem mathmatically a C.V. is considered fixed on the sound wave.

Applying Continuity Eq.

$$\rho a \cancel{A} = (\rho + d\rho)(a + dV)\cancel{A} \qquad \div \rho a$$

$$\Rightarrow \frac{d\rho}{\rho} + \frac{dV}{a} = 0 \qquad \text{——— (1)} \qquad \text{(Derive)}$$

To apply the momentum equation, it must be noted that the thickness of the wave is very small, therefore the viscosity force, which is proportional to the area of C.V. wall, can be neglected, The momentum eq. becomes,

$$PA - (P + dP)A = \dot{m}\left[(a + dV) - a\right]$$

but $\dot{m} = \rho a A$

$$\therefore \; dP + \rho a \, dV = 0 \qquad \text{——— (2)}$$

This equation is similar to Euler's equation for non-viscous fluid through infinitesmal C.V.

By substituting (dV) from eq. ① into eq. ② we get,

$$a^2 = \left(\frac{dP}{d\rho}\right)$$

The gas that a sound wave passes through has experienced an isentropic process since the process is adiabatic reversible.

$$\therefore \quad a = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s}$$

For isentropic flow $\quad \dfrac{P}{\rho^{\gamma}} = \text{constant} = C$

$$P = C\rho^{\gamma} \Rightarrow \left(\frac{\partial P}{\partial \rho}\right)_s = C\,\gamma\,\rho^{\gamma-1} \Rightarrow \left(\frac{\partial P}{\partial \rho}\right)_s = \frac{P}{\rho^{\gamma}}\cdot \gamma \rho^{\gamma-1}$$

$$\Rightarrow \left(\frac{\partial P}{\partial \rho}\right)_s = \frac{\gamma P}{\rho}$$, but for perfect gas $\dfrac{P}{\rho} = RT$,

$$\therefore \left(\frac{\partial P}{\partial \rho}\right)_s = \gamma RT \Rightarrow a^2 = \gamma RT \Rightarrow a = \sqrt{\gamma RT}$$

For air $\gamma = 1.4$ & $R = 287$ J/kg.k,

$$\therefore \quad \boxed{a = 20.04\sqrt{T}} \quad \text{m/s}, \; T \text{ in K units}$$

2.2. Mach Number

It is defined by the following equation,

$$M = \frac{V}{a}$$

where (V) is the fluid velocity

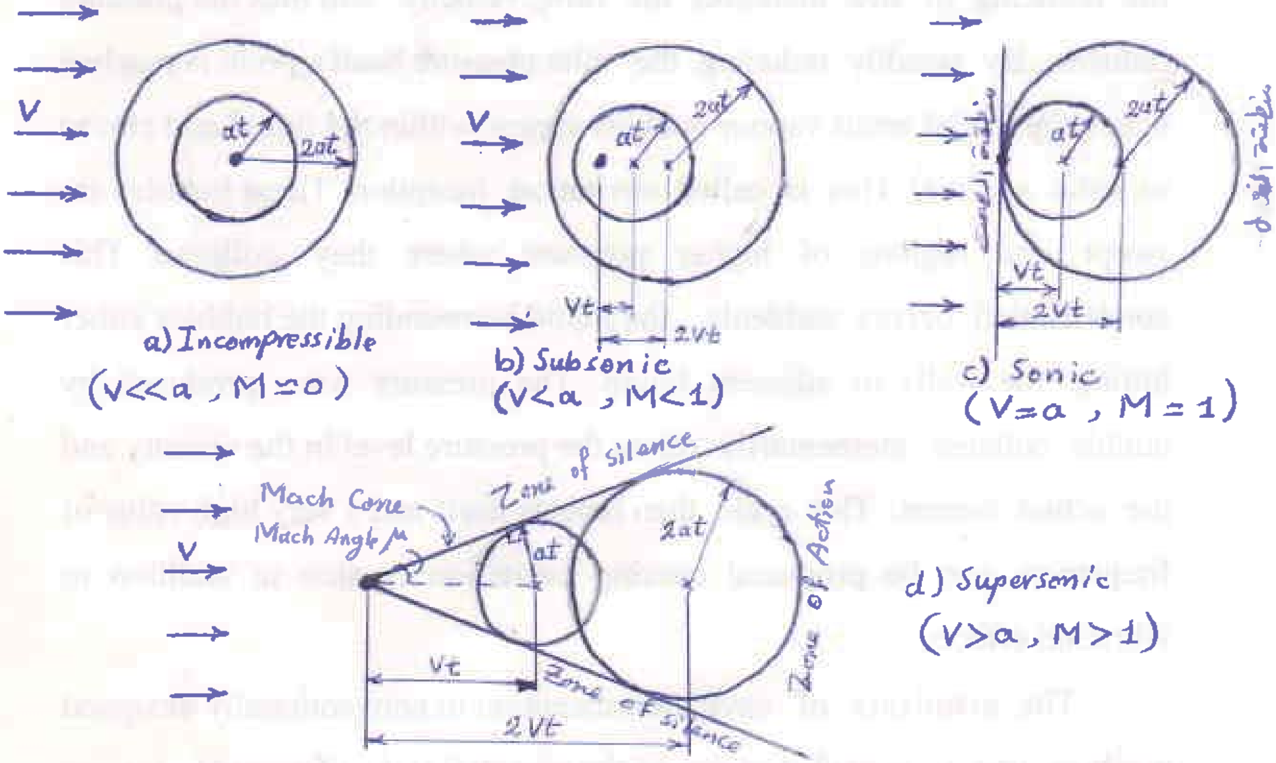
(a) is the sound velocity corresponding to the fluid condition that the fluid velocity (V) is measured.

According to the Mach number the flow is divided into:

Incompressible flow	$M \ll 1 \approx 0$	
Subsonic flow	$M < 1$	
Sonic flow	$M = 1$	Transonic flow
Supersonic flow	$M > 1$	
Hypersonic flow	$M \gg 1$	

* M له فائدتان: ① تقسيم الجريان إلى أنواع ② في التحليل البعدي للجريانات الانضغاطية

2.3. Physical Differences Between The Flow Types



$$\sin \mu = \frac{at}{Vt} = \frac{1}{M} \Longrightarrow \boxed{\mu = \sin^{-1}\frac{1}{M}}$$

2.4. The Adiabat. Steady Flow Ellipse

The adiabatic steady flow energy Eqn

$$h_o = h + \frac{V^2}{2} = \text{const.}$$

$$2C_p(T_o - T) = V^2$$

$$V = \sqrt{2C_p(T_o - T)}$$

or

$$V = \sqrt{\frac{2\gamma R\,(T_o - T)}{\gamma - 1}}$$

when a theoretically $= 0$, $T = 0$

hence

$$V_{max} = \sqrt{\frac{2\gamma R T_o}{\gamma - 1}} = \sqrt{\frac{2}{\gamma - 1}\,a_o^2} \quad \text{——} (1)$$

where $a_o^2 = \gamma R T_o$

Consider a stream tube in which the flow dosenot exchange heat with the fluid in neighboring stream tubes. The steady energy eqn is

$$h + \frac{V^2}{2} = \text{const.} \quad \text{——} (2)$$

For perfect gas

$$h = C_p \cdot T = \frac{\gamma R}{\gamma - 1} T \quad ; \quad RT = \frac{P}{\rho}$$

but

$$a^2 = \gamma R T$$

Therefore,

$$h = \frac{a^2}{\gamma - 1} = \frac{\gamma P}{(\gamma - 1)\rho} \quad \text{——} (3)$$

From eqn's (2) & (3)

$$\frac{a^2}{\gamma-1} + \frac{V^2}{2} = \text{const.} \qquad \text{——— (4)}$$

For stagnation conditions $(V=0)$, eqn (4) will be

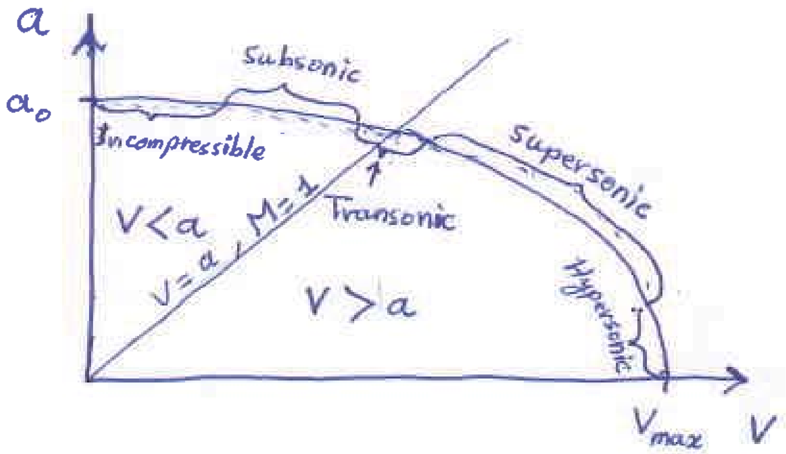
$$\frac{a_o^2}{\gamma-1} = \text{const.} \qquad \text{——— (5)}$$

where a_o is the speed of sound at stagnation conditions. Substitute eqn (1) & (5) in eqn (4) to get,

$$\boxed{\frac{a^2}{a_o^2} + \frac{V^2}{V_{max}^2} = 1} \qquad \text{——— (6)}$$

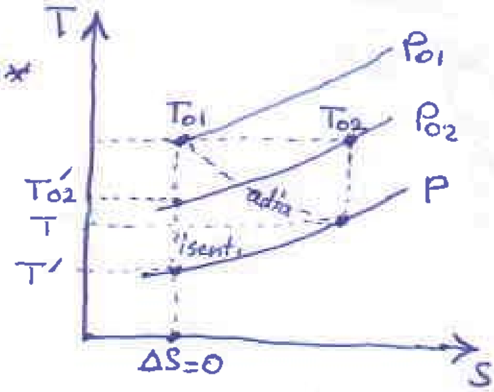
Eqn (6) is called adiabatic Steady flow ellipse which is a useful device to examine the relationship between the flow velocity and speed of sound. The ellipse plotted for $\gamma = 1.4$ is shown below

Incompressible flow $(M < 0.3)$
Subsonic flow $(0.3 < M < 1)$
Transonic flow $(0.8 < M < 1.2)$
Supersonic flow $(1.2 < M < 5)$
Hypersonic flow $(M > 5)$



2.5. Stagnation State

* هي الحالة التي تنتج اذا أوقفنا الجريان تماماً في نفس النقطة بصورة ايسنتروبية.



From 1st law of thermodynamics

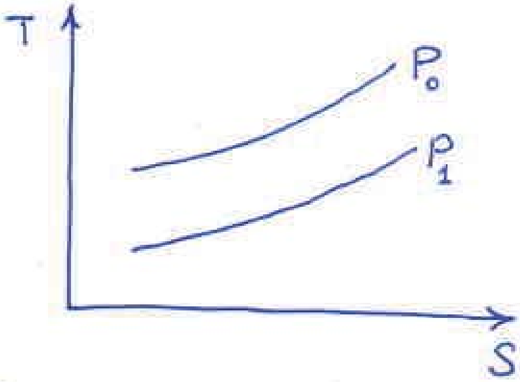
$$\Delta E = \cancel{\Delta W}^{\,0} + \cancel{\Delta Q}^{\,0}$$

$$\Delta E = 0 \Longrightarrow E = \text{const.}$$

$$h_0 + \cancel{\frac{V_0^2}{2}}^{\,0} = h + \frac{V^2}{2} = \text{const.}$$

Adiabatic E.E.

For ideal gas



$$C_p T_0 = C_p T + \frac{V^2}{2}$$

For reversible

$$h_{01} = h_{02} \Longrightarrow C_p T_{01} = C_p T_{02} \Longrightarrow T_{01} = T_{02}$$

Increase of entropy

$$\Delta S = C_p \ln \frac{T_{02}}{T_{01}} - R \ln \frac{P_{02}}{P_{01}}$$

$$\Delta S = -R \ln \frac{P_{02}}{P_{01}}$$

For isentropic flow $\Delta S = 0 \Rightarrow \therefore P_{01} = P_{02}$

$0 \longrightarrow 1$

$$C_p T_0 + \frac{V_0^2}{2} = C_p T_1 + \frac{V_1^2}{2}$$

$$C_p T_0 = C_p T_1 + \frac{V_1^2}{2}$$

$$\boxed{T_0 = T_1 \left(1 + \frac{V_1^2}{2 C_p T_1}\right)}$$

$$T_0 = T_1 \left(1 + \frac{V_1^2 (\gamma - 1)}{2 \gamma R T_1}\right)$$

$$\boxed{T_0 = T_1 \left[1 + \frac{\gamma - 1}{2} M_1^2\right]}$$

Very IMPORTANT

$$\frac{T_o}{T_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)$$

$$\left(\frac{P_o}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_o}{T_1} \Longrightarrow \frac{P_o}{P_1} = \left(\frac{T_o}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\therefore \quad \frac{P_o}{P_1} = \left[1 + \frac{\gamma - 1}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

Derive all relations $\frac{P}{P_o}$, $\frac{\rho}{\rho_o}$, $\frac{T}{T_o}$

2.6. Critical State

Sonic flow ($M=1$) is called also critical state. Critical state is considered an important reference state. The superscript (*) refers to that the property is at critical state.

It is clear that

$$V^* = a^* = \sqrt{\gamma R T^*}$$

because $M = M^* = 1$

Also, it is easy to derive $\frac{P^*}{P_o}$, $\frac{\rho^*}{\rho_o}$ & $\frac{T^*}{T_o}$ by substituting $M = 1$ in $\frac{P}{P_o}$, $\frac{\rho}{\rho_o}$ & $\frac{T}{T_o}$.

Note: P^*, ρ^* & T^* can be found when stagnation properties are known. (How?)